AQA Practice GCSE Examination Paper Higher Set 5 Paper 3 Calculator

Solutions

Q <u>u</u> Nº	Answer	Solutions	Marks		AO	Spec.
1	360°	The sum of the interior angles of any triangle is 180° . All quadrilaterals can be split into two triangles so the sum of the interior angles in any quadrilateral = sum of the angles in both triangles = $2 \times 180 = 360^\circ$	A1	1	2.1a	G3
2	$3^{n+2} + 3^{n+3}$	$9(3^{n} + 3^{n+1}) = 3^{2}(3^{n} + 3^{n+1}) = (3^{2} \times 3^{n}) + (3^{2} \times 3^{n+1})$ $= 3^{n+2} + 3^{n+3}$	A1	1	1.3b	A4
3	$y = (x-3)^2$	A translation of $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ is a horizontal translation of 3 units to the right. $y = f(x)$ becomes $y = f(x-3)$ so $y = x^2$ becomes $y = (x-3)^2$	A1	1	2.3b 2.4a	G24
4	9,000	81 million = 81,000,000 $\sqrt{81 \text{ million}} = \sqrt{81,000,000} = \sqrt{81 \times 10^6} = \sqrt{81} \times \sqrt{10^6}$ = $\sqrt{81} \times \sqrt{10^3 \times 10^3} = 9 \times 10^3 = 9,000$	A1	1	1.3a	N7
5	2.2 cm ³	Volume increases by 65%. Expanded volume is 3.63 cm^3 which is $100 + 65 = 165\%$ of original volume Original volume is $3.63 \div 1.65$ = 2.2 cm^3	M1 M1 A1	3	1.3a	R9
6	Yes (Claire is correct) [with suitable	e.g. A geometric progression is a sequence where the next term is found by multiplying the previous term by a constant value.	A1	1	2.1a	A24
7		C is the midpoint of DE Coordinates of D are $(0, 0)$; coordinates of E are $(4, 6)$ The midpoint of DE is at $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 4}{2}, \frac{0 + 6}{2}\right) = \left(\frac{4}{2}, \frac{6}{2}\right) = (2, 3)$ Coordinates of C are $(2, 3)$ C is also the midpoint of AB. Co-ordinates of B are $(8, 4)$; let co-ordinates of A be (x, y) The midpoint of AB is at $\left(\frac{x + 8}{2}, \frac{y + 4}{2}\right) = (2, 3)$ $\therefore \frac{x + 8}{2} = 2, \frac{y + 4}{2} = 3$ $x + 8 = 4, y + 4 = 6 \rightarrow x = -4, y = 2$ Coordinates of A are $(-4, 2)$	M1 finding coordinates of C M1 method to find coordinates of A	3	2.1a 3.1b 1.3a	A8
8		Factorise $x^2 + x - 2 = 0$ by finding 2 numbers which add to make 1 and multiply to make -2 : $(-1 + 2 = 1)$; $(-1 \times 2 = -2)$ $x^2 + x - 2 = (x - 1)(x + 2) = 0$ $(x - 1) = 0$ or $(x + 2) = 0$ $x = 0 + 1 = 1$ or $x = 0 - 2 = -2$	M1 $(x \pm 1)(x \pm 2)$ M1 $(x - 1)(x + 2)$	3	1.3b	A18

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9		The original price has been reduced by $\frac{3}{8}$				
		$1 - \frac{3}{8} = \frac{5}{8}$; £12.50 is $\frac{5}{8}$ of the original price of the DVD	M1		1.3b	R3
		$\frac{1}{8}$ of the original price is $12.50 \div 5 = £2.50$			1.55	113
		The original price of the DVD is 2.50 × 8	M1			
	£20	= £20	A1	3		
10a	86–90 cm	The modal class interval is the most common waist size. This is 86—90 cm as it has the highest frequency.	A1		2.3a	S4
Ь	88 cm	$\begin{aligned} & \text{mean} = \frac{\text{sum of all waist sizes}}{\text{number of people}} \\ & \text{The sum of all waist sizes is the sum of (midpoint \times frequency).} \\ & \text{Mid-point between 76 cm \& 80 cm is 78 cm} \\ & \text{Mid-point between 81 cm \& 85 cm is 83 cm} \\ & \text{Mid-point between 86 cm \& 90 cm is 88 cm} \\ & \text{Mid-point between 91 cm \& 95 cm is 93 cm} \\ & \text{Mid-point between 96 cm \& 100 cm is 98 cm} \\ & \text{Sum of all waist sizes is } (78 \times 4) + (83 \times 7) + (88 \times 9) + (93 \times 5) + (98 \times 5) = 2640 \\ & \text{The information is about 30 adults.} \\ & \text{The mean waist size is } 2640 \div 30 = 88 \text{ cm.} \end{aligned}$	M1 finding mid-points of intervals M1	4	1.3b	\$ 4
11a	5 <i>a</i>	$AB = 3a - b \qquad BC = 2a + b$ $AC = AB + BC = (3a - b) + (2a + b) = 5a$	A1		1.3b	G25
b	'Prove That' Q ^U working must be shown	Working must be shown $ \overrightarrow{BC} = 2a + b \qquad \overrightarrow{PQ} = \frac{5}{4}a $ $ \overrightarrow{PC} = \frac{1}{4}\overrightarrow{BC} = \frac{1}{4}(2a + b) $ $ \overrightarrow{CQ} = \overrightarrow{CP} + \overrightarrow{PQ} = \overrightarrow{PQ} - \overrightarrow{PC} = \frac{5}{4}a - \frac{1}{4}(2a + b) = \frac{5}{4}a - \frac{2}{4}a - \frac{1}{4}b $ $ = \left(\frac{5}{4} - \frac{2}{4}\right)a - \frac{1}{4}b = \frac{3}{4}a - \frac{1}{4}b = \frac{1}{4}(3a - b) $ $ \overrightarrow{AB} = 3a - b $ $ \overrightarrow{CQ} = \frac{1}{4}(3a - b) = \frac{1}{4}\overrightarrow{AB} $ $ \overrightarrow{CQ} = \frac{1}{4}(3a - b) = \frac{1}{4}\overrightarrow{AB} $ $ \overrightarrow{CQ} = \overrightarrow{AB} = A$	M1 M1		3.1b 1.3b 2.2	G25

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12a	Any correct assumption	e.g. the first counter taken out the bag was replaced before the second counter was chosen	A1	3.4	P8
b	3/20 (oe fraction, decimal or percentage)	If assumption is not true, the counter taken first will not be replaced	M1	3.1c 1.3a	I PX
13	<i>x</i> = 0.6	F is directly proportional to x so $F = kxFind the value of k:When x = 0.2, F = 19.2\therefore 19.2 = 0.2k [\div 0.2]k = 96\therefore F = 96xWhen F = 57.6,57.6 = 96x [\div 96]x = 0.6$	M1 M1 A1	1.3b	R10 R13
14	x = 3, y = 4	Either: 3x - y = 5 [rearrange for y] $y = 3x - 5$	M1 eliminate one variable A1 M1 substitute into a previous equation A1	1.3b	A4 A19

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15		Total number of yellow marbles is $3x$				
		Total number of marbles is $3x + 4x = 7x$				
		P(First marble is yellow) = $\frac{\text{Number of yellow marbles}}{\text{Total number of marbles}} = \frac{3x}{7x}$				
		Total number of marbles 7x				
		P(Second marble is yellow, if first marble is yellow)				
		$= \frac{\text{Number of yellow marbles left}}{\text{Total number of marbles left}} = \frac{3x - 1}{7x - 1}$	M1 correct			
			fractions			
		P(both marbles are yellow) = $\frac{3x}{7x} \times \frac{3x-1}{7x-1} = \frac{15}{91}$				
		$\frac{3x}{7x} \times \frac{3x-1}{7x-1} = \frac{3x(3x-1)}{7x(7x-1)} = \frac{9x^2 - 3x}{49x^2 - 7x} = \frac{9x - 3}{49x - 7} = \frac{15}{91}$	M1 correct			
			equation		2 14	P8
		$\left \frac{9x - 3}{49x - 7} = \frac{15}{91} \right [\div 3]$			3.1d 1.3b	A17
					1100	, , ,
		$\frac{3x-1}{49x-7} = \frac{5}{91} $ [×91]				
		$\left \frac{91(3x-1)}{49x-7} = 5 \right \left[\times (49x-7) \right]$	Ma			
		91(3x-1) = 5(49x-7) [expand brackets]	M1 method to eliminate fractions			
		273x - 91 = 245x - 35 $[-245x]$				
		28x - 91 = -35 [+91]				
		$\begin{vmatrix} 28x = 56 \\ x = 2 \end{vmatrix}$ [÷28]	M1 finding x			
		Total number of yellow marbles is 3x				
		If $x = 2$, the number of yellow marbles William put in the bag original is				
	6	$3 \times 2 = 6$	A1	5		
16a		Height $= x$ cm				
		Length = $2x$ cm	M1 expressions			
		Width = $(x-1)$ cm	for dimensions			
		Surface area = $2(\text{height} \times \text{length}) + 2(\text{width} \times \text{length})$ + $2(\text{height} \times \text{width})$ [substitute in values]				
	'Show That' Q ^U	$= 2(x \times 2x) + 2((x-1) \times 2x) + 2(x \times (x-1))$ [simplify]				
	working must be		M1 begin		2.2 3.1a	A21 G16
	shown	$= 2(2x^{2}) + 2(2x^{2} - 2x) + 2(x^{2} - x)$	process to find surface area		Jila	u i o
		Surface area = 136 cm ²				
		$\therefore 2(2x^2) + 2(2x^2 - 2x) + 2(x^2 - x) = 136$ [÷ 2]	M1			
		$2x^{2} + 2x^{2} - 2x + x^{2} - x = 68$ [simplify]				
		$5x^2 - 3x - 68 = 0$	A1			
b		$5x^2 - 3x - 68 = 0$				
		Using quadratic formula:				
		$3 \pm \sqrt{\left(-3\right)^2 - 4 \times 5 \times \left(-68\right)}$ [simplify]	M 1			
		$x = \frac{3 \pm \sqrt{(-3)^2 + 4 \times 3 \times (-36)}}{2 \times 5}$ [simplify]	M1 quadratic formula (or attempt			
			at factorisation)			
		$x = \frac{3 \pm \sqrt{9 + 1360}}{3 \pm \sqrt{9 + 1360}} = \frac{3 \pm \sqrt{1369}}{3 \pm \sqrt{1369}} = \frac{3 \pm 37}{3 \pm \sqrt{1369}} = \frac{3 \pm \sqrt{1369}}{3 \pm \sqrt{1369}} = \frac{3 \pm 37}{3 \pm \sqrt{1369}} = \frac{3 \pm \sqrt{1369}}{3 \pm \sqrt{1369}} = \frac{3 \pm 37}{3 \pm \sqrt{1369}} = \frac{3 \pm 37}{3 \pm \sqrt{1369}} = \frac{3 \pm 37}{3 \pm \sqrt{1369}} = \frac{3 \pm \sqrt{1369}}{3 \pm \sqrt{1369}} = \frac{3 \pm 37}{3 \pm \sqrt{1369}} = \frac{3 \pm \sqrt{1369}}{3 \pm \sqrt{1369}} = \frac{3 \pm \sqrt{1369}}{3 \pm \sqrt{1369}} = \frac{3 \pm 37}{3 \pm \sqrt{1369}} = \frac{3 \pm \sqrt{1369}}{3 $	M1 simplifying (or successful		1.3b	A2
		10 10 10	factorisation into $(5x + 17)(x - 4)$		3.1b 3.2	A18
		40 _ 4 or r = 34 _ 34			3.3	G16
		$x = \frac{40}{10} = 4 \text{ or } x = -\frac{34}{10} = -3.4$				
		x is a dimension so must be positive, $\therefore x = 4$	M1			
		Volume = $x \times 2x \times (x-1)$ [substitute in $x = 4$]				
		$=4\times8\times3$				
	96 cm ³	= 96 cm ³	A1	8		

Q <u>u</u> Nº	Answer	Solutions	Marks		AO	Spec.
17		Circumference of cone $=2\pi r = 16\pi$ [÷ 2π] $r = 8$ cm	M1			
		Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 8^2 \times h = \frac{64}{3}\pi h$				
		Volume of cone = 320π : $\frac{64}{3}\pi h = 320\pi$ [$\div \frac{64}{3}\pi$]	M1			
		$h = 320 \div \frac{64}{3} = 320 \times \frac{3}{64}$			1.1 1.3b	G17 G20
		h = 15 cm	M1			
		$tanPQX = \frac{opposite}{adjacent} = \frac{15}{8}$ [tan ⁻¹]	M1			
	61.9°	Angle PQX = $\tan^{-1} \left(\frac{15}{8} \right) = 61.9275 = 61.9^{\circ} (1 \text{ d.p.})$	A1	5		
18		Perimeter of square = 20 cm				
		One side of square $=\frac{20}{4}=5$ cm	M1			
		Area of left triangle $=\frac{1}{2} \times (5-1) \times (5-3.5)$				
		$=\frac{1}{2} \times 4 \times 1.5 = 3 \text{ cm}^2$	M1			R9
		Area of right triangle $=\frac{1}{2} \times 5 \times (5-2)$			1.3b 2.1b	G14 G16
		$=\frac{1}{2}\times5\times3=7.5\text{ cm}^2$	M1			
		Area of square $= 5 \times 5 = 25 \text{ cm}^2$ Shaded area $= 3 + 7.5 = 10.5 \text{ cm}^2$				
		Percentage of area that is shaded = $\frac{10.5}{25} \times 100$	M1			
	42%	= 42%	A1	5		
19a		$x^3 + 2x = 2$, so $f(x) = x^3 + 2x - 2$				
	'Show That' Q ^U working must be	$f(0) = 0^3 + 2 \times 0 - 2 = -2$			1.3b 2.2	A2
	shown	$f(1)=1^3+2\times 1-2=1$	M1 values for $x = 0, x = 1$		2.3b	A6
		Sign change signifies that the solution lies between 0 and 1	A1			
b	'Show That' Q ^U	$\begin{vmatrix} x^3 + 2x = 2 \\ 2 & 3 \end{vmatrix}$ [-x ³]			1.3a	A 4
	working must be	$ 2x = 2 - x^3 \qquad [\div 2] $			2.2	A4 A6
	shown	$x = 1 - \frac{x^3}{2}$	A1	3	2.3b	
20		$\frac{1}{-} = \frac{1}{-} - \frac{1}{-}$ [isolate $\frac{1}{-}$]				
		$\begin{bmatrix} x & y & z \end{bmatrix}$				
		$ \frac{1}{z} = \frac{1}{y} - \frac{1}{x} $ [find common denominator on RHS]				A4
		$\frac{1}{1} = \frac{x}{x} - \frac{y}{y} = \frac{x - y}{y}$ [reciprocal]	D4		1.3b	A5
		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B1			
	$z = \frac{xy}{x - y}$	$z = \frac{xy}{x - y}$	B1	2		

Q <u>u</u> Nº	Answer	Solutions	Marks	AO	Spec.
21		Either: $E = \frac{8.99 \times 10^{9} Q}{r^{2}}$ Molecule 1: $E = \frac{8.99 \times 10^{9} \times 6.41 \times 10^{-19}}{\left(3.98 \times 10^{-9}\right)^{2}}$ $= 3.64 \times 10^{8}$ Eatio $E_{2}: E_{1} = 1: \frac{E_{2}}{E_{1}}$ $\frac{E_{2}}{E_{1}} = \frac{8.99 \times 10^{9} Q_{2}}{r_{2}^{2}} \div \frac{8.99 \times 10^{9} Q_{1}}{r_{1}^{2}}$ $= \frac{E_{2}}{E_{1}} = \frac{8.99 \times 10^{9} Q_{2}}{r_{2}^{2}} \div \frac{8.99 \times 10^{9} Q_{1}}{r_{1}^{2}}$ $= \frac{Q_{2}}{r_{2}^{2}} \times \frac{Q_{1}}{r_{1}^{2}} [invert fraction]$ $= \frac{Q_{2}}{r_{2}^{2}} \times \frac{Q_{1}}{r_{1}^{2}} = \frac{Q_{2}r_{1}^{2}}{Q_{1}r_{2}^{2}}$		1.3b	A2 R4
	1 : 56.3		M1 A1 3		
22	15:3:1	Ratio of goldfish: tetras is $5:1$ Ratio of tetras: swordtails is $3:1$ Ratio of goldfish: tetras is $5:1=5\times3:1\times3=15:3$ [multiply by 3 to make tetras the same in both ratios] Ratio of goldfish: tetras: swordtails is $15:3:1$	M1 A1 2	1.3a 3.1c	R4
23	5 -4	y = 2 $y = 3$ $y = 3x + 2$	M1 one line drawn correctly M1 more than one line drawn correctly A1 region shaded correct lines A1 all lines and shaded region correct	2.3b	A9 A22

Q <u>u</u> Nº	Answer	Solutions	Marks		AO	Spec.
24a	8.1 — 8.2 m/s	At 11 seconds, the distance travelled is 30 m. At 17 seconds, the distance travelled is 79 m. Average speed = $\frac{\text{distance}}{\text{Time}}$ = gradient = $\frac{\text{change in } y}{\text{change in } x}$ = $\frac{79 - 30}{17 - 11}$ = $\frac{49}{6}$ = 8.16 m/s	M1		2.3a 1.3a	A15
b	2.5 m/s – 3.5 m/s	Average speed is gradient, m , of tangent at 8 seconds $ \begin{array}{cccccccccccccccccccccccccccccccccc$	M1 drawing tangent at 8 seconds M1 finding gradient of tangent A1	5	2.3a 1.3a	A15
25	Yes (Jeff is correct) [with reason]	Reasoning must be given $16a \div 4a = 4$ 4 is a whole number : Jeff is correct, $4a$ is always a factor of $16a$	A1	1	2.4a	N4
26a	5.72 × 10 ⁻⁵	In standard form a number between 1 & 10 is multiplied by 10^n 5.72 is between 1 and 10; the decimal moves 5 places to the left. $0.0000572 = 5.72 \div 100,000 = 5.72 \times \frac{1}{100000} = 5.72 \times 10^{-5}$	A1		1.3b	N9
b	3,200	$3.2 \times 10^3 = 3.2 \times 1000 = 3,200$ The decimal point moves 3 places to the right.	A1 otal Marks:	2 80	1.3a	N9