Maths

A Level | Edexcel | 9MA0

2017 specification
first exams in 2018

Practice Exams

for A Level Edexcel Paper 1: Pure Mathematics

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POD 9438

Contents

Teacher's Introduction

This pack contains five examination-style question papers and solutions (Sets A–E) for the Pure part of the Edexcel A Level Maths 2017 specification.

Written for the 2017 A Level specification, these papers are designed to be used by your students in preparation for their examinations. In line with the real examinations, every paper examines a large part of the specification, including taking into account the following particular changes:

- there is now a greater emphasis on **harder problem-solving and modelling** questions
- **three new assessment objectives** replace the old five
- the use of a **calculator** is now required and expected in every paper

Remember!
Always check the exam board website for new information, including changes to the specification and sample assessment material.

Non-write-on question papers
The papers have been provided individually in a condensed **A4 non-write-on format** to keep photocopying costs to a minimum.

Mark schemes
The mark schemes have been provided as A4 masters. They include fully worked solutions and common alternative solutions, making them helpful and easy for students to follow. Where the final answer is numerical, this has been provided in a separate answer column to speed up the marking process. Questions that ask the student to establish a given result have been labelled as either a 'Show that' question or as a 'Prove that' question in this column, indicating that it is the student's working that is of interest here, not their final answer.
Each solution starts with any relevant information from the question or any relevant formulae – this is useful to see how the solution starts and is a good habit for students to follow.

Formula book
The Edexcel formula book is not provided with this resource, but a copy of it should be provided to students as they will need to become familiar with its contents.

Suggested use of this resource
With five papers at your disposal, there is a variety of flexible ways to use this resource:

- Set papers A and B as open-book tests, so that students can get used to examination-style questions, then take the plunge and use papers C–E in exam-style conditions.
- Space out the five papers over the whole revision period so students can build their confidence and see their hard work paying off in the form of ever-improving marks.

Also available from ZigZag Education
This pack contains five 'Paper 1' exams – to prepare fully, a pack of five 'Paper 2' exams* and a pack of five 'Paper 3' exams are also available. Each of sets A–E can then be used to mimic an entire A Level Maths examination consisting of the three papers – the sets have the correct overall weighting of assessment objectives.

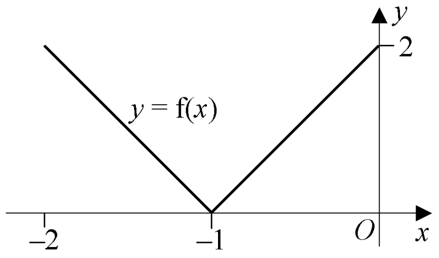
* If you have both Paper 1 and Paper 2, then the A papers go together to cover different areas of the specification (likewise for the B papers, the C papers, and so on).

A Level Edexcel Exam Set B Practice Paper 1	Ti
Do **not** write on this paper. Give your answers to **3 significant figure**	

1. Calculate the following integrals:
   (a) $\int \frac{x^4}{5}\,dx$
   (b) $\int x\left(x^{-1/2} - x^{1/2}\right)dx$

2. (a) Simplify fully the expression $\frac{\cot\theta}{\tan\theta + \cot\theta}$ for $\theta \neq \frac{k\pi}{2}$, $k \in \mathbb{Z}$
   (b) Hence or otherwise simplify fully the expression $\frac{\cot^2\theta}{\tan^2\theta + \mathrm{co}}$

3. The diagram below shows the graph of $y = \mathrm{f}(x)$



   (a) Sketch the graph of $y = \mathrm{f}(x - 3)$
   (b) Describe the transformation that transforms the graph of $y =$ f
   $y = \mathrm{f}(-x)$
   The point (3, 2) lies on the graph of $y = \mathrm{g}(x)$
   (c) State the coordinates of the image of this point when $y = \mathrm{g}(x)$
   following graphs:
   (i) $y = 6\mathrm{g}(x)$
   (ii) $y = \mathrm{g}(6x)$

4. The points A, B and C have position vectors $2\mathbf{i} - 7\mathbf{j} + \mathbf{k}$, $4\mathbf{i} + 5\mathbf{j} - 7$
   The point M is the midpoint of the line segment AB.
   (a) Show that the magnitude of $\overrightarrow{OM}$ is $\sqrt{19}$
   The point D is such that $\overrightarrow{AB} = \overrightarrow{CD}$
   (b) Show that the position vector of D is [illegible] $+ 13\mathbf{j} - 3\mathbf{k}$

5. Charles investigates the population of finches on an island. He mod
   on the island by the formula $F = 80\mathrm{e}^{0.2t}$, where t is the time in y
   (a) Using Charles's model:
   (i) state the population of finches on the island on 1st Janu
   (ii) predict the population of finches on the island on 1st Ja
   (b) According to Charles's model, at what value of t will the pop
   to 2000? Give your answer to 1 decimal place.
   (c) Give one reason why Charles's model is not appropriate for l
   On the same island, Charles determines that the population of ants
   formula $A = 2400\mathrm{e}^{0.1t}$, where t is the time in years since 1st January
   (d) Determine the year during which the population of finches w
   of ants according to Charles's models.

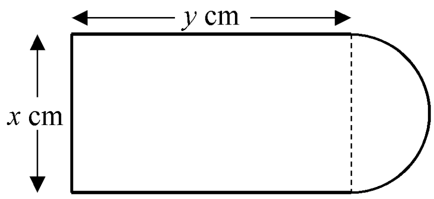
6. A triangle includes an obtuse angle θ. The two shorter sides of the
and 7.5 cm; the area of the triangle is 10 cm^2.
Find the exact value of $\tan\theta$.

7. Prove or disprove each of the following statements:
   (a) If n is an integer, then $3n^2 - 11n + 13$ is a prime number.
   (b) If x is a real number, then $x^2 - 8x + 17$ is positive.
   (c) If p and q are irrational numbers, then pq is irrational.

8. Prove from first principles that the derivative of $\sin x$ with respect
measured in radians.
You may assume that $\lim\limits_{h\to 0}\frac{\sin h}{h} = 1$ and $\lim\limits_{h\to 0}\frac{\cos h - 1}{h} = 0$ and use w
formulae for sine.

9. (a) Given that $\frac{5x^2+6x-2}{(x+1)(x-2)} = A + \frac{B}{x+1} + \frac{C}{x-2}$, find the integers
   (b) Hence or otherwise expand $\frac{5x^2+6x-2}{(x+1)(x-2)}$ in ascending powe
   the term in x^2

10. The shape below is a rectangle attached to a semicircle. It has area



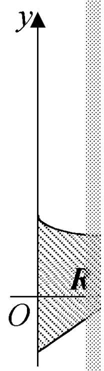
   (a) If $P = 100$ cm, then show that $A = 50x - \frac{x^2}{2} - \frac{\pi x^2}{8}$ cm^2
   (b) Find the value of x for which the area is maximal, giving you
   You must clearly show that your value is a maximum.

11. Consider the function $f(x) = x^6 - x^2 - 1$. The equation $f(x) = 0$ has
$(n, n + 1)$ where n is a positive integer.
   (a) Find n, showing detailed reasoning for your answer.
   (b) Starting with $x_0 = n$, use an iterative formula based on the equ
   to 3 decimal places.
   (c) Show that $\alpha = 1.151$ is correct to 3 decimal places.

12. The circle C has equation $x^2 - 6x + y^2 + 16y = 8$
   (a) Find the centre and radius of C
   (b) PQ is a chord of C whose midpoint is the origin. Show that
   where k is an integer to be found.

13. Jenny is training for a marathon. In the first week of the year she r
week she plans to run for 5 minutes longer than in the previous we
of the year she will run for 15 minutes, in the third week for 20 mi
(a) For how long will Jenny run during the 52nd week of the yea
(b) How long will Jenny have spent running in total over the 52
to the nearest hour.
Jenny's brother is training for a half marathon. He starts the year b
and he plans to increase the time he spends running each week by 5
(c) How long will Jenny's brother have spent running in total aft
Give your answer to the nearest hour.

14. The diagram to the right shows the curve C with equation
$y = x^2 - 4\sqrt{x} + 11,\ x \geq 0$
The line L is the tangent to C at the point $P(4, 19)$
The shaded region R is bounded by C, L and the y-axis
Show that the area of R is 24 **(10)**



Preview of Questions Ends Here

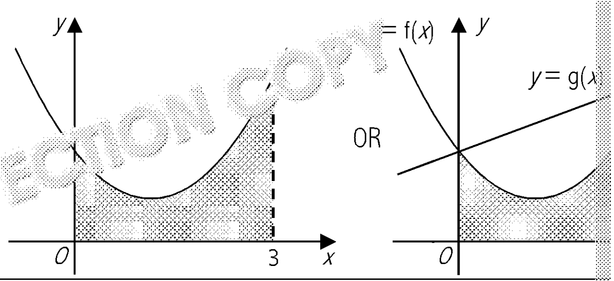
This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Qu №	Answer	Solutions	
1	, $y=-1$ $x=4$, $y=2$	$x^2+y^2=4x+2y,\ x-y=2$	
		Either: $x=y+2$ $\Rightarrow\ (y+2)^2+y^2=4(y+2)+2y$ $\therefore 2y^2-2y-4=0$ $\therefore y^2-y-2=0$ $\therefore (y+1)(y-2)=0$ $\therefore y=-1$ or $y=2$ $y=-1\ \Rightarrow\ x=1$ $y=2\ \Rightarrow\ x=4$	**Or:** $y=x-2$ $\Rightarrow\ x^2+(x-2)^2=$ $\therefore 2x^2-10x+8=0$ $\therefore x^2-5x+4=0$ $\therefore (x-1)(x-4)=0$ $\therefore x=1$ or $x=4$ $x=1\ \Rightarrow\ y=-1$ $x=4\ \Rightarrow\ y=2$
2	$k=3$	$f(x)=kx^3-8x^2+kx+2$ $(x-2)$ is a factor of $f(x)$, so by the factor theorem $f(2)=0$ $\therefore k\times2^3-8\times2^2+k\times2+2=0$ $\therefore 8k-32+2k+2=0$ $\therefore 10k=30$ $\therefore k=3$	
3ai	$15x^4-45x^2$	$y=3x^5-15x^3-24$ $\frac{d(x^n)}{dx}=nx^{n-1}$ $\therefore \frac{dy}{dx}=15x^4-45x^2$	
aii	$\frac{d^2y}{dx^2}=60x^3-90x$		
b	**'Show that' question – Answer given in question**	$x=\sqrt{3}\ \Rightarrow\ \frac{dy}{dx}=15\times(\sqrt{3})^4-45\times(\sqrt{3})^2=15\times9-45$ $=0$, so this is a stationary point	
c	minimum	$x=\sqrt{3}\ \Rightarrow\ \frac{d^2y}{dx^2}=60\times(\sqrt{3})^3-90\times\sqrt{3}$ $=155.884\ldots>0$, so the stationary point is a local minimum	

Qu №	Answer	Solutions
4a	$x - 2y + 18 = 0$ $(p = -2,\ q = 18)$	l_1 passes through $(-4, 7)$ and $(-18, 0)$ $\therefore$ Gradient of $l_1 = \dfrac{7-0}{-4-(-18)} = \dfrac{1}{2}$ Use $y - y_1 = m(x - x_1)$ with $m = \dfrac{1}{2},\ x_1 = -18,\ y_1 = 0$ $\therefore y - 0 = \dfrac{1}{2}(x+18)$ $\therefore x - 2y + 18 = 0$
b	$y = -2x - 1$	l_2 is perpendicular to l_1, so gradient of $l_2 = \dfrac{-1}{\frac{1}{2}} = -2$ Use $y - y_1 = m(x - x_1)$ with $m = -2,\ x_1 = -4,\ y_1 = 7$ $\therefore y - 7 = -2(x+4)$ $\therefore y = -2x - 1$
c	$(0, -1)$	y-intercept of $l_2 = -1$, so $C = (0, -1)$
d	70	**Either:** By Pythagoras' theorem, $\lvert AB \rvert = \sqrt{14^2 + 7^2} = 7\sqrt{5}$ $\lvert AC \rvert = \sqrt{4^2 + 8^2} = 4\sqrt{5}$ $\therefore$ area $= \dfrac{1}{2} \times 7\sqrt{5} \times 4\sqrt{5} = 70$ **Or:** Let $D(0, 9)$ be the y-in Then area $\Delta BCD = \frac{1}{2}$ Area $\Delta ACD = \dfrac{1}{2} \times 10 \times$ $\therefore$ area $= 90 - 20 = 7$
5a	$5\mathbf{i} - 15\mathbf{j}$	$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (9\mathbf{i} - 7\mathbf{j}) - (4\mathbf{i} + 8\mathbf{j})$ $= 5\mathbf{i} - 15\mathbf{j}$
b	$5\sqrt{10}$	$\lvert x\mathbf{i} + y\mathbf{j} \rvert = \sqrt{x^2 + y^2}$ $\lvert \overrightarrow{PQ} \rvert = \sqrt{5^2 + (-15)^2}$ $= 5\sqrt{10}$
6	'Prove that' question – Answer given in question	$LHS \equiv \tan^2\theta - \sin^2\theta \equiv \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$ $\equiv \sin^2\theta\left(\dfrac{1}{\cos^2\theta} - 1\right) \equiv \sin^2\theta\left(\sec^2\theta - 1\right)$ $\equiv \sin^2\theta\tan^2\theta \equiv RHS$

Qu №	Answer	Solutions
7a	'Show that' question – Answer given in question	$u_2 = u_1 r$, so $u_1 = \frac{u_2}{r} = \frac{120}{r}$ For geometric series with common ratio r, $\sum_{n=1}^{\infty} u_n = \frac{u_1}{1-r}$ $\therefore \frac{120/r}{1-r} = 490$ $\therefore \frac{120}{r} = 490 - 490r$ $\therefore 120 = 490r - 490r^2$ $49r^2 - 49r + 12 = 0$
b	$\frac{3}{7}, \frac{4}{7}$	$49r^2 - 49r + 12 = 0$ By the quadratic formula, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 49$, $c = 12$ $\therefore r = \frac{49 \pm \sqrt{(-49)^2 - 4 \times 49 \times 12}}{2 \times 49}$ $= \frac{49 \pm 7}{98} = \frac{3}{7}$ or $\frac{4}{7}$
c	210	$r > \frac{1}{2}$ so $r = \frac{4}{7}$ $u_1 = \frac{120}{r} = \frac{120}{4/7} = 210$
d	17	For geometric series, $\sum_{n=1}^{N} u_n = \frac{a(1-r^N)}{1-r}$. Here $a = 210$, r $\therefore \sum_{n=1}^{N} u_n = \frac{210\left(1-\left(4/7\right)^N\right)}{1-4/7} = 490\left(1-\left(\frac{4}{7}\right)^N\right)$ So we need $490\left(1-\left(\frac{4}{7}\right)^N\right) > 489.95$ $\left(\frac{4}{7}\right)^N < 1 - \frac{489.95}{490} = \frac{0.05}{490}$ $N \ln\left(\frac{4}{7}\right) < \ln\left(\frac{0.05}{490}\right)$ $N > \frac{\ln\left(0.05/490\right)}{\ln\left(4/7\right)} = 16.4222\ldots$ So smallest integer N is 17

Qu №	Answer	Solutions
8a	$(x+4)^2+(y-7)^2=64$ $(a=-4,\ b=7,\ c=64)$	$x=8\cos t-4 \ \Rightarrow\ \cos t=\dfrac{x+4}{8}$ $y=8\sin t+7 \ \Rightarrow\ \sin t=\dfrac{y-7}{8}$ $\cos^2 t+\sin^2 t=1 \ \Rightarrow\ \left(\dfrac{x+4}{8}\right)^2+\left(\dfrac{y-7}{8}\right)^2=1$ $\therefore (x+4)^2+(y-7)^2=64$
b	$\dfrac{14\pi}{3}$ $(m=14,\ n=3)$	C is part of a circle of radius $\sqrt{64}=8$ Circumference of full circle is $2\pi r=2\pi\times 8$ C subtends an angle of $\dfrac{\pi}{3}-\left(-\dfrac{\pi}{4}\right)=\dfrac{7\pi}{12}$ So length $=\dfrac{7\pi/12}{2\pi}\times 2\pi\times 8$ $=\dfrac{14\pi}{3}$
9a	$4x^2-28kx+36k^2$ $(a=4,\ b=-28k,\ c=36k^2)$	$\text{f}(x)=x^2-8kx+3k^2$, $\text{g}(x)=2x-3k$ $\therefore \text{fg}(x)=(2x-3k)^2-8k(2x-3k)+3k^2$ $=4x^2-12kx+9k^2-16kx+24k^2+3k^2$ $=4x^2-28kx+36k^2$
b	$\text{fg}(x)\geq -13k^2$	$\text{fg}(x)=4x^2-28kx+36k^2$ $=4\left(x^2-7kx+9k^2\right)=4\left(\left(x-\dfrac{7k}{2}\right)^2-\left(\dfrac{7k}{2}\right)^2+9k^2\right)$ $=4\left(x-\dfrac{7k}{2}\right)^2-13k^2$ $\therefore \text{fg}(x)\geq -13k^2$
c	3	$\text{fg}(3)=4\times 3^2-28k\times 3+36k^2=36-84k+36k^2=108$ $\therefore 36k^2-84k-72=0$ $\therefore 3k^2-7k-6=0$ $\therefore (3k+2)(k-3)=0$ $\therefore k=-\dfrac{2}{3}$ or $k=3$, but k is an integer, so $k=3$
d	For any $y>-13k^2(=-117)$ there are two values of x such that $\text{fg}(x)=y$ For example, if $\text{fg}(x)=108$, then $x=3$ or $x=18$ So fg does not have an inverse	

Qu №	Answer	Solutions
10a	Integral ... $\frac{33}{2}$ S... for sketch	$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c$ so $\int_0^3 \left(x^2 - x + 4\right) dx = \left[\frac{x^3}{3} - \right.$ $= \left(\frac{3^3}{3} - \frac{3^2}{2} + 4 \times 3\right) - 0$ $= \frac{33}{2}$ 
b	$3x - x^2$	$h(x) = g(x) - f(x) = (2x + 4) - (x^2 - x + 4)$ $= 3x - x^2$
11a	36.87°, 131.81°, 228.19°, 323.13°	$6\cos^2 x - 2\cos x + 1 = 9\sin^2 x$ $\sin^2 x = 1 - \cos^2 x$ $\therefore 6\cos^2 x - 2\cos x + 1 = 9(1 - \cos^2 x)$ $\therefore 15\cos^2 x - 2\cos x - 8 = 0$ $\therefore (5\cos x - 4)(3\cos x + 2) = 0$ $\therefore \cos x = \frac{4}{5}$ or $\cos x = -\frac{2}{3}$ $\therefore x = \arccos\left(\frac{4}{5}\right) = 36.87°$ or $x = 323.13°$ or $x = \arccos\left(-\frac{2}{3}\right) = 131.81°$ or $x = 228.19°$ (2 d.p.)
b	3.43°	$6\cos^2(2\theta + 30) - 2\cos(2\theta + 30) + 1 = 9\sin^2(2\theta + 30)$ $\therefore 2\theta + 30 = 36.8698\ldots$ $\therefore \theta = \frac{36.8698\ldots - 30}{2} = 3.43°$ (p.)
12a	'Show that' q... Ans... in question	$f(x) = e^x + $ ... ... $= -$ ...$1\ldots < 0$, $f(2.5) = 8.80\ldots > 0$ Change of sign, and f is continuous in the interval [2, 2.5], s this interval
b	2.19	Use Newton–Raphson formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ with $x_0 =$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{-3.6109}{19.39}$ $= 2.19$ (3 s.f.)

Qu №	Answer	Solutions
c	'Show that' question – Answer given in question	$f(x)=e^x+x^3-19$ $\frac{d(x^n)}{dx}=nx^{n-1},\ \frac{d}{dx}(e^{kx})=ke^{kx}$ $\therefore\ f'(x)=e^x+3x^2>0$ for all x So f(x) is an increasing function, i.e. it has no turning points a… most one root
13a		$\frac{d}{dx}(a^x)=a^x\ln a$, so $u=3^x \Rightarrow \frac{du}{dx}=3^x\ln 3$
b	$\frac{4\sqrt{2}}{3\ln 3}$	$\int_0^1 3^x\sqrt{3^x-1}\,dx$ **Either:** Let $u=3^x$, then $\frac{1}{\ln 3}du=3^x\,dx$ $x=0 \Rightarrow u=1$ $x=1 \Rightarrow u=3$ $\int_0^1 3^x\sqrt{3^x-1}\,dx=\int_1^3\frac{1}{\ln 3}\sqrt{u-1}\,du$ $=\left[\frac{1}{\ln 3}\times\frac{2}{3}(u-1)^{3/2}\right]_1^3$ $=\frac{1}{\ln 3}\times\frac{2}{3}\times 2^{3/2}-0$ $=\frac{4\sqrt{2}}{3\ln 3}$ **Or:** Note that $3^x\sqrt{3^x-1}=\frac{1}{\ln 3}(3^x \ldots$ $\int f'(x)[f(x)]^n\,dx= \ldots$ By part (a), if $f(x)= \ldots$ $f'(x)=3^x\ln 3$ $\therefore\ \int_0^1 3^x\sqrt{3^x-1}\,dx$ $=\frac{1}{\ln 3}\int_0^1(3^x\ln 3)(3^x \ldots$ $=\frac{1}{\ln 3}\left[\frac{(3^x-1)^{3/2}}{3/2}\right]_0^1$ $=\frac{1}{\ln 3}\left(\frac{(3-1)^{3/2}}{3/2}- \ldots\right.$ $=\frac{2}{3\ln 3}\times 2^{3/2}=\frac{4\sqrt{2}}{3\ln 3}$
14a	$\ln\left(\frac{2}{3}\right)$	$F=15(1-e^{kt}),\ t=1,\ F=5$ $\therefore\ 5=15(1-e^k)$, and so $1-e^k=\frac{1}{3}$ $\therefore\ e^k=\frac{2}{3}$ $\therefore\ k=\ln\left(\frac{2}{3}\right)$
b	1 370 000	$t=6,\ k=\ln\left(\frac{2}{3}\right) \Rightarrow F=15\left(1-e^{6\times\ln(2/3)}\right)=13.6831\ldots$ So 1 370 000 followers (3 s.f.) or 1 400 000 (2 s.f.)

Qu №	Answer	Solutions
c	'Show that' question – Answer given in question	$e^{kt} > 0$ for all t, so $1 - e^{kt} < 1$ for all t So $F = 15\left(1 - e^{kt}\right) < 15$ for all t Hence the number of followers is always less than 1 500 000
d	$A = 6.08$	$\dfrac{d}{dx}\left(e^{ax}\right) = a\,e^{ax}$ $F = 15\left(1 - e^{kt}\right) \Rightarrow \dfrac{dF}{dt} = -15k\,e^{kt}$ $= -15 \times \ln\left(\dfrac{2}{3}\right)e^{kt} = 6.08e^{kt}$
15	$\dfrac{1}{7}\ln\left(\dfrac{10}{3}\right)$	Let $\dfrac{1}{(x+3)(2x-1)} = \dfrac{A}{x+3} + \dfrac{B}{2x-1}$ $\therefore 1 = A(2x-1) + B(x+3)$ Let $x = \dfrac{1}{2}$, so $1 = \dfrac{7}{2}B$, hence $B = \dfrac{2}{7}$ Let $x = -3$, so $1 = -7A$, hence $A = -\dfrac{1}{7}$ $\therefore A = -\dfrac{1}{7},\ B = \dfrac{2}{7}$ $\displaystyle\int_1^3 \frac{1}{(x+3)(2x-1)}\,dx = \int_1^3 \left(\frac{-1/7}{x+3} + \frac{2/7}{2x-1}\right)dx$ $\displaystyle= \frac{1}{7}\int_1^3 \left(\frac{-1}{x+3} + \frac{2}{2x-1}\right)dx$ $\displaystyle\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln\lvert ax+b\rvert + c$ $\displaystyle\therefore \frac{1}{7}\int_1^3 \left(\frac{-1}{x+3} + \frac{2}{2x-1}\right)dx = \frac{1}{7}\Big[-\ln\lvert x+3\rvert + \ln\lvert 2x-1\rvert\Big]_1^3$ $= \dfrac{1}{7}\Big[(-\ln 6 + \ln 5) - (-\ln 4 + \ln 1)\Big]$ $\ln a + \ln b = \ln ab$ and $\ln a - \ln b = \ln\dfrac{a}{b}$ $\dfrac{1}{7}\Big[(-\ln 6 + \ln 5) - (-\ln 4 + \ln 1)\Big] = \dfrac{1}{7}\ln\left(\dfrac{5 \times 4}{6}\right) = \dfrac{1}{7}\ln\left(\dfrac{10}{3}\right)$

Preview of Answers Ends Here