2015 specification
first exams in 2017

Mechanics 'Tricky Topics' Worksheets

for A Level Edexcel Physics

Exam skills prep pack

zigzageducation.co.uk

POD 9385

Contents

TEACHER'S INTRODUCTION

Understanding mechanics is essential to understanding a range of areas in A Level Physics and mechanics topics require the application of a range of maths skills, which many students can find difficult. This activity pack is designed to prepare students for the exam by developing the skills essential to answering even the trickiest mechanics questions.

The resource opens with a student introduction followed by 13 worksheets, each of which covers one or more skills outlined below.

The content covers a range of topics across the Edexcel A Level Physics course, specifically including content from **Module 2: Mechanics, Module 4: Materials, Module 6: Further mechanics** and **Module 13: Oscillations**. The range of content is chosen to familiarise students with the many equations in these sections of the course, and test a variety of required maths skills, to both give students the grounding they need for the exam and test them to prepare them for more difficult questions.

Remember!
Always check the exam board website for new information, including changes to the specification and sample assessment material.

Although each worksheet does cover a specific section of the Edexcel A Level specification, the exercises are primarily skills-focused. These skills are all relevant to the A Level Edexcel examinations. The competencies which are developed and tested by this resource include:

- Resolving, combining, and otherwise performing calculations with vectors, using both graphical and numeric methods
- Analysing motion in one and two dimensions, including manipulation of the equations of motion
- Using graphical methods to calculate quantities of motion, and understanding the motion represented by these graphs
- Understanding Newton's laws to analyse situations involving multiple forces, and relating this to the motion produced.
- Understanding problems involving extended bodies and how objects' centres of mass can be used in these problems, and analysing support systems in terms of forces involved
- Understanding how forces and displacement relate to work done
- Performing calculations involving the conservation of energy, including in processes which are not 100 % efficient
- Performing calculations involving solids submerged in fluids, including how density, pressure and upthrust affect these solids
- Understanding how the properties of a material affect the way it interacts with forces and energy, including performing calculations and interpreting graphs
- Performing calculations relating to the outcomes of collisions and interactions using the law of conservation of momentum
- Analysing and predicting the forces involved in circular motion
- Understanding and analysing situations involving simple harmonic motion, using the various equations that define these oscillations
- Analysing systems undergoing simple harmonic motion, such as masses on springs and pendulums, and relate these to effects such as damping and resonance

Each worksheet contains **a short section of background information**, followed by **worked examples** and then **in-depth questions** to test students' knowledge (an answers section can be found at the end of the resource). The questions are split into three levels of increasing difficulty, illustrated using the headings *Setting off*, *Speeding up* and *Top speed.*

At the back of the pack is a short **GCSE refresher and quiz**, intended for students who may not be confident with their existing knowledge going into the pack.

The pack also includes four **full exam-style questions**, which closely replicate the style of question found in an examination paper.

We hope this resource will be useful to your teaching, and help your students to tackle an area of physics which many find challenging, so that each student gains a deeper, holistic understanding of the subject.

March 2019

STUDENT INTRODUC

Mechanics describes the way that objects move – their motion, and the forces that c
their speed or direction. Mechanics can describe stable systems, like a rock carefull
cliff, or dynamic systems, such as a rocket firing into space. Mechanics aren't only u
their own – mechanics can keep track of multiple objects colliding and exerting forc

Since ancient times, philosophers have tried to understand the world by considering ho
century, Galileo Galilei performed a series of experiments which began to quantify a
mathematical way, and introduced the idea of reference frames. In the seventeenth ce
his three laws, which describe how forces, inertia and momentum affect the motion of o
expanded on in the eighteenth century by Leonhard Euler, who applied them to extend
particles.

Considering the mechanics of different systems has led to the development of entire
physics, special and general relativity, and quantum mechanics.

Mechanics is used to develop the technologies we rely on every day. When buildin
mechanics are essential to understand the forces stopping a building falling apart.
is designed with careful deliberation about the forces that the different parts will e
By understanding momentum and acceleration, athletes can predict the path of a b

Mechanics comes up in a few different sections of your Edexcel A Level course, in a
applications. Mechanics topics include a lot of equations that can be tricky to use, c
understanding of the principles involved. To get top marks in questions on mechanic
rearrange equations, see how they're related and recognise how the different equa

This pack will help you develop several core skills related to mechanics, including:

- Resolving, combining, and otherwise performing calculations with vectors, using bo
- Analysing motion in one and two dimensions, including manipulation of the equatio
- Using graphical methods to calculate quantities of motion, and understanding the m
- Understanding Newton's laws to analyse situations involving multiple forces, and re
- Understanding problems involving extended bodies and how objects' centres of m and analysing support systems in terms of forces involved
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- Analysing and predicting the forces involved in circular motion
- Understanding and analysing situations involving simple harmonic motion, using the these oscillations
- Analysing systems undergoing simple harmonic motion, such as masses on springs a effects such as damping and resonance

At the back of this pack is an appendix which is intended as a refresher of GCSE p
pack. If you don't feel confident before attempting any of the worksheets, you
consolidate your background knowledge before working through each worksheet.

1. VECTORS

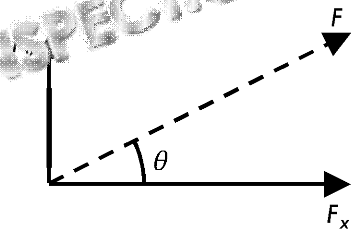
2: MECHANICS

BACKGROUND

When discussing motion and forces, it's important to take into account direction. This is because displacement, velocity, acceleration, force and momentum are all **vectors** – they have direction.

EXAM
Typical
upward
acting
negativ
Always
frame

The diagram below shows two vectors acting at right angles to each other, F_y and F_x and their resultant (combined) vector, F.



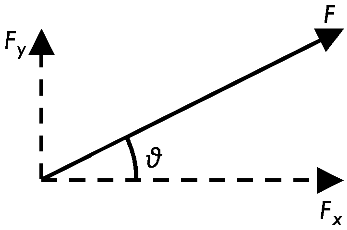
The magnitude of the resultant vector is given by

$$F=\sqrt{F_x^2+F_y^2}$$

The direction of the r
acts in) is given by

$$\vartheta$$

The diagram below shows a single vector acting at an angle – it would be
vectors acting at right angles, horizontally and vertically.



The magnitude of F_x is given by

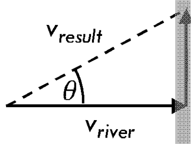
$$F_x=F\cos\vartheta$$

The magnitude of F_y

$$F$$

Example 1
A river flows at a velocity of 1.2 m s⁻¹ directly to the east. A duck swims north, relative to the current, at 0.65 m s⁻¹.
Calculate the velocity of the duck relative to the ground, and the angle it travels in compared to the river.

First, draw all the vectors given end the resultant vector:



This creates a triangle.
The duck's resultant velocity, v_{result}, is triangle and its magnitude is given b

$$v_{result}=\sqrt{v^{2}_{river}}$$

Putting in numbers:

$$v_{result}=\sqrt{1.2^2+0.65^2}$$

$$\boldsymbol{v_{result}=1.4\ \mathrm{m\ s^{-1}}}$$

Velocity is a vector, though, so it ne by

$$\tan\vartheta=\frac{v_{d}}{v_{ri}}$$

so

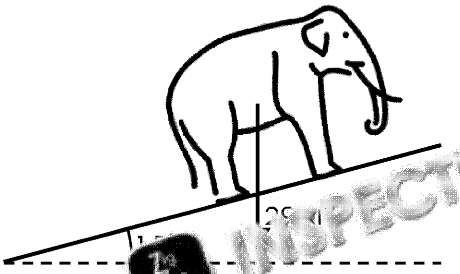
$$\vartheta=\tan^{-1}\frac{v_{duck}}{v_{river}}$$

Putting in numbers

$$\vartheta=\tan^{-1}\frac{0.65}{1.2}$$

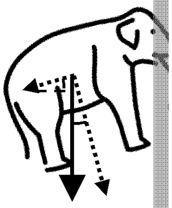
$$\boldsymbol{\vartheta=28°}$$

Example 2
An elephant is standing on a ramp which is on an incline of 15° to the ground.
The elephant weighs 29 kN.
Resolve the elephant's weight into two vectors – one down the ramp and one at right angles to the ramp's surface.



Using weight in these directions is useful for finding things such as the normal reaction force and friction acting on the elephant.

The elephant's weight can be resolv seen here.



To find these vectors' magnitudes, th of as making two triangles with the

Down the ramp:

$F_{ramp}=F\sin\vartheta$ [this is sin bec force opposit

$$F_{ramp}=29\times10^{3}\times\sin15$$

$$\boldsymbol{F_{ramp}=7.5\times10^{3}\ \mathrm{N}}$$

At right angles to the ramp:

$F_{normal}=F\cos\vartheta$ [we use cos the force ac

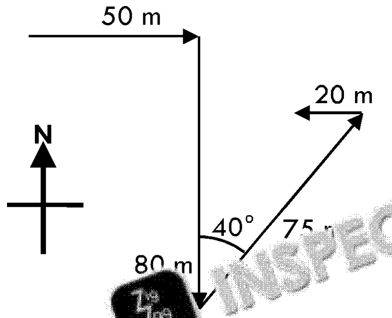
$$F_{normal}=29\times10^{3}\times\cos15$$

$$\boldsymbol{F_{normal}=28\times10^{3}\ \mathrm{N}}$$

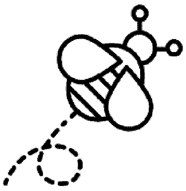
Example 3

A bee flies from her hive due east for 50 m, then due south for 80 m, then at a bearing of 40° from the north for 75 m, and finally due west for 20 m.

The bee's flight path is shown below.



Calculate the displacement of the bee from her hive.



It's important to first make sure that
vectors are resolved into components
right angles to each other – here tha
means resolving the vector at 40° to
north.

The length of the eastward compon
s_{East}, is given by

$s_{East}=s \sin\vartheta=75 \sin 40$

$s_{East}=48.2$ m due east

The length of the northward compone

$s_{North}=s\cos\vartheta=75 \cos 40$

$s_{North}=57.5$ m due north

Now that all vectors are at right ang
add them.

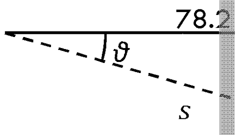
$s_x=50-20+48.2$

$s_x=78.2$ m

$s_y=-80+57.5$

$s_y=-22.5$ m

And now we can combine these vect



The magnitude of s is given by

$$s=\sqrt{s_x^2+}$$

$$s=\sqrt{78.2^2+(-22.5)^2}$$

$\boldsymbol{s=81}$ **m**

And the direction is given by

$$\tan\vartheta' =$$

$$\vartheta'=\tan^{-1}$$

$$\vartheta'=\tan^{-1}\frac{-22.5}{78.2}$$

$\vartheta'=-16.1°$

(This is the angle below due east – f

$\vartheta=90+\vartheta'=90+16.1$)

$\boldsymbol{\vartheta=106.1°}$

EXAM TIP

Bearings a
north, unle
you sketch
see all the
for yoursel

QUESTIONS

Setting off

1. A cyclist is initially travelling at a speed of 6.5 m s^{-1}, when a gust of wind blo track.

   The gust of wind blows at right angles to the cyclist's velocity at a wind speed

   Calculate the cyclist's resultant speed, and the direction they are travelling in their initial path.

2. A cannonball is fired at a velocity of 54 m s^{-1} at an angle of 12° from the gro

   Resolve the cannonball's velocity into horizontal and vertical vectors.

Speeding up

3. A ball rolls onto a conveyor belt which is moving at a steady rate of 4.30 m [illegible]he ball moves onto the conveyor belt at a right angle to the convey[illegible]

   After rolling onto the conveyor belt, the ball rolls at a speed of 5.21 m s^{-1} relative to the ground.

   a) Calculate the initial speed of the ball as it rolls onto the conveyor belt.

   b) Calculate the angle the ball rolls at relative to the conveyor belt's motion.

   Challenge: Try to do part b) both with and without using your answer from

4. In an arcade game, the main character can only walk along a grid of squares.

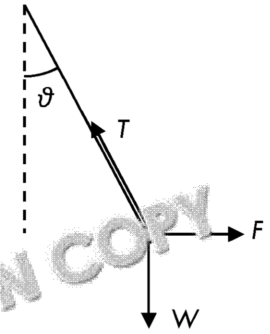
   The character walks up 9 squares, left 3 squares, right 8 squares, up 11 squa squares, down 1 square and then right 2 squares.

   Calculate the total displacement of the character.

Top speed

5. A stone on a string is pulled with a force F at an angle of ϑ to vertical.

   The forces acting on the stone are seen below.

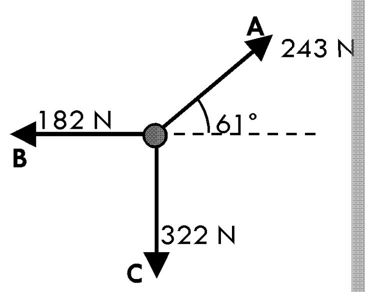


   The ten[illegible] th[illegible], T, is perfectly balanced by the weight of the stone, W,

   W= 4.[illegible]nd F= 3.89 N.

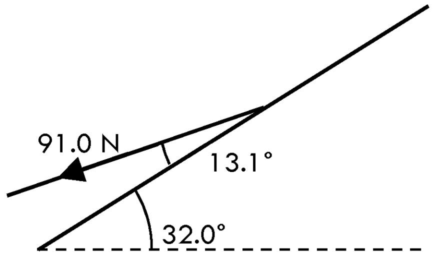
   Calculate the angle the string makes to vertical.

6. Three cars are attached to a post by ropes and each pulls.
The three forces exerted on the post by the cars are seen below.



Calculate the total force experienced by the post.

7. A rope is attached to a board.
The board is at an angle of 32.0° to the ground, and the rope pulls with 91.0 N
Resolve the tension in the rope into vertical and horizontal components.



2. LINEAR AND PROJECTILE

2: MECHANICS

BACKGROUND

An object's motion can be described by discussing its **displacement**, **velocity** and **acceleration**.

$$v=\frac{\Delta s}{\Delta t} \qquad a=\frac{\Delta v}{\Delta t}$$

For situations involving constant acceleration, the following equations can be used:

$$v=u+at \qquad s=\left(\frac{u+v}{2}\right)t$$

$$v^2=u^2+2as \qquad s=ut+\frac{1}{2}at^2$$

Projectile motion describes motion where there is a vertical acceleration do
zero horizontal acceleration.
In projectile motion, an object's horizontal and vertical motions are **indepen**

On Earth, all object's fall with an acceleration equal to g = 9.81m s^{-2}.

Example 1

A car accelerates from 7.0 m s^{-1} at an acceleration of 3.2 m s^{-2} up to a final speed of 13 m s^{-1}.

Calculate the distance the car travels in this time.

First, list out all the variables that ha
s = looking for, u = 7.0 m s^{-1}, v = 1

u, v and a are all given, and s is ne
The equation with all of these variab
$v^2=u^2+2as$
Rearrange for s

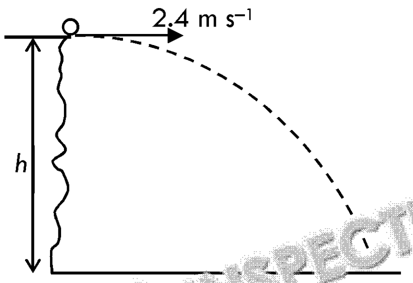
$$s=\frac{v^2-u^2}{2a}$$

$$s=\frac{13^2-7.0^2}{2\times 3.2}$$

s= 19 m

Example 2
A 0.43 kg football is kicked horizontally off a cliff at an initial speed of 2.4 m s^{-1}. The ball lands 11 m from the cliff.

Calculate the height of the cliff.



For projectile motion, the horizontal
are completely independent.

Assuming that there is no air resistan
horizontal component of the ball's v

$$v=\frac{\Delta s}{\Delta t}$$
$$\Delta t=\frac{\Delta s}{v}=\frac{11}{2.4}$$
$$\Delta t=4.58 \text{ s}$$

so the ball spends 4.58 s in the air.

For the vertical motion, we know tha
$u=0$ m s^{-1}, $a=9.81$ m s^{-2}, $t=4.58$ s,

The equation of motion containing u,
$$s=ut+\frac{1}{2}at^2$$
$$s=\frac{1}{2}\times 9.81\times 4.58^2$$
$$\boldsymbol{s=100} \textbf{ m}$$

Example 3
Anna and Ben are in a long-distance race.

Anna stops to tie her shoelaces and Ben runs past her.

When Ben is 26.4 m ahead of Anna, Anna stands up and begins to run again, accelerating at a constant 0.900 m s^{-2}.

Ben is running at 4.40 m s^{-1} and starts to decelerate at 0.200 m s^{-2} at the exact moment that Anna stands up and begins to run.

How long after Anna stands up will she overtake Ben?

First, write out all the information gi
$u_A=0$ m s^{-1}, $a_A=0.900$ m s^{-2}, $t_A=$ lo
$u_B=4.40$ m s^{-1}, $a_B=-0.200$ m s^{-2}, $t_B=$

At first glance it looks as if there isn
of the equations!
However, we do have information f
runners are linked:

When Anna catches up with Ben, the
amount of time

$$t_A=t_B=$$

When Anna gets up, Ben is 26.4 m
to run an extra 26.4 m to reach the

$$s_A=s_B+2$$

Use this information to produce two
Anna
$$s_A=u_at+\frac{1}{2}a_at^2$$
$$s_B+26.4=0.450\times t^2$$
$$s_B=0.450\times t^2-26.4$$
$$0.450\times t^2-26.4=4.40t-0.100\times t^2$$
$$0.550\times t^2-4.40\times t-26.4=0$$
$$t^2-8t-48=0$$
$$(t-12)(t+4)=0$$
$t=12$ s or $t=-4$ s
can't have negative time so
$\boldsymbol{t=12}$ **s**

QUESTIONS

Setting off

1. A cyclist slows down from 14 m s^{-1} to 3.2 m s^{-1} over 12 s.

   Calculate the acceleration of the cyclist.

2. A jogger runs 210 m in 45 s. The jogger is initially running at 3.7 m s^{-1} but inc acceleration.

   Calculate the jogger's final velocity.

3. A student throws a ball up from a height of 1.6 m. The ball lands at a velocity

   Calculate the initial speed the ball was thrown up at

Speeding up

4. A car accelerates from an initial speed of 11 m s^{-1} at an acceleration of 9.0 m

   Calculate how long it takes the car to travel 88 m.

5. A shot is thrown by a 1.91 m tall shot-putter, at an angle of 25.1° and an initi velocity of 10.4 m s^{-1}.

   Calculate the distance from the shot-putter the shot lands.

Top speed

6. In 2099 on Mars, a golf ball is hit off a tee, at an angle of ϑ and an initial vel

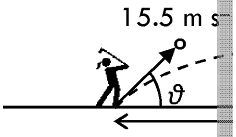
   The golf ball lands 31.5 m from the tee.

   Find the angle ϑ that the ball is hit from the tee.

   Use $g_{Mars} = 3.71$ m s^{-2}.

   You will have to use the trigonometric identity

   $2 \sin x \cos x = \sin 2x$



7. A motorbike speeds past a police car. The police car sets off 1.08 s after the accelerates at 5.15 m s^{-2}.

   The motorbike is initially travelling at 29.5 m s^{-1}, and after it passes the police 0.314 m s^{-2}.

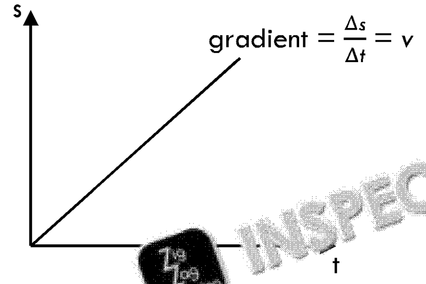
   Calculate the velocity of the police car when it catches up to the motorbike.

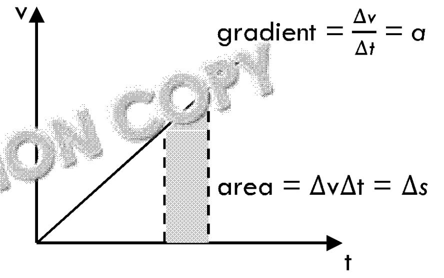
3. MOTION AND GRA

2: MECHANICS

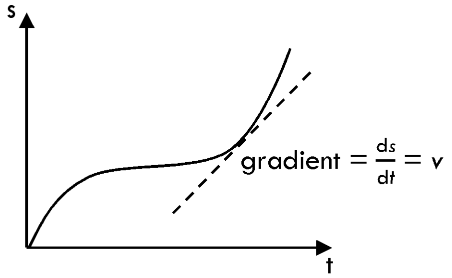
BACKGROUND

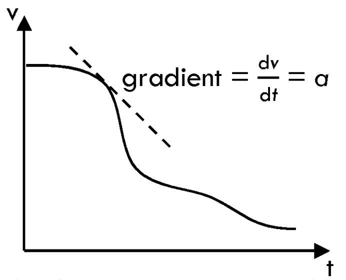
An object's motion can be displayed on **displacement–time, velocity–time** c
These show graphically how an object's displacement, velocity or accelerati





For a graph that is **non-linear** (curved), the gradient can be calculated by drawing a **tangent** to the line at a specific point and calculating the gradie of this tangent.

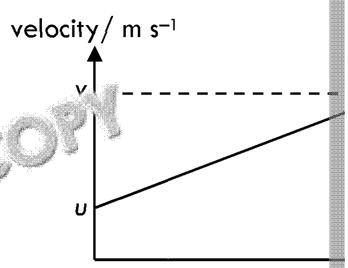




This can give **instantaneous velocity** (from a displacement–time graph) or **instantaneous acceleration** (from a velocity–time graph).

Example 1
From a velocity–time graph, derive the equation of motion
$v = u + at$

From a simple velocity–time graph f
from velocity u to v over a time t:



The gradient of the graph gives the

This means that

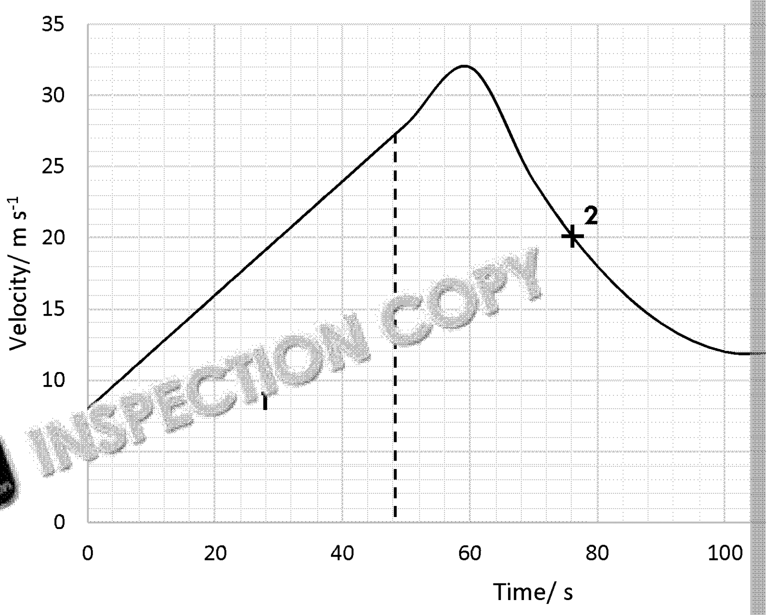
$a = \frac{\Delta y}{\Delta x} = \frac{v - u}{t}$

which rearranges to

$\boldsymbol{v = u + at}$

Example 2

The graph below shows the velocity–time graph for a car.



a) Calculate the distance travelled by the car in the section marked 1.

b) Calculate the acceleration of the car at the point marked 2.

a) The distance travelled by the car is given by the area under the veloc
The area can be found by splitting the area into a rectangle (with a h
of 48 s) and a triangle (with a height of 19 m s^{-1} and a width of 48 s)

Total area = $8\times48+\frac{1}{2}\times19\times48$

Distance = 840 m

b) The acceleration is found by drawing a tangent to the curve and findi
An example gradient would be:

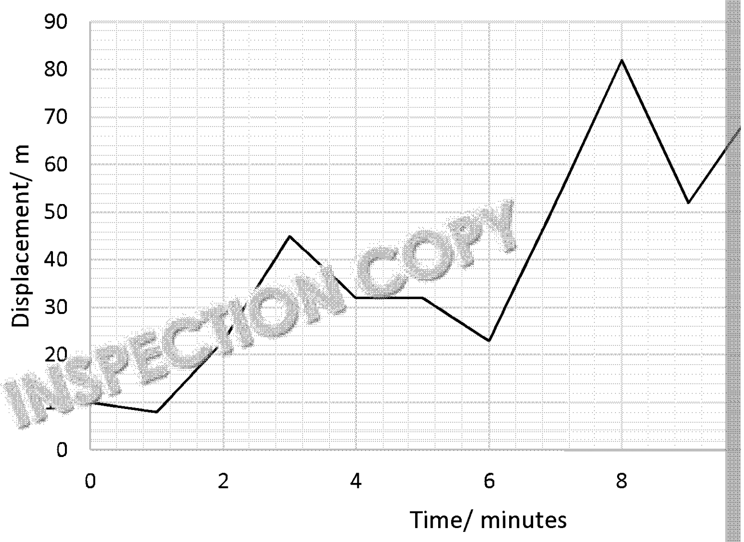
$$gradient=\frac{\Delta y}{\Delta x}=\frac{0\text{-}29}{112\text{-}60}$$

acceleration = -0.56 m s^{-2}

QUESTIONS

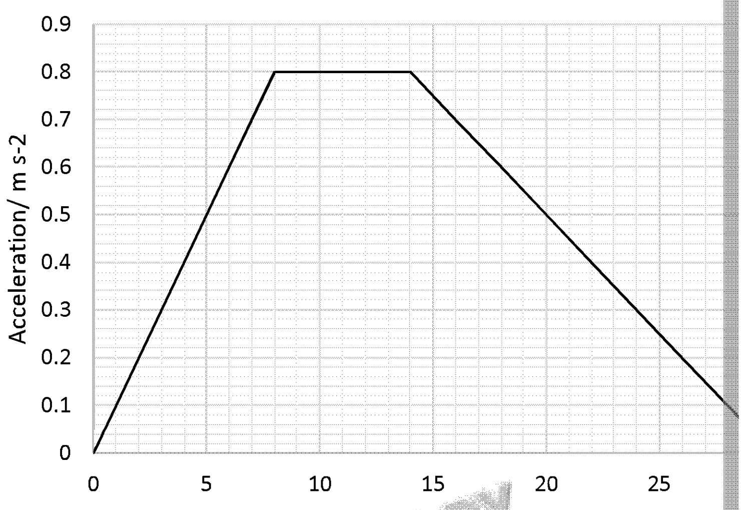
Setting off

1. The displacement of a cat (called Beetroot) is measured every minute for 10 m
   The displacement–time graph of Beetroot's motion is shown below.



   a) Calculate the average velocity of Beetroot.
   b) Calculate the velocity of Beetroot after 7 minutes.

2. The acceleration of a car is measured for 5 minutes. The acceleration–time gr



   Calculate the total change in velocity of the car.

3. A pebble is dropped into a lake. Sketch displacement–time, velocity–time an
   pebble's motion as it accelerates through the water, and then reaches terminal
   Annotate the graphs to explain what happens at each stage.

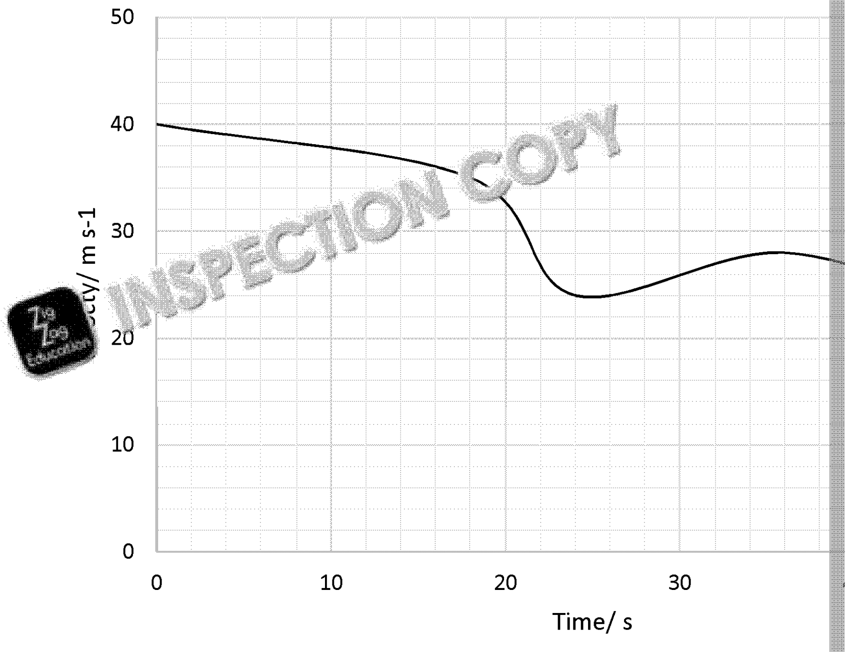
Speeding up

4. a) From a velocity–time graph, derive the equation of motion

$$s=\frac{1}{2}(u+v)t$$

b) From the equations $v = u + at$ and $s=\frac{1}{2}(u+v)t$, derive the other two equat

5. A car travels along a road. The velocity–time graph below shows the car's jou



a) Calculate the acceleration of the car at 10.5 s.

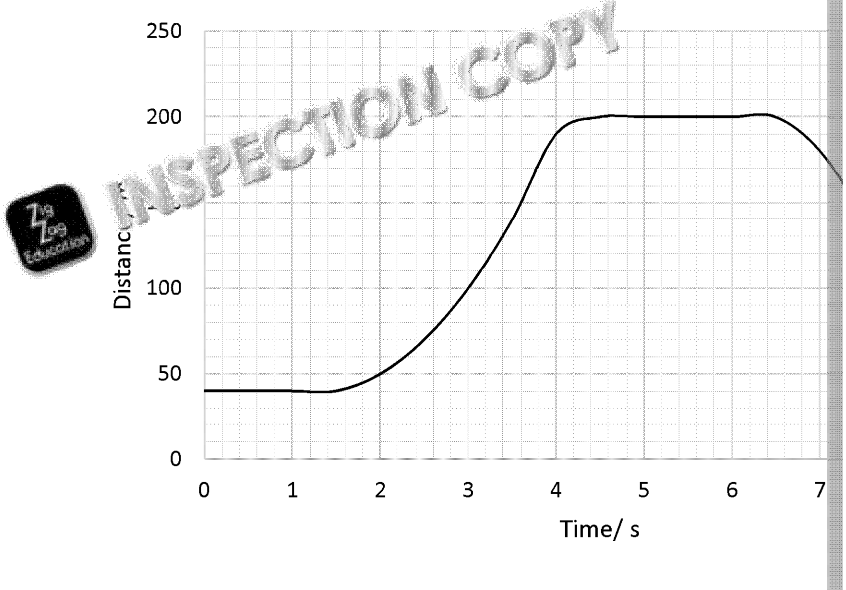
b) Estimate the displacement of the car over the entire 48 s.

6. A ball is dropped, and then bounces several times. Sketch displacement–time, time graphs of the ball's motion after it is dropped.

Top speed

7. A bike that is initially travelling at 6 m s^{-1} accelerates at 0.8 m s^{-2} for 12 s, then d
Draw a velocity–time graph of the bike's motion and calculate its displacemen bike's journey.

8. Use the distance–time graph of a test rocket below to calculate the average a between 3.5 s and 8.5 s.



4. NEWTON'S LAWS AND

2: MECHANICS

BACKGROUND

Newton's first law

An object with **no external force** acting on it will remain at **rest** or at a **con**
In other words: an acceleration requires a force to act.

Newton's second law

The force exerted on an object is **proportional** to the **mass** of the object an
the object.

As an equation this can be written as

$$F = ma$$

when mass is constant.

Newton's third law

When a force is exerted on an object, a **reaction force** of the same **size** an
direction to the exerted force.

Example 1

A coin with a mass of 9.0 g is dropped into water. The coin experiences drag forces of 35 mN. Calculate the acceleration experienced by the coin.

The acceleration of an object due t
Newton's second law:

$$F=ma$$

F refers to the resultant force, the su

Here, F is given by the coin's weight
acting on the coin.

$$F=mg–F_{dr}$$

So

$mg-–F_{drag}=ma$

Rearranging

$$a=\frac{mg–F_{drag}}{m}$$

$$a=\frac{9.0\times10^{-3}\times9.81–35\times10^{-3}}{9.0\times10^{-3}}$$

$\mathbf{a=5.9\ m\ s^{-2}}$

Example 2
Two pebbles are dropped into a pond. Both pebbles have the same shape and size, but different densities.
The pebbles' accelerations decrease, and then remain at zero.
The denser pebble reaches a higher final velocity.
Explain the pebbles' motion using Newton's laws.

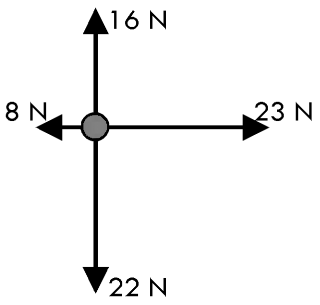
Initially, the pebbles accelerate dow their weight is greater than the drag is described by **Newton's first law**.

As their velocities increase, the pebb molecules. As the pebbles exert a fo the water molecules exert a force ba decreasing their acceleration, as in N

Eventually the drag forces balance w stop accelerating. This is **Newton's**

The denser pebble has a larger weig forces acting over its surface. This m force, and has a higher final velocity **second law**.

Example 3
Four forces act on a 3.4 kg mass, as shown below.



Calculate the magnitude and direction of the acceleration of the mass.

The resultant force needs to be calcu and vertical directions, and then reso

Horizontally
$F_H=F_1+F_2$
$F_H=23-8$
$F_H=15$ N to the right

Ve
F_V
F_V
F_V

These components then need to be co

$$F=\sqrt{F_V^2+F_H^2}$$
$$F=\sqrt{15^2+6^2}$$
***F=16* N**

$$\vartheta=\tan^{-1}\frac{F_V}{F_H}$$
$$\vartheta=\tan^{-1}-\frac{6}{15}$$
ϑ= -21° (from horizontal)

QUESTIONS

Setting off

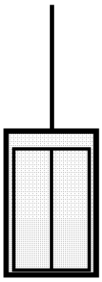
1. a) A 530 g toy car is pushed with a force of 6.1 N.
   Calculate the toy car's acceleration.
   b) After the car is accelerated, another force of 6.1 N acts in the opposite di
   Describe and explain what happens to the car's motion after this second fo

2. For each of the following forces, state the reaction force acting, as per Newto
   a) The weight of a student pressing down on the floor.
   b) A rope pulling on a post with a tension of 30 N.
   c) The upthrust on a pineapple in a pool of water.

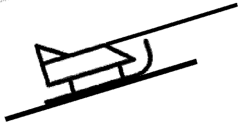
Speeding up

3. Draw free body diagrams for each of the following situations.

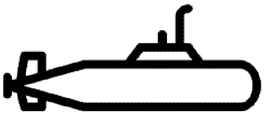
   a) A lift travelling upwards at a steady speed.



   b) A sledge being pulled up a hill while accelerating

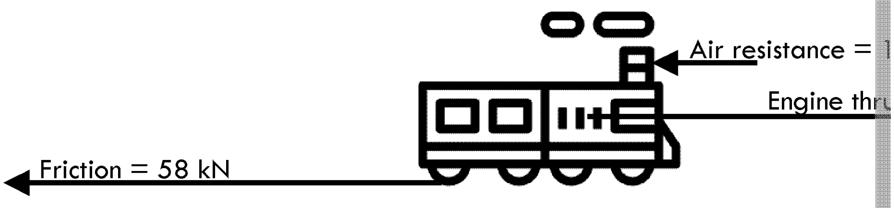


   c) A pressurised submarine underwater, which is stationary and at a constant



4. A train travels along a track, and decelerates at a steady rate of 4.4 m s^{-2}.

   The forces acting on the train are seen below.



   Calculate the mass of the train.

Top speed

5. A heavy rock is pulled uphill with a force of 8[illegible] N.

   The rock has a mass of 390 kg, and the hill has a slope of 11°.

   A frictional force of 12[illegible] acts on the rock as it is being pulled.

   Calculate the acceleration of the rock.

6. Two identical sheets of paper are dropped from the same height at the same

   One of the sheets of paper is crumpled up, and the other is flat.

   Compare the motions of the sheets of paper as they fall.

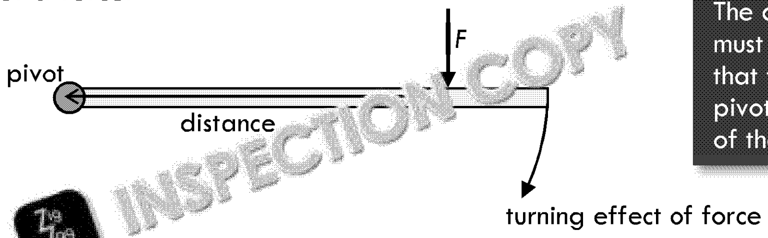
5. MOMENTS

2: MECHANICS

BACKGROUND

Many processes use **moments** to transfer forces. A lever can increase the effect of a force, or a gear can change the direction of a force.

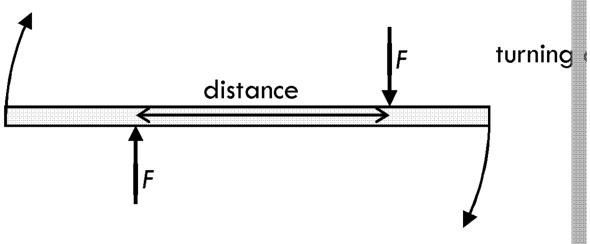
EXAM TIP
The distanc
must be the
that the lin
pivot make
of the forc



The moment of a force is given by

$$moment = F \times (distance\ from\ pivot)$$

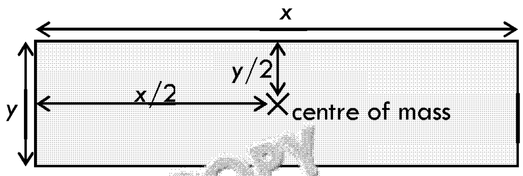
A **couple** is a pair of forces that have the same magnitude but act in opposi line, causing a turning force.



The moment of a couple is given by

$$moment = F \times (distance\ between\ forces)$$

The weight of an object acts through its **centre of mass** – an imaginary poin mass in the object lies.



For a uniform and regular solid, its centre of mass is at its centre.

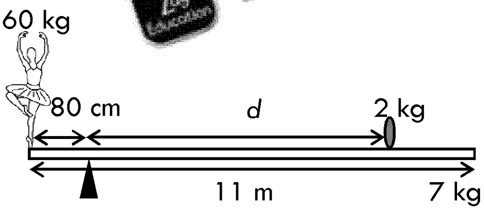
Example 1

Some Physics students go to a modern art gallery. Not enthused about the artwork, they decide to investigate the physics behind each installation.

In an art installation called *The Unbearable Lightness of Ballet*, a 60 kg ballerina stands on a plank, 80 cm from a pivot. On the other side of the pivot is a 2 kg stone. The plank is regular and uniform and has a mass of 7 kg and a total length of 11 m.

The plank is perfectly balanced.

Calculate the distance of the stone away from the pivot.



For the plank to be perfectly balanc
each side of the pivot need to be eq

To the left of the pivot, the only forc
ballerina.

$F_{left}=W_{ballerina}=m_{ballerina}g$

$F_{left}=60\times9.81=589$ N

The moment caused by the ballerina

$moment_{left}=F_{left}\times$ distance

$moment_{left}=589\times0.80=471$ N m

To the right of the pivot, there are tw
of the stone, and the weight of the p

$moment_{right}=moment_{pl}$

The centre of mass of the plank acts
5.5 m from either end of the plank a

$moment_{plank}=7\times9.81\times4.7=323$ N m

The moment caused by the weight of

$moment_{stone}=moment_{right}-moment_{plank}$

We know that $moment_{right}=moment_{lef}$

$moment_{stone}=471-323=148$ N m

The distance of the stone from the pi

$$d=\frac{moment_{stone}}{weight_{stone}}=\frac{148}{2\times9.81}$$

$\boldsymbol{d=7.5}$ **m**

Example 2

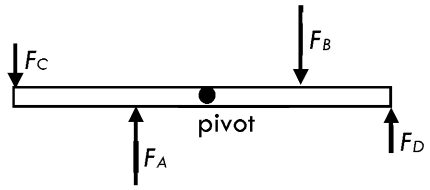
In an art installation called *The Odd Couple*, four fireworks are attached to a rod with a pivot at its centre.

Fireworks A and B are each 45 cm from the pivot, and exert a force on the rod of 18 N each, in opposite directions.

Fireworks C and D each exert a force of 12 N, also in opposite directions, and are the same distance away from the pivot as each other.

The fireworks are all lit at the same time, and the rod is held perfectly still.

Calculate the distance of fireworks C and D from the pivot.



In the example, use $F_A=F_B=F_{AB}$ and
For the rod to be balanced, the two
fireworks need to balance.

The couple generated by fireworks
$couple_{AB}=F_{AB}\times distance\ between\ A\ an$
$couple_{AB}=18\times0.45\times2=16.2$ N

This balances with the couple genera
$couple_{CD}=F_{CD}\times distance\ between\ C\ an$

$$d_{CD}=\frac{couple_{CD}}{F_{CD}}$$

$$d_{CD}=\frac{16.2}{12}=1.35\text{ m}$$

This gives the force between the fire
the pivot to each is

$\boldsymbol{d_{pivot}=0.68}$ **m**

Example 3

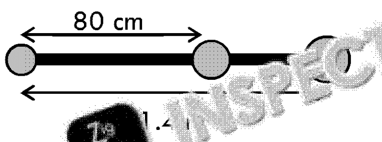
In an art installation called *Life in Balance*, three metal balls are arranged along a light rod, which balances on a single point.

The first 320 g ball is placed at the end of the rod.

The second 680 g ball is placed 80 cm from the end of the rod.

The third 910 g ball is placed 1.4 m from the end of the rod.

Calculate where the point that the rod balances on is.



The rod will balance on its centre of

The centre of mass is given by

$$m_{total}d_{CoM}=m_1d_1+m_2d_2+m_3d_3$$

$$d_{CoM}=\frac{m_1d_1+m_2d_2+m_3d_3}{m_{total}}$$

$$d_{CoM}=\frac{m_1d_1+m_2d_2+m_3d_3}{m_1m_2m_3}$$

$$d\;=\frac{(0.32\;[illegible])+0.68\times0.8+0.91\times1}{0.32+0.68+0.91}$$

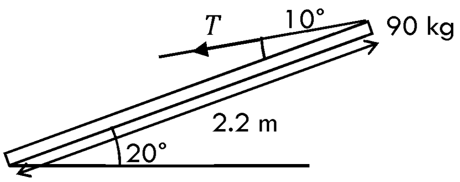
d_{CoM}=0.95 m along the rod (from le

Example 4

In an art installation called *Balancing the Books*, an enormous 2.2 m tall, 90 kg book is held at an angle of 20° from the ground by a rope, which is at 10° to the book at its top.

A side view of the balancing book is shown below.

Calculate the tension in the rope.



It is easiest to solve this problem by

the book itself. The pivot in this case

the ground.

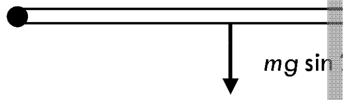
There are two forces acting on the b

itself, and the tension in the rope.

The weight acts down at the book's

the pivot, and the tension acts up at

the pivot.



For the moments to balance

$$T\sin 10\times2.2=90\times9.81\times\cos 20\times1.$$

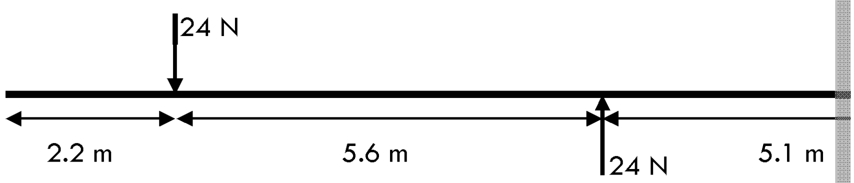
$$T=\frac{90\times9.81\times\cos 20\times1.1}{\sin 10\times2.2}$$

T= 2400 N

QUESTIONS

Setting off

1. Two forces act on a bar, as shown below.



Calculate the moment of the forces and state where the centre of the bar's res

2. Lucie and Elvira are balanced on a see-saw.

Lucie is 1.1 m from the see-saw's pivot and has a mass of 50 kg.

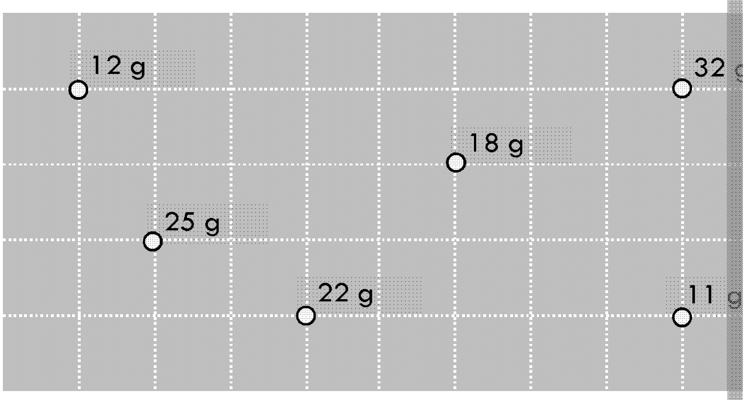
Elvira has a mass of 65 kg.

Calculate Elvira's distance from the see-saw.

Speeding up

3. Several stones are placed on a large flat sheet, with their positions and masse

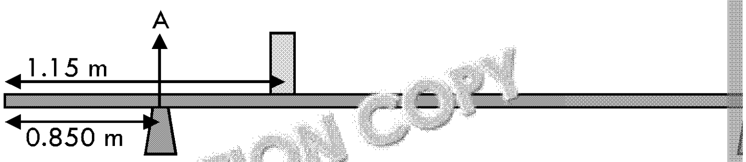
The sheet itself is uniform, has a mass of 45 g and has 1 cm markings along its



Calculate the centre of mass of the stones and sheet.

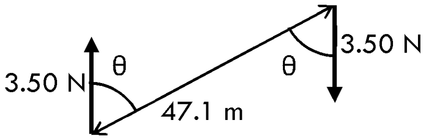
4. A plank has a mass of 8.10 kg and a total length of 2.60 m.

A 4.30 kg mass is placed 1.15 m from the left end of the plank, between two



Calculate the magnitudes of the normal contact forces labelled A and B.

5. The forces shown below create a moment of 67.0 N m.



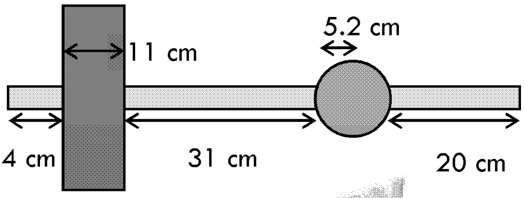
Calculate θ.

Top speed

6. A metal rod has a circular cross section, with a radius of 2.4 cm.

   The metal rod passes through the centres of a metal block with dimensions 11x
   radius of 5.2 cm.

   The density of the rod is 7.9 g cm^{-3}, the density of the block is 8.3 g cm^{-3}, and
   7.4 g cm^{-3}.



   Calculate the centre of mass of the rod, block and sphere.

7. A plank has a mass of [illegible] and a length of 83.0 cm, and rests at an angle of [illegible]. [illegible] along the plank, a rope pulls with a tension of 24 N at an angle of 18.1°. A second rope 62.1 cm along the plank pulls with tension *T* at an angle of 77.1°.

   Calculate *T*.

6. DOING WORK

2: MECHANICS

BACKGROUND

When a force causes a displacement, it does **work**.

In this example, a force F is pulling on the block at an angle ϑ – but becaus
block's weight), the block is only displaced horizontally by s.

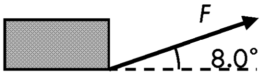
$$W = Fs\cos\vartheta$$

The work done by a force is equal to the **energy transferred.**

Power is the rate of doing work or rate of transferring energy, and is given

$$P = \frac{\Delta W}{\Delta t} = Fv$$

Example 1
A rope pulls a crate, at an angle of 8.0° from horizontal.
The rope is pulled by a winch that uses 5400 J to drag the box 2.2 m across the ground.
Calculate the force that the winch pulls with.



The box is pulled along the ground,
the force acting horizontally is impo

$$W=Fs\cos\vartheta$$

which rearranges to

$$F=\frac{W}{s\cos\vartheta}$$

$$F=\frac{5400}{2.2\times\cos 8.0}$$

$$\boldsymbol{F=2500}\ \textbf{N}$$

EXAM TI
Remembe
60 kW =
1.5 kN = 1
The excep
SI unit for

Example 2

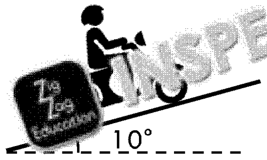
A scooter's engine can provide a maximum power of 60 kW.

The scooter and driver have a combined mass of 200 kg.

The frictional force acting on the scooter along level ground at its maximum speed is 1.5 kN.

a) Calculate the maximum speed of the scooter.

b) The scooter climbs up a hill at a 10° incline.
   Calculate the maximum speed of the scooter up the hill.



a) The scooter has to overcome 1.
constant velocity

$P=Fv$

$$v=\frac{P}{F}=\frac{60\times10^3}{1.5\times10^3}$$

$\boldsymbol{v=40}$ **m s^{-1}**

b) On an incline, the scooter has to
forces as on level ground, but a

$F=F_{resistive}+mg\sin\vartheta$

$F=1.5\times10^3+200\times9.81\times\sin 10$

$F=1840$ N

$$v=\frac{P}{F}=\frac{60\times10^3}{1840}$$

$\boldsymbol{v=33}$ **m s^{-1}**

QUESTIONS

Setting off

1. A motor uses 930 J to pull a rope attached to a small crate. The crate moves
   A constant resistive force of 120 N acts against the crate's motion.
   Calculate the tension in the rope.

2. A speedboat's engine provides a power of 52 kW, and is 85 % efficient.
   At a speed v, 3.6 kN of resistive forces act against the speedboat.
   Calculate the speed v.

Speeding up

3. A frog's legs can do 0.668 J of work, providing 3.02 N of force.
   If the frog jumps at an angle of 5.11 ° to horizontal, calculate the distance cove

4. A 9450 kg truck travels down an incline. The truck's engine provides a total p
   and is 53.8 % efficient. 48.1 kN of resistive forces act against the truck.
   The truck travels at 44 m s^{-1}. Calculate the angle of the slope.

Top speed

5. A stone block slides down an 18.[illegible] [illegible]. The block experiences a friction of
   The stone block slides [illegible] distance of 3.55 m and is initially at rest.
   The stone block [illegible] mass of 43.3 kg.
   Calculate [illegible] final velocity of the stone block.

6. A rocket has a mass of 2.06 Gg. The rocket's engine provides a power of 56
   The rocket is launched at an angle of 11.6° to vertical. This is caused by wind
   resistance, which produces a downwards force of 42.6 kN at angle of 38.8° t

   Assuming that the rocket's speed and power and all relevant forces are consta
   after take-off, calculate the time it takes for the rocket to leave the atmosphe
   10 000 km above the surface of Earth.

7. ENERGY AND EFFIC

2: MECHANICS

BACKGROUND

Energy is always conserved, meaning that it can't be created or destroyed. Energy can be converted between different forms, though, either usefully or

When a process dissipates energy into non-useful forms of energy such as th we can calculate the efficiency of that process using

$$\text{efficiency} = \frac{\text{useful output power}}{\text{input power}} = \frac{\text{useful output energy}}{\text{input energy}}$$

Power and energy are linked by

$$P = Et$$

Kinetic energy, the energy stored by an object due to its motion, is given by

$$E_k = ½\ mv^2$$

Gravitational potential energy, the energy stored by an object because of its place in a uniform gravitational field, is given by

$$\Delta E_p = mg\Delta h$$

E
E
p
A
c
s

P = pov
E = ene
t = time
E_k = kin
E_p = gra
m = ma
v = spe
Δh = ch
g = gra

Example 1
A ball with a mass of 350 g is fired directly upwards at a speed of 5 m s^{-1}. Calculate the maximum height the ball reaches.

At the highest point of the ball's path, al
has been transferred to gravitational po

$$\Delta E_p = \Delta E_k$$

which gives

$$mg\Delta h = ½ m$$

Rearranging

$$\Delta h=\frac{v^2}{2g}$$

Substituting in values

$$\Delta h=\frac{5^2}{2\times 9.81}$$

$$\Delta h = 1.27 \text{ m}$$

Example 2
A car's engine provides 60 kW total power, which produces a force of 5 kN.
The engine is only 80 % efficient. Calculate the velocity of the car.

The useful power provided by the e

$$efficiency=\frac{useful\ o}{inpu}$$

Rearranging
useful power output = efficiency x input

Substituting in values
useful power output = 0.80 x 60 x 10^3
useful power output = 48 x 10^3 W

which gives us P for
$P = Fv$

$$v=\frac{P}{F}$$

$$v=\frac{48\times 10^3}{5\times 10^3}$$

v= 9.6 m s^{-1}

QUESTIONS

Setting off

1. Two light bulbs each produce the same amount of light, 4.8 W.

   One of the light bulbs is a traditional incandescent bulb, which is 3.4 % efficie

   The other light bulb is an energy-saving LED bulb, which is 8.1 % efficient.

   Calculate how much more power the traditional bulb uses, compared to the LE

2. A waterfall is 38 m high. At the top of the waterfall, the water travels at 5.4

   Assuming there are no energy losses, use conservation of energy to calculate t
   bottom of the waterfall.

Speeding up

3. A boulder with a mass of 26[illegible] kg rolls down a 72.9 m hill, and then up a smal

   When the boulder reaches the top of the smaller hill, it bangs into a smaller ro
   comes to a stop, sending the rock rolling away at 7.14 m s^{-1}.

   93.8 % of the kinetic energy of the boulder is dissipated in the collision.

   Calculate the height of the smaller hill.

4. An engine uses a constant power of 4.13 kW for 164 s to accelerate a 919 k

   The engine is 55.1 % efficient.

   Calculate the final speed of the car.

Top speed

5. A motor uses 0.94 J to drag a 13 g block up a slope by a horizontal distance
   distance of 22 cm. A constant frictional force of 0.55 N acts on the block.

   Calculate the efficiency of the motor.

6. A spring has a spring constant of 12 N m^{-1} and is compressed by 6.1 cm.

   Elastic potential energy is given by $E_e = \frac{1}{2}kx^2$ where k is the spring constant a

   The energy in the spring is used to accelerate a 34 g hammer, which raises by
   bearing.

   93 % of the energy in the spring is used to accelerate the hammer, and 82 %
   transferred to the ball bearing.

   Calculate the speed of the ball bearing after it is struck by the hammer.

7. Wind enters a wind turbine at a rate of 8.75 kg m^{-2} s^{-1}, and leaves at
   5.59 kg m^{-2} s^{-1}.

   Over a period of 60.0 s, 18 % of the wind's kinetic energy is converted to he
   wind turbine's axels.

   All other energy is converted to electrical energy.

   A single one of the wind turbine's blades has a length of 35.4 m. The density
   1.225 kg m^{-3}.

   Calculate the efficiency of the wind turbine.

8. DENSITY AND PRES

4: MATERIALS

BACKGROUND

Every material has a density, which is how much mass there is in a given volume.

$$\rho = \frac{m}{V}$$

The pressure exerted on a surface due to a force is given by

$$p = \frac{F}{A}$$

The pressure in a fluid is given by

$$p = h\rho g$$

An object in a fluid will experience an upthrust, due to the difference in pressure be
surfaces of the object.

The upthrust an object experiences in a fluid is equal to the **weight of fluid displac**
The upthrust is also equal to the difference in pressure between the top and bottom

How easily an object falls through a fluid is dependent on the fluid's viscosity. This i

As a spherical object falls through a fluid, the drag forces acting on it are given by

$$F_{drag} = 6\pi r\eta v$$

EXAM TIP
The symbol for pressure, p, and the symbol for density, ρ, can look very similar!
Double-check which one you're working with when looking at an equation.

Example 1

A triangular prism sits on a table.

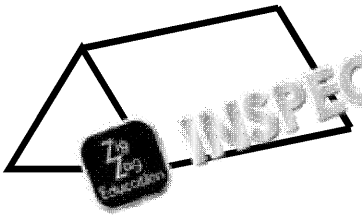
The triangular cross section of the prism has a base of 36.0 mm, and a height of 28.0 mm.

The length of the prism is 209 mm.

The prism has a density of 1.33 g cm^{-3}.

a) Calculate the mass of the prism.

b) The prism sits with the base of its triangle and its length on the table. Calculate the pressure exerted on the table by the prism.



a) First, find the volume of the pris

The cross section of the prism ha

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{area} = \frac{1}{2} \times 36.0 \times 10^{-3} \times 28.0$$

$$\text{area} = 5.04 \times 10^{-4}\ \text{m}^2$$

The volume of the prism is given

volume = cross-sectional area ×

volume = 5.04 × 10^{-4} × 209 ×

volume = 1.053 × 10^{-4} m^3

The density is given in g cm^{-3} –

useful in the form kg m^{-3}.

$$1.33\ \text{g cm}^{-3} = 1.33 \times 10^{-3} \div ($$

The density of the prism is given

$$\rho = \frac{m}{V}$$

which rearranges to

$$m = \rho V$$

$$m = 1330 \times 1.053 \times 10^{-4}$$

$$\boldsymbol{m = 0.140\ \text{kg}}$$

(0.1402 kg to 4 significant figures)

b) The pressure is the force exerte

which is in contact with the table

Here, the only force acting is th

$$A = \text{base} \times \text{length}$$

$$A = 36.0 \times 10^{-3} \times 209 \times 10^{-3}$$

$$A = 7.524 \times 10^{-3}\ \text{m}^2$$

$$F = mg$$

$$F = 0.1402 \times 9.81$$

$$F = 1.375\ \text{N}$$

$$p = \frac{F}{A}$$

$$p = \frac{1.375}{7.524 \times 10^{-3}}$$

p = 183 Pa (or N m^{-2})

Example 2

A beach ball is halfway submerged into water.

The beach ball has a radius of 18.0 cm. Water has a density of 997 kg m^{-3}.

a) Calculate the difference in pressure between the bottom and top of the ball.

b) Calculate the upthrust on the ball.

a) The top of the ball is only affe
pressure. The value of atmosph
here, but it is important that it is
size of the beach ball.
Atmospheric pressure adds to th

The pressure due to the depth o
$p=h\rho g$

Here, the depth of water, h, is g
ball.

$p = 0.180 \times 997 \times 9.81$

$p = 1760$ Pa (or N m^{-2})

b) The upthrust on the ball is equal
displaced by the ball.
This means that the upthrust is gi

$\text{upthrust} = \frac{1}{2} \times \text{volume of ball} \times$

Only half the ball's volume is us
submerged.

$\text{upthrust} = \frac{1}{2} \times \frac{4}{3}\pi r^3 \times \rho \times g$

$\text{upthrust} = \frac{2}{3}\pi \times 0.180^3 \times 997$

$\text{upthrust} = 119 \text{ N}$

QUESTIONS

Setting off

1. A metal sphere has a density of 4.506 g cm^{-3} and a mass of 0.6711 kg.

   Calculate the radius of the sphere.

2. A wooden cube has a side length of 3.00 cm and sits on a table.

   An upward force of 0.0930 N acts on the cube, so that the pressure of the cub

   Calculate the mass of the wooden cube.

3. A plastic cylinder has a cross-sectional area of [illegible] 0 [illegible]^2. It is submerged in me
   13.6 g cm^{-3}, and experiences an upthrust of [illegible] N.

   Calculate the length of the cylinder.

Speeding up

4. A block of balsam wood floats in the Dead Sea.

   The block has a width of 1.34 m, a length of 4.66 m, and a height of 0.882 m.

   x m of the block's height is submerged.

   The block floats at a constant depth.

   Water in the Dead Sea has a density of 1.24 g cm^{-3}.

   Balsam wood has a density of 160 kg m^{-3}.

   Calculate the value of *x*.

5. A steel ball bearing is dropped through a thick syrup, and travels at a termina

   The ball bearing has a diameter of 2.30 cm. The syrup has a density of 1.41

   Calculate the viscosity of the syrup.

Top speed

6. Saturn's moon Titan has lakes made out of liquid methane.

   A sphere is entirely submerged into one of these lakes and slowly sinks at a co
   0.00832 m s^{-2}.

   The sphere has a radius of 2.04 m, and has a spherical hollow cavity at its cen

   Titan has a gravitational field strength of 1.35 m s^{-2}.

   Liquid methane has a density of 0.656 g cm^{-3}.

   Calculate the density of the sphere.

7. A pipe stands on its end. The pipe has an outer radius of 4.10 cm and a leng

   The pipe has a density of [illegible].

   The pipe is pushed down with a force of 11.5 N, so that it exerts a pressure of

   Calculate the inner radius of the pipe.

9. MATERIALS

4: MATERIALS

BACKGROUND

One of the most important characteristics of a material is how much force is r
and how much force it can withstand before breaking.

Hooke's law describes the extension of an object

$$F = k\Delta L$$

Elastic strain energy, the energy stored by an object due to its extension is g

$$E_e = \tfrac{1}{2} F\Delta L = \tfrac{1}{2} k(\Delta L)^2$$

The stress caused by an extension is given by

$$stress = \frac{F}{A}$$

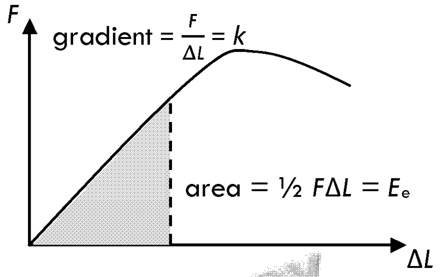
and the strain by

$$strain = \frac{\Delta L}{L}$$

The Young modulus is a property of each material, and is independent of a

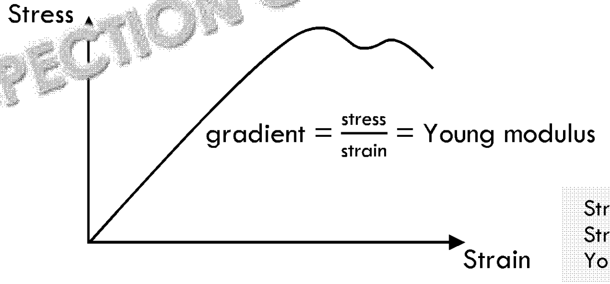
$$\text{Young modulus} = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{FL}{A\Delta L}$$

For a force–extension graph, the gradient gives the stiffness and the area u
energy stored.



F = fo
k = sti
L = ler
E_e = el
A = cr

For a stress–strain graph, the gradient gives the Young modulus.

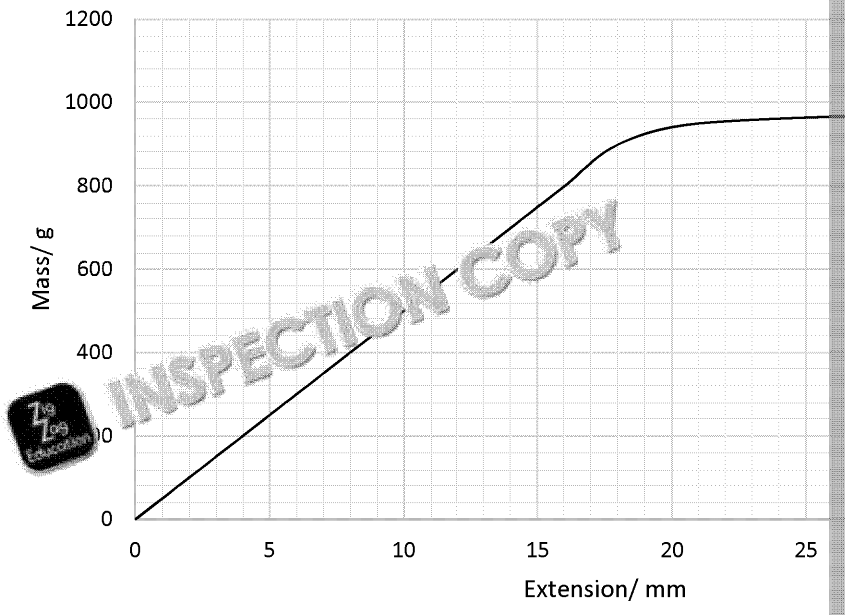


Stress
Strain
Young

EXAM TIP
Young moduli tend to be ve
units such as MPa (10^6 Pa) o
Some materials such as diam
can be measured in TPa (10

Example 1

Masses are hung from a wire and the extension is measured, producing the
below.



a) Calculate the stiffness of the wire.

b) Calculate the work done on the wire before it reaches its limit of propor

a) The stiffness of a wire can be determined using the equation

$F=k\Delta L$

Rearranging, $k=\frac{F}{\Delta L}$

However, the graph gives mass instead of force, and the extension is in

Reading from the graph

extension, ΔL = 18 mm = 0.018 m

mass = 900 g

which gives $F=0.9\times9.81=8.83$ N

so

$k=\frac{8.83}{0.018}$

$k=490$ N m^{-1}

EXA
Thes
any
the g
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maki
easi

b) The work done in stretching a wire is given by

$E_e=\frac{1}{2}F\Delta L$

Here, we're interested in the wire's limit of proportionality: the portion o
curve.

As before,

extension, ΔL = 18 mm = 0.018 m

mass = 900 g and $F=0.9\times9.81=8.83$ N

$E_e=\frac{1}{2}\times8.83\times0.018$

$E_e=0.079$ J

EX
Ano
calc
E$_e$=
fro

Example 2

An iron bar has a Young modulus of 190 GPa. The iron bar has a square cross section, where this square has side length 0.80 cm. A force of 65 kN is applied to the iron bar, and it compresses by 4.0 mm.

a) Calculate the original length of the bar.

b) Calculate the strain of the bar when the force is applied.

a) The Young modulus of a materia

$$Young\ modulus = \frac{stress}{strain} = \frac{FL}{A\Delta L}$$

Rearranging for the original leng

$$L = \frac{Young\ modulus \times A\Delta L}{F}$$

$$L = \frac{190\times 10^{9}\times\left(0.80\times 10^{-2}\right)^{2}\times 4.0\times 10^{-3}}{65\times 10^{3}}$$

L = 0.75 m

b) Strain is given by

$$strain = \frac{\Delta l}{L}$$

or

$$strain = \frac{Young\ modulus}{stress} = \frac{Young\ modulus}{F/A}$$

Hint: This example uses the second wrong with using the first, using the out for rounding errors!

$$strain = \frac{190\times 10^{9}}{65\times 10^{3}\div\left(0.80\times 10^{-2}\right)^{2}}$$

$strain$ = 190

Remember! Strain is unitless.

QUESTIONS

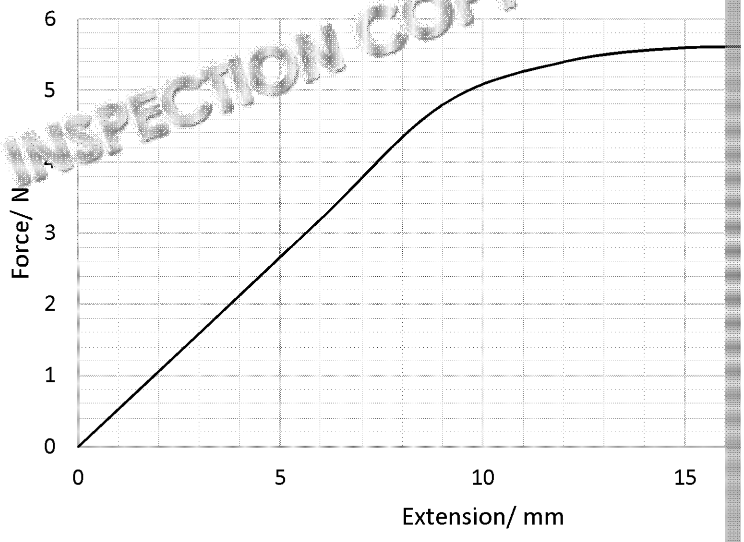
Setting off

1. A spring has a spring constant of 405 N m^{-1}. A force of 23.1 N compresses th
   Calculate the change in length of the spring.

2. A nylon wire has a Young modulus of 2.38 GPa, and is extended from 38.1 c
   Calculate the stress in the nylon wire.

Speeding up

3. A wire is extended until it breaks, with the force–extension graph for the wire

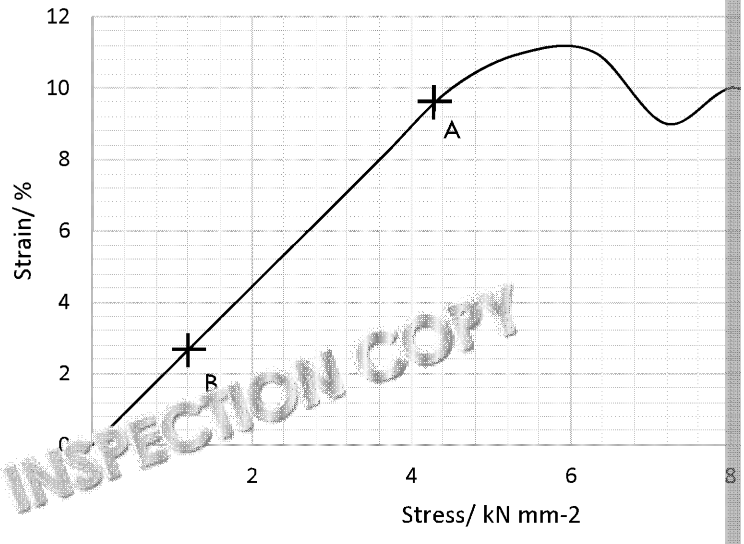


   a) Calculate the work done on the wire before it breaks.
   b) Calculate the stiffness of the wire up until its limit of proportionality.

4. A carbon nanotube has a Young modulus of 1380 GPa, and a radius of 4.14
   A force extends the carbon nanotube from a length of 8.00 cm to a length of
   Calculate the magnitude of the force on the carbon nanotube. Model the car

Top speed

5. The stress–strain graph below is for a wire.



a) Calculate the Young modulus of the wire.

b) The extension of the wire at A is 3.0 cm. Calculate the original length of th

c) The force exerted on the wire at B is 840 N. Calculate the cross-sectional

6. A wire has a Young modulus of 321 GPa, an original length of 42.2 cm and a circumference of 0.888 mm. Calculate the stiffness, ***k***, of the wire.

10. MOMENTUM

6: FURTHER MECHANICS

BACKGROUND

Momentum is a property of all moving objects.
Momentum is a vector, given by

momentum = mass × velocity

Momentum is always **conserved** – the momentum of a system of objects or p collision as afterwards.

Elastic collisions are ones in which kinetic energy is the same before and af
Inelastic collisions are ones in which kinetic energy is dissipated during the

For an elastic collision
$K_{E,before}=K_{E,after}$

For an i
$K_{E,b}$

The force exerted on an object is equal to the **rate of change in momentu**

$$F=\frac{\Delta(mv)}{\Delta t}$$

Impulse is equal to the force exerted over time, or the change in momentu

$$F\Delta t=\Delta(mv)$$

Example 1
A 70 kg student stands stationary on a 3.0 kg skateboard.
The student jumps off the skateboard to the right-hand side at a velocity of 0.45 m s⁻¹.
Calculate the velocity of the skateboard.

Momentum is conserved – this mean before and after the student jumps same.

Before the jump, the student and sk their combined momentum is zero.
This means that the combined mome skateboard after the jump must also

The momentum of the student after t
$momentum_{student}=m_{student}v_{student}$
$momentum_{student}=70\times0.45=31.5$ kg

This means that the momentum of the same in the opposite direction, since
$momentum_{total}=momentum_{student}+mom$

$$v_{skateboard}=\frac{momentum_{skateboard}}{m_{skateboard}}$$
$$v_{skateboard}=\frac{-31.5}{3.0}$$
$$v_{skateboard}=-11 \text{ m s}^{-1}$$

Example 2

A rower rows a rowing boat. The oars exert a constant force of 360 N when they are in the water, causing the rowing boat to accelerate from 1.5 m s^{-1} to 2.3 m s^{-1}. The combined mass of the rowing boat and rower is 190 kg. Calculate the amount of time the oars are in the water for.

Use the equation

$F\Delta t = \Delta(mv)$

and rearrange for Δt

$$\Delta t = \frac{\Delta(mv)}{F}$$

$$\Delta t = \frac{190 \times (2.3 - 1.5)}{360}$$

$$\Delta t = 0.42 \text{ s}$$

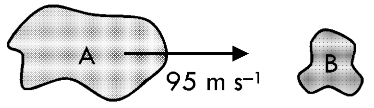
Example 3

Two asteroids collide in the asteroid belt between Mars and Jupiter. Asteroid A has a mass of 5300 kg and is initially moving to the right at a velocity of 95 m s^{-1}. Asteroid B has a mass of 1200 kg and is initially at rest.

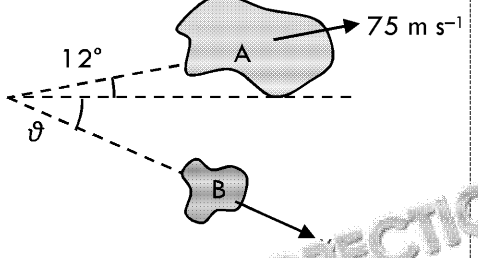
After the asteroids collide, Asteroid A travels at 75 m s^{-1}, at an angle of 12° to its initial path.

Calculate the velocity and direction of travel of Asteroid B after the impact.

Before



After



Momentum must be conserved in bo

Before, the horizontal momentum of

$momentum_{before} = m_A u_A = 5300 \times 9$

This is the combined momentum of b collision.

After the collision, horizontally

$momentum_{before,\,x} = m_A v_{A,\,x} + m_B v_{B,x}$

$$v_{B,x} = \frac{momentum_{before,\,x} - m_A v_{A,x}}{m_B}$$

$(v_{A,x} = v_A \cos \vartheta_A)$

$$v_{B,x} = \frac{504\,000 - 5300 \times 75 \cos 12}{1200}$$

$v_{B,x} = 96.0 \text{ m s}^{-1}$

The vertical momentum of the astero zero, so

$m_A v_{A,y} = -m_B v_{B,y}$

$$v_{B,y} = \frac{-m_A v_{A,y}}{m_B}$$

$v_{A,y} = v \sin \vartheta_A$

$$v_{B,y} = \frac{-5300 \times 75 \sin 12}{1200}$$

$v_{B,y} = -68.9 \text{ m s}^{-1}$

Combining the horizontal and vertic magnitude of

$$v_B = \sqrt{v_{B,x}^2 + v_{B,y}^2} = \sqrt{96.0^2 + 68.9^2}$$

$v_B = 120 \text{ m s}^{-1}$

$$\vartheta = \tan^{-1} \frac{v_{B,y}}{v_{B,x}} = \tan^{-1} \frac{-68.9}{96.0}$$

$\vartheta = -36°$

EXAM TIP

As in previous sheets, u is the initial velocity and v is the final velocity. Subscripts such as A, B, x and y help keep track of which asteroid and which direction are currently being used.

Once we have to consider multiple objects and multiple directions, it can get a lot more complicated keeping track of everything!

QUESTIONS

Setting off

1. A 4.5 g paper boat travels over a pond at a steady speed of 0.068 m s^{-1}. A penny is dropped into the paper boat from directly above.

   Calculate the velocity of the paper boat after the penny has been dropped i

2. A space shuttle fires its thrusters for 135 s, providing a constant force of 64.2

   The space shuttle has a mass of 20.3×10^{6} kg.

   Calculate the change in speed of the space shuttle.

Speeding up

3. A mine cart travels at an initial speed of [illegible]00 m s^{-1}.

   Sand falls out of the mine cart at a steady rate of 280 g s^{-1}.

   The initial mass of the cart is 31.5 kg.

   Calculate the speed of the cart after 1 minute.

4. Two balls roll towards each other. Ball A has a mass of 236 g and ball B has

   Initially ball A is travelling to the right at 2.10 m s^{-1} and ball B is travelling to

   The two balls collide head on, and ball A rolls back to the left at 0.810 m s^{-1}.

   a) Calculate the velocity of ball B after the collision.

   b) Show that this collision is inelastic.

Top speed

5. A tennis ball is dropped from a height of 1.25 m.

   The ball bounces on the floor, and is in contact with the floor for 0.0680 s, bef of 1.08 m.

   The tennis ball has a mass of 58.5 g.

   Calculate the force exerted on the floor by the tennis ball.

6. Two curling stones slide across an ice floor and collide.

   The curling stones before and after the collision are shown below, with their sp path.

   Curling stone A has a mass of 19.[illegible] kg, curling stone B has a mass of 17.4 kg.

   Calculate v_B and θ_B.

   Show that this collision is inelastic.

7. Two cars are involved in a collision.

   Car A has a mass of 2350 kg and is stopped at a red light.

   Car B has a mass of 1870 kg and goes into the back of Car A. After collidin together.

   The brakes of both cars together provide a total force of 6630 N.

   Calculate the initial velocity of car B.

11. CIRCULAR MOTI

6: FURTHER MECHANICS

BACKGROUND

For an object to travel in a circular path, the object must accelerate towards path. This acceleration is caused by a **centripetal force** towards the centre c

Angular speed is a measure of how frequently an object moves through a ci

$$\omega=\frac{v}{r}=2\pi f$$

and

$$f=\frac{1}{T}$$

EXA
The p
circu
for th
circle
positi

The acceleration towards the centre of circular motion is given by

$$a=\frac{v^2}{r}=\omega^2 r$$

so that the centripetal force is given by

$$F=\frac{mv^2}{r}=m\omega^2 r$$

Example 1
A fairground carousel takes 15 s to make a complete revolution.

a) Calculate the angular speed of the carousel.

b) The carousel has a diameter of 18 m.
Calculate the linear speed of the carousel.

a) For the angular speed, use

$$\omega=2$$

But frequency, f, hasn't been gi
Frequency is given by

$$f=$$

so

$$\omega=\frac{2\pi}{T}$$

$$\omega=\frac{2\pi}{15}$$

$\boldsymbol{\omega=0.419}$ **s**

For the linear speed, use

$$\omega=$$

which rearranges to

$v=\omega r$

The question gives the diamete diameter is double the radius.

$v=0.419\times9$

$\boldsymbol{v=3.8}$ **m s^{-1}**

Example 2
A car with a mass of 800 kg travels over a hill.
The hill is a section of a circle with a radius of 30 m.
Calculate the maximum speed the car can travel at to not lose contact with the road.

For the car not to lose contact at the normal reaction force on the car mus equal to the centripetal force on the

In this case, the normal reaction forc and

$F_{centripetal}=\frac{mv^2}{r}$

so

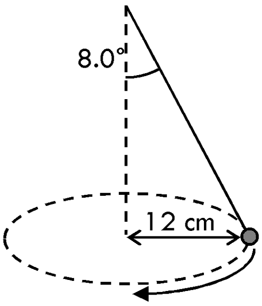
$mg=\frac{mv^2}{r}$

$v=\sqrt{gr}$

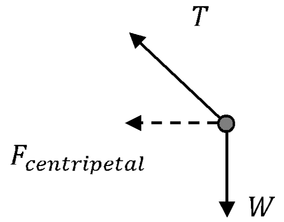
$v=\sqrt{9.81\times30}$

$v=17$ m s^{-1}

Example 3
A conker is spun around on a string, at an angle of 8.0°.
The conker has a mass of 60 g and moves in a circle with a constant radius of 12 cm.
Calculate the angular speed of the conker.



First, draw a free body diagram of the conker.



The conker moves in a circle with a constant radius, so the vertical forces balance and the horizontal forces produce only centripetal acceleratio

This means that the weight of the co vertical component of the tension

$W=T\cos\vartheta$

$mg=T\cos$

and the centripetal force comes fror of the tension

$F_{centripetal}=T\,\mathrm{si}$

$m\omega^2 r=T\,\mathrm{sin}$

Dividing one equation by the other

$\frac{m\omega^2 r}{mg}=\frac{T\sin\vartheta}{T\cos\vartheta}$

$\frac{\omega^2 r}{g}=\tan\vartheta$

Rearranging

$\omega=\sqrt{\frac{g\tan\theta}{r}}$

$\omega=\sqrt{\frac{9.81\tan 8.0}{0.12}}$

$\omega=3.4$ rad s^{-1}

EXA
This
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You
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QUESTIONS

Setting off

1. A record on a record player makes 45 revolutions per minute, and has a radi
   a) Calculate the angular speed of the record.
   b) Calculate the linear speed of the outer edge of the record.

2. A fairground ride consists of a hollow cylinder with a radius of 5.5 m which ro causing its occupants to stick to the wall.

   The ride produces a centripetal acceleration of 3 g.

   The ride runs for 18 s.

   Calculate the number of rotations the ride makes while it runs.

Speeding up

3. A car sits on a banked road, driving around the corner.

   The car travels [illegible] km h^{-1} and the road's bend is an arc of a circle with a

   Calculate the angle of the bank of the road.

4. Europa is one of the moons of Jupiter.

   Europa orbits Jupiter at a radius of 671 Mm, and has a mass of 4.80 x 10^{22} k

   Jupiter has a mass of 1.90 x 10^{27} kg.

   Gravitational force is given by

$$F=\frac{Gm_1m_2}{r^2}$$

   Calculate the time it takes for Europa to orbit Jupiter.

Top speed

5. A boat rides around the edge of a whirlpool with a radius of 23.4 m, travelli current of water.

   The sides of the whirlpool make an angle of 62.1 ° to the horizontal.

   The water at the edge of the whirlpool travels at 24.5 m s^{-1}.

   The boat has a mass of 3220 kg and the boat's engine can provide a force o

   Calculate the power that is needed from the boat's engine for the boat to not

6. A roller coaster contains a vertical loop which is a perfect circle with a radius

   Calculate the minimum speed that the roller coaster needs to enter the vertical be able to sit on the roller coaster without falling out of their seat, purely from experienced.

   Hint: Consider the forces on the passenger and the energy changes the carriage expe

12. SIMPLE HARMONIC

13: OSCILLATIONS

BACKGROUND

Simple harmonic motion (SHM) occurs in a system where a restoring force re
the particle or object to return to a central equilibrium point.

The condition for simple harmonic motion is that acceleration is proportional
opposite direction.

$$a \propto -x$$

$$a = -\omega^2 x$$

The displacement of an object undergoing simple harmonic motion is given b

$$x = A \cos\omega t$$

The velocity of an object undergoing simple harmonic motion is given by

$$v = \pm\omega\sqrt{A^2 - x^2}$$

An object undergoing SHM has a maximum speed as it passes through its e
maximum acceleration at its greatest displacement

$$\text{Maximum speed} = \omega A$$

$$\text{Maximum acceleration} = \omega^2 A$$

Example 1

A buoy on the surface of a still lake bobs up and down with simple harmonic motion.
The buoy's motion has an amplitude of 2.5 cm and a maximum speed of 0.080 m s^{-1}.

a) Calculate the angular frequency of the buoy.

b) Calculate the maximum acceleration of the buoy.

c) State at which point in the buoy's oscillation that it has maximum speed and maximum acceleration.

a) The angular velocity of the buo
Maximum spe

which rearranges to

$$\omega = \frac{\text{Maximum speed}}{A}$$

$$\omega = \frac{0.080}{2.5 \times 10^{-2}}$$

$$\omega = 3.2 \text{ rad } s^{-1}$$

b) The maximum acceleration of th
Maximum acceleration = $\omega^2 A$
Maximum acceleration = 3.2^2 ×
Maximum acceleration = 0.26

c) The buoy's maximum speed occ
through its central equilibrium p
The buoy's maximum accelerati
changes direction, when its disp
amplitude.

Example 2

A pendulum swings with simple harmonic motion.

The pendulum has an amplitude of 6.0 cm, and an angular frequency of $\frac{4}{3}\pi$.

a) Calculate the displacement of the pendulum after 10 s.

b) Calculate the speed of the pendulum after 20 s.

a) The pendulum's displacement is

$$x=A\,c$$

$$x=6.0\times10^{-2}\times\cos\left(\frac{4}{3}\pi\times10\right)$$

$x=0.045$ m

b) The pendulum's speed at a give

$$v=\pm\omega\sqrt{}$$

To get this equation in terms of

equation for displacement

$$v=\pm\omega\sqrt{A^2-A^2\cos^2\omega t}$$

$$v=\pm\frac{4}{3}\pi\sqrt{\left(6.0\times10^{-2}\right)^2\times\left(1-\left(c\right.\right.}$$

$v=0.25\ \text{m s}^{-1}$

QUESTIONS

Setting off

1. A particle, moving with simple harmonic motion, has a maximum speed of 1.1 mm.
   a) State the minimum speed of the particle, and where this occurs.
   b) Calculate the angular speed of the particle.

2. An atom moves with simple harmonic motion in an electric field.
   When the atom has a displacement of 220 nm, the acceleration of the atom is
   a) Calculate the linear frequency of the atom.
   b) The maximum acceleration of the atom is [illegible]. Calculate the amplitu

Speeding up

3. A particle moves with simple harmonic motion with an amplitude of 6.20 mm a 15.2 [illegible] Calculate the time after it is initially displaced that its displacem

4. When a particle has a displacement of -3.55 cm, its velocity is 5.67 m s^{-1} and
   Calculate the amplitude of the particle's motion.

Top speed

5. A particle oscillates with a linear frequency of 98.1 Hz and an amplitude of 2
   Calculate the time after the particle is at maximum displacement at which its a

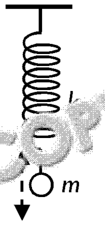
6. A particle moves with simple harmonic motion. The particle's motion has a per amplitude of 22.1 μm.
   Calculate the time at which the velocity of the particle is 1.28 mm s^{-1}.

13. SIMPLE HARMONIC SYSTEMS A

13: OSCILLATIONS

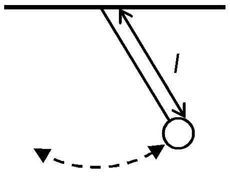
BACKGROUND

For a mass–spring system undergoing simple harmonic motion,



$$T=2\pi\sqrt{\frac{m}{k}}$$

For a simple pendulum,



$$T=2\pi\sqrt{\frac{l}{g}}$$

When a simple harmonic system is damped, energy is dissipated.

A periodic force can be exerted on a simple harmonic system to change the
is called a driving force.

If the driving force is at a specific frequency, called the natural frequency, t
oscillations greatly increases. The natural frequency of a system is a freque
system.

Example 1
A particle on a spring oscillates with simple harmonic motion.
The spring constant of the spring is 4.0 N m^{-1} and the linear frequency of the particle is 0.50 Hz.
Calculate the mass of the particle.

The period of the particle's oscillatio

$$T=2\pi\sqrt{\ }$$

Instead of the particle's period, we h

$$T=\frac{1}{f}=\frac{1}{0.5}=2.0\text{ s}$$

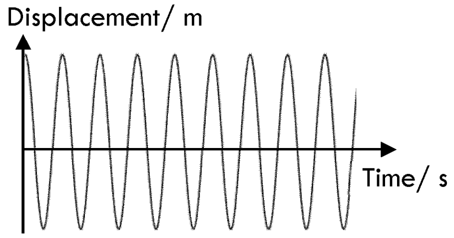
Rearranging the equation above

$$m=\left(\frac{T}{2\pi}\right)^2\times k$$

$$m=\left(\frac{2.0}{2\pi}\right)^2\times 4.0$$

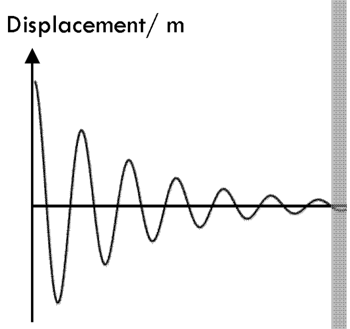
$$m=0.41\text{ kg}$$

Example 2
The graph below shows the displacement of a particle undergoing simple harmonic motion.

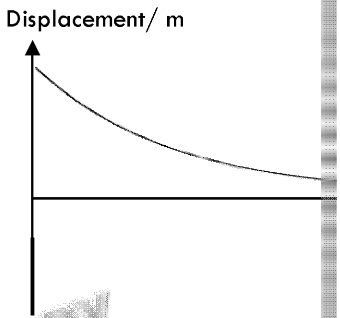


Damping is added to the particle.
Sketch graphs showing the effect on the displacement of the particle for light, heavy and critical damping.

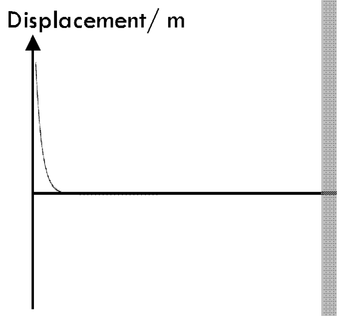
For a system with **light damping**, th
oscillate for several periods, with the
oscillations decreasing:



For **heavy damping**, the system doe
instead slowly returns to its equilibri



For a **critically damped** system, ther
an unchanged period, which returns
position, where it then stays:

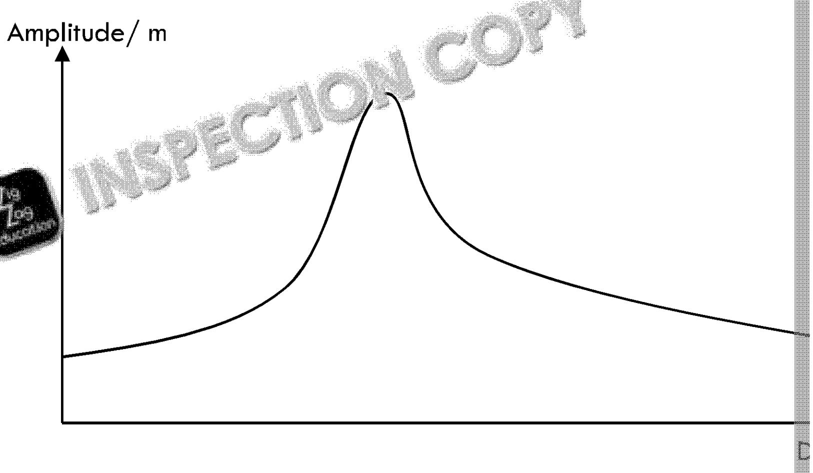


QUESTIONS

Setting off

1. Draw displacement–time graphs for a system which has been:
   a) lightly damped
   b) heavily damped
   c) critically damped
   Suggest an application for each type of damping.

2. The graph below shows the amplitude of a systems oscillation with the freque



Draw what the graph would look like if the system were damped.

Speeding up

3. A pendulum has a frequency of 2.33 Hz.
   Calculate the length of the pendulum.

4. A force of 0.261 N is used to extend a particle on a spring by 0.994 mm.
   The particle is released, and the spring oscillates with an angular speed of 8.
   Calculate the mass of the particle.

5. Two iron balls oscillate with the same frequency.
   One ball is on the end of a 3.64 m pendulum, and the other is attached to a s
   Both the balls have masses of 1.98 kg.
   Calculate the spring constant of the spring.

Top speed

6. By considering the motion of a pendulum, derive the equation for the period o

7. By considering the energy of a mass–spring system undergoing simple harmoni
   potential and total energy, derive the equation for the velocity of a simple har

EXAM-STYLE QUESTI

1 **Figure 1** shows a track that a ball bearing follows.
The ball bearing is released from point **A**, travels to point **B**, which is the
travels to point **C** at the end of the track. At point **C**, the ball bearing le
through the air.

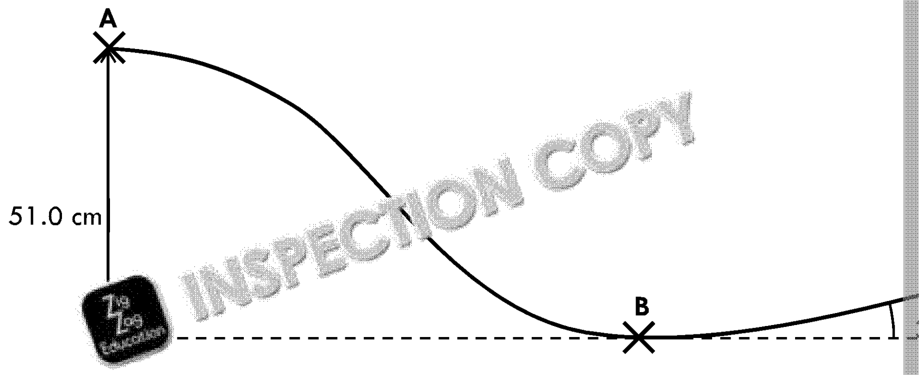


Figure 1

1.1 Sketch a velocity-time graph of the horizontal component of the ball be

1.2 Show the speed of the ball bearing at point **C** is 2.30 m s^{-1}.

1.3 Calculate the horizontal and vertical components of the ball bearing's v

1.4 Calculate the horizontal distance from point **C** that the ball bearing lan

2 At a fair, two bumper cars, A and B, collide.
The total mass of bumper car A and its passengers is 215 kg.

2.1 Before the collision, bumper car A is accelerated from rest by a force o
Calculate the momentum of bumper car A after being accelerated.

2.2 After being accelerated, bumper car A collides with bumper car B, whic
direction at 1.41 m s^{-1}.
After the collision, car A rebounds in the opposite direction at 0.333 m s
0.536 m s^{-1}.
Calculate the mass of bumper car B and its passengers.

2.3 Explain how the safety features such as cushioned bumpers in the bumpe
passengers safe.

3 A spring is extended by adding masses to the end. The data shown in

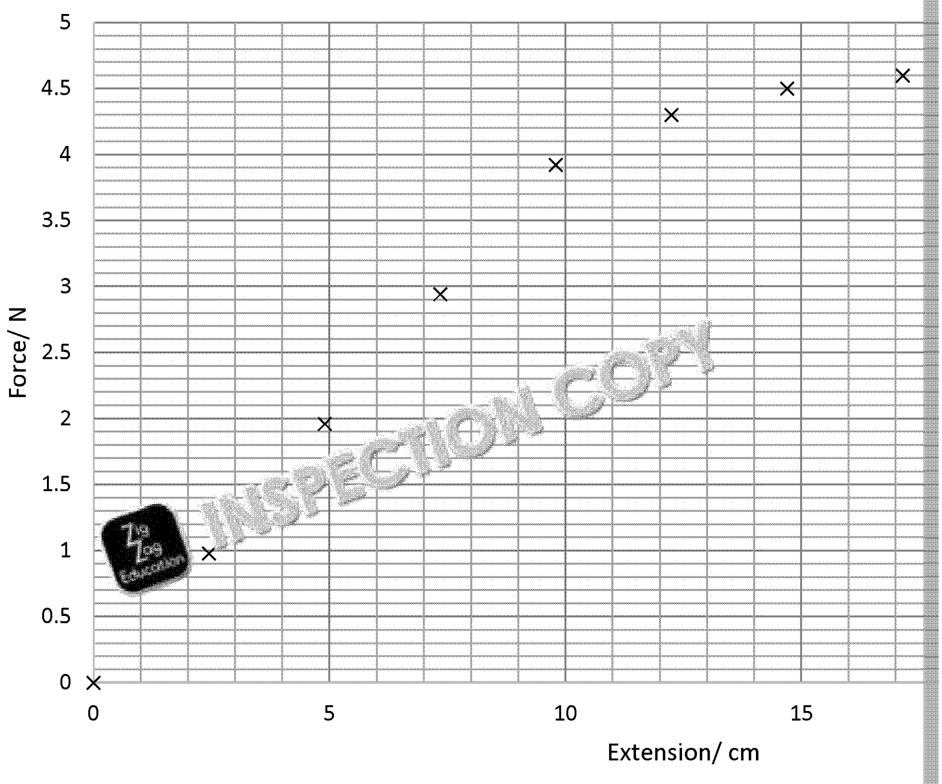


Figure 2

3.1 Describe how the data shown in **Figure 2** could be collected.

3.2 Explain the shape of the graph shown in **Figure 2**.

3.3 Calculate the spring constant of the spring.

3.4 A motor has a power rating of 0.432 W and pulls on the spring for 3.0
The spring extends by 8.71 cm.
Calculate the efficiency of the motor.

4 **Figure 3** shows a roller-coaster cart. The section of track the cart is on i of 40.3° and makes an arc of a circle with a radius of 21.0 m.

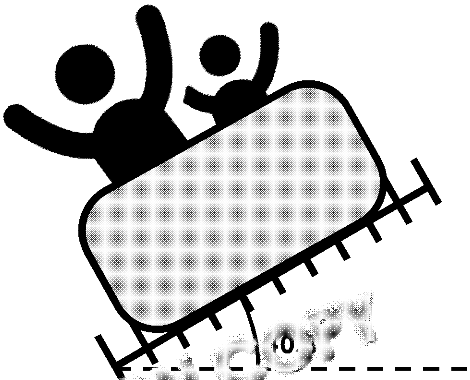


Figure 3

4.1 Comp [illegible] 3 by adding the forces experienced by a passenger i

4.2 The roller coaster is designed so that a passenger with a mass up to 15 at all on the section of the track shown in **Figure 3**.
Calculate the minimum speed the roller-coaster cart must travel around be possible.

4.3 The force provided by the engine for a given velocity is given by

$$F=5.51v^2$$

Calculate the power provided by the engine as it travels around the se **Figure 3**.

APPENDIX – GCSE MEC
REFRESHER

If you studied GCSE Physics, you'll probably be familiar with some aspects o
come across some of the forces in this pack, such as gravity, tension or frictio
combine in different ways. You'll probably have learnt about the different c
displacement, velocity and acceleration. You may have also learnt about m
of energy, and how both are conserved.

The next few pages are a quick summary of what you may have already com
memory and consolidate your knowledge before moving on to the more diffic
not feeling confident, you should first go slowly through this appendix and sp
foundation of knowledge and understanding before building on these topics w

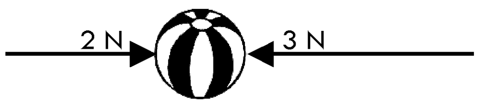
SCALARS AND VECTORS

Measurements can either be **scalars** or **vectors.**

Scalars have magnitude (or size), but not direction.
Vectors have both magnitude and direction.

Scalars	V
Distance	Displ
Speed	Ve
Mass	Acce
Temperature	F
Time	

Imagine two forces pushing on a ball – one 2 N and the other 3 N. This c
but what if the forces were acting in different directions?



In this case, the forces add up to a net force of 1 N! This is why giving c
Here, we would say the forces are + 2 N and – 3 N, giving a total forc
The negative sign means the force is acting to the left – usually positive

MOTION

An object's motion can be described in terms of its displacement, velocity an

Displacement is how far away an object is from a specific point.

Velocity is how quickly an object moves – its rate of change of displacemen

$$v = \frac{\Delta s}{\Delta t}$$

Acceleration is how quickly an object's velocity changes – the rate of chang

$$a = \frac{\Delta v}{\Delta t}$$

FORCES

A force is something that causes an acceleration, changing an object's motio
object's speed or direction.
Since forces are vectors, they can add and balance, cancelling out to zero.

Forces are measured in newtons, given the symbol N.

Some common forces are:

Weight	The force caused by gravity – the attraction between d On Earth, an object with a mass m experiences weight t Earth of $W = mg$ where g is the gravitational field strength of Earth, g =
Tension	A force in a rigid object caused by pulling on the objec
Friction	A force which resists the motion of an object
Normal reaction force	A force at right angles to a surface, which is equal to a a force making contact with the surface

NEWTON'S LAWS

Newton's laws describe forces and their effects on matter.

Newton's first law
An object at rest or travelling at constant velocity will not accelerate or dec

> An arrow fired from a bow only accelerates while the bow string exert
> on the arrow.
> When the arrow leaves the bow, and no force acts on it, the arrow cont
> travel at the same speed it was before.
> The arrow will only slow down due to another force acting in the oppos
> direction, such as air resistance, or when it hits a target – the target ex
> normal contact force on the arrow, bringing it to a stop.

Newton's second law
A force exerted on an object produces an acceleration which is proportiona

$$\text{Force} = \text{mass} \times \text{acceleration}$$

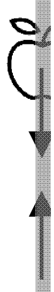
> A space shuttle's rockets exert a force, accelerating the rocket. A spac
> shuttle with half the mass with rockets that can exert the same force wo
> accelerate twice as fast.
>
> For a space shuttle with a mass of 2000 kg accelerating at 5 m s^{-2}, the
> provided by the rockets is
> Force = mass x acceleration = 2000 × 5 = **10 000 N**

Newton's third law

Any force on an object causes the object to exert a force of the same type a direction.

An apple falls to Earth because Earth exerts an attractive force on the apple.
Earth is also pulled up towards the apple, with a force equal to the weight of the apple.

When the apple reaches the ground, it stops accelerating because the normal contact force the apple exerts on the ground is balanced by the normal contact force the ground exerts on the apple.



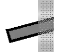
MOMENTS

A lever or gear can transfer a force over a distance, turning a linear force ir
A moment is given by

moment = force × distance

A crowbar is 2 m long. A force of 4 N is applied to one end of the cro

The moment on the other end of the crowbar is
moment = force x distance = 4 x 2 = **8 N m**



MOMENTUM

All moving objects have momentum.

momentum = mass × velocity

Momentum is always conserved in any interaction.

A 60 kg cannon fires a 5 kg cannonball at 12 m s^{-1}.

The momentum of the cannonball is
momentum = mass x velocity = 5 x 12 = 60 kg m s^{-1}

Because momentum is conserved, the cannon must have a momentum of – 60 kg m s^{-1}.

The velocity of the cannon is
velocity = $\frac{momentum}{mass} = \frac{-60}{60} =$ **-1 m s^{-1}**.

ENERGY

Energy is a property which gives an object the ability to do work or exert a force.

Energy is always conserved, so the total energy in a closed system stays the same, from one store to another.

Some common forms of energy are:

Kinetic energy	The store of energy in an object because of its motion. The kinetic energy stored by an object is given by kinetic energy = ½ × mass × ve
Gravitational potential energy	The energy an object stores because of its position in a In a uniform gravitational field, the gravitational potenti object is given by gravitational potential energy = mass × gravitatio
Thermal energy	The energy an object stores due to the vibration of its m
Electrical energy	The energy transferred by an electric current.
Elastic potential energy	The energy stored by an object because of being stretc force. This is the energy stored between particles being stretch together.
Chemical energy	The energy stored by a material due to its chemical pro released via chemical reactions.

These are just some of the forms that energy can be stored or transferred as.

EFFICIENCY

Even though energy is always conserved, energy can dissipate, or be wasted. This the system has been converted into non-useful forms.

The efficiency of a process is a measure of how much energy is transferred into use

The efficiency of a process is given by

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\ \%$$

A light bulb transfers 40 J of energy per second.

8 J of energy is wasted as heat by the light bulb.

This means that 32 J of the energy is used usefully.

The efficiency of the light bulb is

$$\textit{efficiency} = \frac{\textit{useful output energy}}{\textit{total input energy}} \times 100\ \% = \frac{32}{40} \times 100\ \% = \mathbf{80\ \%}$$

POWER

Power is the rate at which energy is transferred by a process.

Power is given by

$$\text{power} = \frac{\text{energy transferred}}{\text{time}}$$

A kettle has a power rating of 350 W and takes 2 minutes to boil.

The energy used by the kettle is given by

energy transferred = power × time = 350 × 120 = **42 000 J (= 42 kJ)**

MATERIALS

The strength of a material can be described in terms of the amount of distortion th
on that object.

The force required to extend or compress an object by a certain amount is given b

force = spring constant × change in length

The elastic potential energy stored in an object due to a force is given by

elastic potential energy = ½ × spring constant × (change i

A spring has a spring constant of $15\ \text{N m}^{-1}$ and is extended by 4.0 cm.

The force exerted on the spring is given by
force = spring constant × change in length = 15 × 0.040 = **0.60 N**

The energy stored in the spring is given by
elastic potential energy = ½ × spring constant × (change in length)^2 = ½ × 1

CIRCULAR MOTION

For an object to travel in a circle, a force must act perpendicular to the object's dir
centre of its circular path.

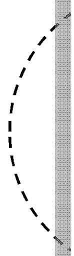
An object travelling in a circle is always accelerating towards the centre of its path,
changing, even if its speed stays the same.

As Earth travels around the Sun, it is attracted by gravity.

Earth has a constant speed around the Sun, but the Sun's gravity causes Earth to accelerate towards the Sun.

This causes Earth to travel in a circle, with the Sun at the centre.

(In reality Earth travels in an ellipse, a sort of stretched out circle. This is because its velocity changes as it travels round the Sun.)



GCSE POP QUIZ

1. State whether the following quantities are scalars or vectors:
   a) Distance
   b) Velocity
   c) Weight
   d) Temperature
   e) Tension
   f) Brightness

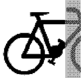
2. Three forces act on a trolley, as shown below.

Calculate the resultant force on the object, and state its direction.

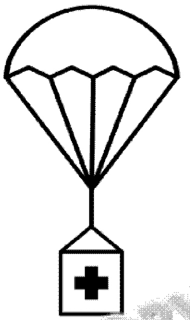
3. 

A sea otter floats 80 m in 140 s.
Calculate the sea otter's speed.

4. A bike is initially travelling at 1.0 m s^{-1} and accelerates at 0.60 m s^{-2}.
How much time does it take the bike to reach 5.0 m s^{-1}?



5. Label each of the forces acting on the parachute and its package below.



6. On Earth, an object has a weight of 34 N. On Jupiter, the same object has a
Calcul[illegible] gravitational field strength on Jupiter.
Use g_{Ear}[illegible].8 N kg^{-1}

7. Explain the following situations, with reference to Newton's laws:
   a) A ball rolling down a slope until it hits a wall, when it stops.
   b) A boat's engine causes the boat to go faster, until it reaches a certain spe

8. A 13 N force pushes a 2.0 kg stone, producing an acceleration of 6.2 m s^{-2}.
Calculate the resistive forces acting on the stone.

9. A lever is pulled with a force of 29 N, producing a moment of 11 N m.
Calculate the distance between the force and the lever's pivot.

10. A 55 kg student throws a 0.40 kg ball away at a speed of 3.2 m s^{-1}.
Describe what happens to the student's motion. Include any relevant numbers, where appropriate.

11. A ball is thrown directly up at a speed of 5.4 m s^{-1}.
Calculate the maximum height reached by the ball.
Use $g=9.81$ m s^{-2}

12. A battery powered motor uses 7.1 J of energy, and 6.8 J of that energy is use
    a) Describe the changes in energy occurring in the motor.
    b) Calculate the efficiency of the motor.

13. A light bulb has a power rating of 40 W.
How much energy does the light bulb use in one hour?

14. A spring with a stiffness of 67 N m^{-1} is compressed by 8.1 cm.
    a) Calculate the force exerted on the spring.
    b) Calculate the energy stored in the spring.

ANSWERS

1. VECTORS

Setting off

1. $v=\sqrt{v_{bike}^2+v_{wind}^2}$

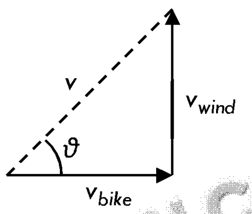
$v=\sqrt{6.5^2+3.2^2}$

$v=7.2$ m s^{-1}

$\vartheta= \tan^{-1} \frac{v_{wind}}{v_{bike}}$

$\vartheta= \tan^{-1} \frac{3.2}{6.5}$

$\vartheta= 26°$



2. $v_x=v\cos\vartheta$

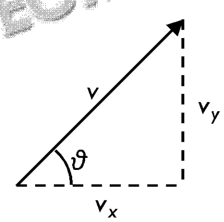
$v_x=54\times$

$v_x=53$

$v_y=v\sin\vartheta$

$v_y=54\times \sin 12$

$v_y=11$ m s^{-1}



Speeding up

3. a) $v=\sqrt{v_{belt}^2+v_{ball}^2}$

$v_{ball}=\sqrt{v^2-v_{belt}^2}$

$v_{ball}=\sqrt{5.21^2-4.30^2}$

$v_{ball}=2.94(2)$ m s^{-1}

b) Using answer from part a):

$\tan\vartheta = \frac{v_{ball}}{v_{belt}}$

$\vartheta= \tan^{-1} \frac{v_{ball}}{v_{belt}}$

$\vartheta= \tan^{-1} \frac{2.942}{4.30}$

$\vartheta=34.4°$

Without using answer from part b):

$\cos\vartheta = \frac{v_{belt}}{v}$

$\vartheta= \cos^{-1} \frac{v_{belt}}{v}$

$\vartheta= \cos^{-1} \frac{4.30}{5.21}$

$\vartheta=34.4°$

4. $s=s_1+s_2+s_3+\ldots+$

$s_x=-3+8-15+2$

$s_x=-8$ squares

$s_y=9+11-4-1$

$s_y=15$ squares

$s=\sqrt{s_x^2+s_y^2}$

$s=\sqrt{(-8)^2+15^2}$

$s=17$ squares

$\vartheta'= \tan^{-1} \frac{s_x}{s_y}$

$\vartheta'= \tan^{-1} \frac{-8}{15}$

$\vartheta'=-28.1°$, or

$\vartheta=332°$ from di

Top speed

5. Forces balance s

and $T_y=W$

$F=T\sin\vartheta$

$W=T\cos\vartheta$

$\frac{T\sin\vartheta}{T\cos\vartheta} = \tan\vartheta$

$\tan\vartheta= \frac{F}{W}$

$\vartheta= \tan^{-1} \frac{3.89}{4.11}$

$\vartheta=43.4°$

6. $F_{A,x}=F_A\cos\vartheta$

$F_{A,x}=243\times \cos 6$

$F_{A,x}=117.8$ N

$F_{A,y}=F_A\sin\vartheta$

$F_{A,y}=243\times \sin 61$

$F_{A,y}=212.5$ N

$F_x=F_{A,x}+F_B$

$F_x=117.8-182$

$F_x=-64.2$ N

$F_y=F_{A,y}+F_C$

$F_y=212.5-322$

$F_y=-109.5$ N

$F=\sqrt{F_x^2+F_y^2}$

$F=\sqrt{(-64.2)^2+(}$

$F=127$ N

$\vartheta'= \tan^{-1} \frac{F_y}{F_x}$

$\vartheta'= \tan^{-1} \frac{-109.5}{-64.2}$

$\vartheta'=59.6°$ from d

$\vartheta=210°$ from du

7. 32.0° from horizontal is

$180-90-32.0=58.0°$ from vertical

Angle of tension from vertical is

$58+13.1=71.1°$

$F_x=F\sin\vartheta$

$F_x=91.0\times\sin 71.1$

$\boldsymbol{F_x=86.1}$ **N**

$F_y=F\cos\vartheta$

$F_y=91.0\times\cos 71.1$

$\boldsymbol{F_y=29.5}$ **N**

2. LINEAR AND PROJECTILE MOTION

Setting off

1. $v=u+at$

$a=\frac{v-u}{t}$

$a=\frac{3.2-14}{12}$

$\boldsymbol{a=-0.9}$ **m s^{-2}**

2. $s=\left(\frac{u+v}{2}\right)t$

$v=\frac{2s}{t}-u$

$v=\frac{2\times 210}{45}-3.7$

$\boldsymbol{v=5.6}$ **m s^{-1}**

3. $v^2=u^2+2as$

$u=\sqrt{v^2-2as}$

$u=\sqrt{6.1^2-2\times 9.81\times 1.6}$

$\boldsymbol{u=2.4}$ **m s^{-1}**

Speeding up

4. $s=ut+\frac{1}{2}at^2$

$\frac{1}{2}at^2+ut-s=0$

$4.5t^2+11t-88=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$t=\frac{-11\pm [illegible] 4\times 88\times 4.5}{2\times 4.5}$

$t=3.4$ m s^{-1} or $t=-5.8$ m s^{-1}

Can't have negative time, so

$\boldsymbol{t=3.4}$ **s**

5. Time of flight fro

$s=ut+\frac{1}{2}at^2$

$\frac{1}{2}at^2+ut-s=0$

$-\frac{1}{2}\times gt^2+10.4\sin$

$-4.905t^2+4.412t$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$t=\frac{-4.412\pm\sqrt{4.4}}{-2}$

$t=-0.3195$ s or t=

Distance from ho

$v=\frac{\Delta s}{\Delta t}$

$\Delta s=v\Delta t$

$\Delta s=10.4\cos 25.1$

$\boldsymbol{\Delta s=11.5}$ **m**

Top speed

6. **Horizontally**

$v=\frac{\Delta s}{\Delta t}$

$t=\frac{\Delta s}{v}$

$t=\frac{31.5}{15.5\ \cos\vartheta}$

Vertically

$s=ut+\frac{1}{2}at^2$

$s=0$

$u+\frac{1}{2}at=0$

$15.5\sin\vartheta-\frac{1}{2}\times 3.7$

$15.5\sin\vartheta-\frac{1}{2}\times 3.7$

$15.5\sin\vartheta=\frac{58.43}{15.5\cos}$

$\sin\vartheta\cos\vartheta=\frac{58.43}{15.5^2}=$

$\sin 2\vartheta=0.4864$

$2\vartheta=\sin^{-1} 0.4864$

$\boldsymbol{\vartheta=14.6°}$

7.

Police, P	**Motorbike, M**
$v_P=$ wanted	$u_M=29.5$ m s^{-1}
$u_P=0$ m s^{-1}	$a_M=0.314$ m s^{-2}
$s_P=s_M$ $a_P=5.15$ m s^{-2}	$t_M=t_P+1.08$

$s=u_M t_M+\frac{1}{2}a_M t_M^2$

$s=u_P t_P+\frac{1}{2}a_P t_P^2=\frac{1}{2}a_p(t_M-1.08)^2=\frac{1}{2}a_p(t_M^2-2.16t_M+1.166)$

$u_M t_M+\frac{1}{2}a_M t_M^2=\frac{1}{2}a_p(t_M^2-2.16t_M+1.166)$

$\left(\frac{1}{2}a_M-\frac{1}{2}a_P\right)t_M^2+\left(1.08a_p+u_M\right)t_M-5.083a_p=0$

$-2.418t_M^2+35.06t_M-3.003=0$

Using the quadratic equation or otherwise

$t_M=14.$[illegible]$_M=0.08617$ s

Only interested in the value greater than 1.08

$v_P=u_P+a_p(t_M-1.08)=a_P(t_M-1.08)$

$v_P=5.15\times(14.41-1.08)$

$\mathbf{v_P=68.6}$ **m s^{-1}**

3. MOTION AND GRAPHS

Setting off

1. a) Average velocity = $\frac{\text{final displacement}-\text{initial displacement}}{\text{time}}$

   $\text{Average velocity}=\frac{72-10}{10\times60}$

   $\text{Average velocity}=0.10$ m s^{-1} (or 6.2 m minute^{-1})

   b) Velocity = gradient at point = $\frac{\Delta s}{\Delta t}$

   $\text{Velocity}=\frac{82-23}{(8-6)\times60}$

   $\text{Velocity}=0.49$ m s^{-1} (or 30 m minute^{-1})

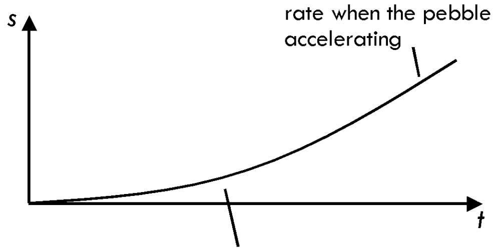
2. Change in velocity = area under acceleration–time graph

   Area of a trapezium = $\frac{1}{2}(a+b)h$

   $\text{Change in velocity}=\frac{1}{2}\times(30+6)\times0.8$

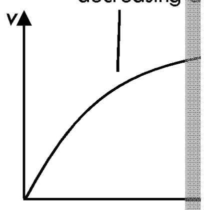
   Change in velocity = 14.4 m s^{-1}

3.

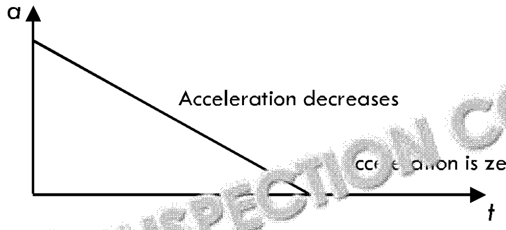


Displacement increases at a constant rate when the pebble is no longer accelerating

Displacement increases at an increasing rate while the pebble accelerates



Velocity incr
decreasing a



Acceleration decreases

Acceleration is zero

You may also have dec
downwards direction is
be flipped.

Speeding up

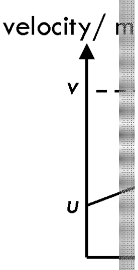
4. a) Displacement, s, is given by the area under a displacement–time graph.

The area of a trapezium is given by

$$area=\frac{1}{2}(a+b)h$$

so

$s=\frac{1}{2}(u+v)t$



b) Inserting $v=u+at$ into $s=\frac{1}{2}(u+v)t$

$s=\frac{1}{2}(u+u+at)t$

$s=\frac{1}{2}(2u+at)t$

$\boldsymbol{s=ut+\frac{1}{2}at^2}$

Rearrange $v=u+at$ for t

$t=\frac{v-u}{a}$

And substitute this into $s=ut+\frac{1}{2}at^2$

$s=\frac{u(v-u)}{a}+\frac{a(v-u)^2}{2a^2}$

$s=\frac{uv-u^2+\frac{1}{2}v^2-uv+\frac{1}{2}u^2}{a}$

$sa=\frac{1}{2}v^2-\frac{1}{2}u^2$

$2sa=v^2-u^2$

$\boldsymbol{v^2=u^2+2as}$

5. a) acceleration = gradient of tangent at point = $\frac{dy}{dx}$

acceleration = $\frac{30-40}{44-0}$

acceleration = – 0.23 m s^{-2} (accept range -0.20 to -0.26 m s^{-2})

b) displacement = area below curve

This is not as easy as previous examples – the easiest method is to count squares

There are around 370 squares below the graph

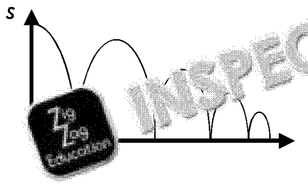
Each square is 2 s across and 2 m s^{-1} up – an equivalent of 4 m travelled

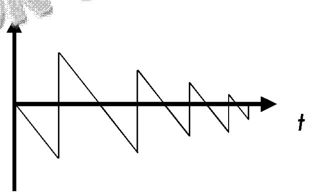
Area = number of squares × value of each square

Displacement = 370 × 4

Displacement = 1480 m

6.





7. To draw the velocity–time graph, we need: velocity after first period of acceleration; acceleration; time taken during second period of acceleration

After first period of acceleration:

$v=u+at=v_1=6+0.8\times 12$

v_1= 15.6 m s^{-1}

After second period of acceleration:

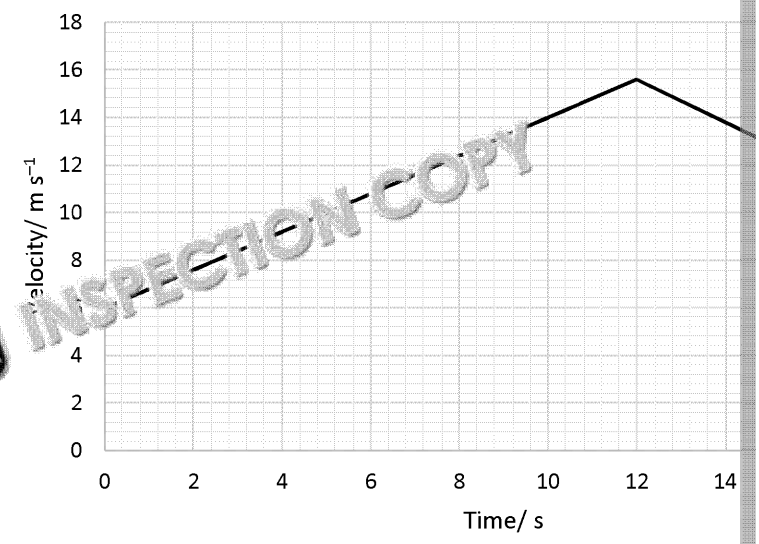
$v^2=u^2+2as$

$v=\sqrt{u^2+2as}=v_2=\sqrt{15.6^2-2\times 0.9\times 60}$

$v_2=11.6$ m s^{-1}

$t=\frac{(v-u)}{a}=\frac{11.6-15.6}{-0.9}$

$t_2=4.44$ s



Total displacement = 60 m + area under first section

Total displacement = $60+\frac{1}{2}\times(6+15.6)\times 12$

Total displacement = 190 m

8. $a=\frac{v_2-v_1}{t}$

v= gradient of tangent at point

$v_1=\frac{150}{3.8-1.9}=78.9\text{ m s}^{-1}$

$v_2=\frac{-200}{8.5-7.4}=-95.2\text{ m s}^{-1}$

$a=\frac{-95.2-78.9}{5}$

$a=-18\text{ m s}^{-2}$

4. NEWTON'S LAWS AND FORCES

Setting off

1. a) $F=ma$

$a=\frac{F}{}$ [illegible] $\frac{6.1}{10}$

$a=$ [illegible]

b) Toy car carries on travelling at same speed, as resultant force = 0, so no accelerat

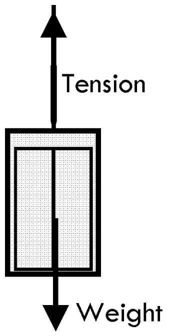
2. a) The gravitational pull on Earth by the student.

b) A tension in the post of 30 N in the opposite direction.

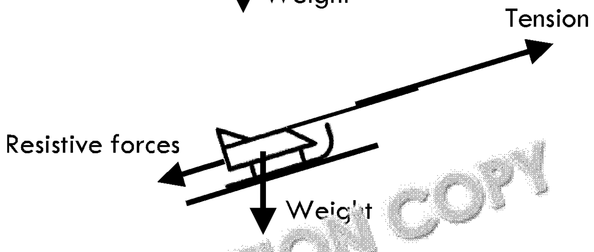
c) Upthrust is due to the difference in pressure on the pineapple, so the correspondin pineapple on the surrounding water.

Speeding up

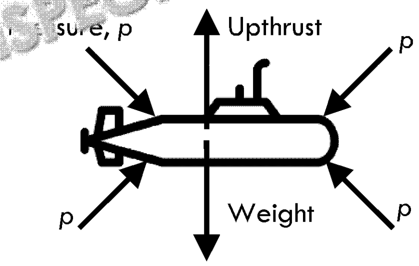
3. a)



b)



c)



You may also include the pressure acting outwards, balancing the pressure inward

4. $F=ma$

$m=\frac{F}{a}=\frac{F_{engine}-F_{friction}-F_{air}}{a}$

$m=\frac{62\times10^3-58\times10^3-14\times10^3}{-4.4}$

$m=2300$ kg

Top speed

5. The weight of the rock acting downhill is

$W_\vartheta=mg\sin\vartheta=390\times9.81\times\sin 11$

$W_\vartheta=730$ N

$F=ma$

$a=\frac{F}{m}=\frac{F_{pull}-W_\vartheta-F_{friction}}{m}$

$a=\frac{8300-7[illegible]-12[illegible]}{[illegible]}$

$a=16$ n[illegible]

6. Both sheets of paper have the same mass, and so the same weight.

In a vacuum, both would accelerate to the ground at the same rate, and land at the s

However, the crumpled sheet of paper has a lower effective surface area than the fl
an effect on the crumpled sheet.

The air resistance on both sheets increases with velocity.

As the air resistance is consistently higher for the crumpled sheet, it accelerates more sl

Eventually, both sheets of paper reach terminal velocity and travel at constant speed.

5. MOMENTS

Setting off

1. moment of couple = force × distance

   moment of couple = 24 × 5.6

   moment of couple = 134 N m

   Centre of rotation is halfway between forces

   i.e 2.2 + $\frac{5.6}{2}$ = **5.0 m** from the left end OR

   5.1+ $\frac{5.6}{2}$ = **7.9 m** from the right end

2. On Lucie's side of the see-saw

   $moment_L = m_Lg \times d_L$

   $moment_L = 50 \times 9.81 \times 1.1$

   $moment_L = 540$ N m

   To bala[illegible] must be equal to the moment on Elvira's si[illegible] the see-saw

   $moment_L = moment_E$

   $moment_E = m_Eg \times d_E$

   $$d_E = \frac{moment_E}{m_Eg}$$

   $$d_E = \frac{540}{65 \times 9.81}$$

   d_E = 0.85 m

Speeding up

3. $M_{total}d_{CoM} = m_1d_1 + m_2d_2 + m_3d_3 + \ldots + m_nd_n$

   $$d_{CoM} = \frac{m_1d_1+m_2d_2+m_3d_3+\ldots+m_nd_n}{m_1+m_2+m_3+\ldots+m_4}$$

   The centre of mass of the sheet on its own is 5.5 cm from the left and 2.5 cm from the bottom (at the centre of the sheet).

   (All distances quoted as from left edge to right and from bottom edge up.)

   Left to right

   $$d_{CoM-x} = \frac{12\times1+25\times2+22\times4+18\times6+32\times9+11\times9+45\times5.5}{12+25+22+18+32+11+45}$$

   d_{CoM-x}= 5.4 cm from left

   (taking distances from right gives 5.6 cm)

   Bottom to top

   $$d_{CoM-y} = \frac{12\times4+25\times2+22\times1+1[illegible]4+[illegible]+45\times2.5}{12[illegible]2[illegible]3+[illegible]11+45}$$

   d_{CoM-y}= [illegible] from bottom

   (taking [illegible]es from top gives 2.4 cm)

4. Centre of mass of right of A, 0.95 m

   The mass sits 0.30

   Around A

   $F_Bd_{A-B} = W_{plank}d_{A-}$

   $$F_B = \frac{W_{plank}d_{A\text{-}CoM}+W}{d_{A-B}}$$

   $$F_B = \frac{8.10\times9.81\times0.45+}{1.}$$

   F_B=34.6 N

   Around B

   $F_Ad_{A-B} = W_{plank}d_{B-}$

   $$F_A = \frac{W_{plank}d_{B\text{-}CoM}+W}{d_{A-B}}$$

   $$F_A = \frac{8.10\times9.81\times0.95+}{1.4}$$

   F_A=87.1 N

5. moment of couple

   Need F and d to

   $moment = F \sin\vartheta \times$

   $$\sin\vartheta = \frac{moment}{Fd}$$

   $$\vartheta = \sin^{-1}\frac{moment}{Fd}$$

   $$\vartheta = \sin^{-1}\frac{67.0}{3.50\times47.1}$$

   ϑ=24.0°

Top speed

6. $m_{rod} = V_{rod} \times \rho_{rod}$

   $m_{rod} = \pi \times 2.4^2 \times (8$

   m_{rod}= 11600 g

   $m_{block} = V_{block}\rho_{block}$

   $m_{block} = 11 \times 15 \times 2$

   m_{block}=27400 g

   $m_{sphere} = V_{sphere}\rho_{sph}$

   $m_{sphere} = \frac{4}{3}\pi \times 5.2^3$

   m_{sphere}=4360 g

   $m_{total}d_{CoM} = m_{rod}d_r$

   $$d_{CoM} = \frac{m_{rod}d_{rod}+m_{blo}}{m_{rod}+}$$

   (all distances are

   $$d_{CoM} = \frac{11600\times40.4+}{1160}$$

   d_{CoM}=25 cm from

7. $moment_{clockwise} = m$

   $T \sin 77.1 \times 62.1 \times$

   $$T = \frac{5.12\times9.81\times\cos 24.}{\sin 7}$$

   T=27.4 N

6. DOING WORK

Setting off

1. $W=Fd$

$$Tension-resistive\ forces=\frac{W}{d}$$

$$Tension=\frac{W}{d}+resistive\ forces$$

$$Tension=\frac{930}{2.8}+120$$

$\boldsymbol{Tension=450}$ **N**

2. $P=Fv$

$P=output\ power=input\ power\times efficiency$

$output\ power=52\times10^{3}\times0.85$

$output\ power=44.2\times10^{3}$ W

$$v=\frac{output\ \text{[illegible]}}{\text{[illegible]}}$$

$$v=\frac{44.2\times\text{[illegible]}}{3.6\times10^{3}}$$

$\boldsymbol{v=12}$ **m s^{-1}**

Speeding up

3. $W=Fd\cos\vartheta$

$$d=\frac{W}{F\cos\vartheta}$$

$$d=\frac{0.668}{3.02\times\cos 5.11}$$

$\boldsymbol{d=0.22}$ **m**

4. $P=Fv$

$input\ power\times efficiency=(F_{resist}-mg\sin\vartheta)v$

$$F_{resist}-\frac{input\ power\times efficiency}{v}=mg\sin\vartheta$$

$$\vartheta=\sin^{-1}\left(\left(F_{resist}-\frac{input\ power\times efficiency}{v}\right)\div mg\right)$$

$$\vartheta=\sin^{-1}\left(\left(48.1\times10^{3}-\frac{111\times10^{3}\times0.538}{44}\right)\div(9450\times9.81)\right)$$

$\boldsymbol{\vartheta=30.3°}$

Top speed

5. $W=Fd$

$$W=(mg\sin\vartheta-F_{resist})\times\frac{h}{\sin\vartheta}$$

$$W=\frac{1}{2}mv^{2}$$

$$v=\sqrt{\frac{2}{m}(m\text{[illegible]}\vartheta-\text{[illegible]})\times\frac{\text{[illegible]}}{\sin\vartheta}}$$

$$v=\sqrt{\frac{2}{43.3}\times(\text{[illegible]}3.3\times9.81\times\sin 18.0-84.2)\times\frac{3.55}{\sin 18.0}}$$

$\boldsymbol{v=5.00}$ **m s^{-1}**

6. $P=Fv$

$P=(F_{wind}\cos(\vartheta'-\vartheta$

$$v=\frac{P}{(F_{wind}\cos(\vartheta'-\vartheta)+m}$$

$$t=\frac{s}{v}=\frac{h\cos\vartheta}{v}$$

$$t=\frac{h\cos\vartheta\times(F_{wind}\cos(\vartheta'}{P}$$

$$t=\frac{10\times10^{6}\times\cos 11.6\times}{\,}$$

$\boldsymbol{t=346}$ **s**

7. ENERGY A

Setting off

1. $efficiency=\frac{useful\ en}{total\ en}$

total energy input

For the LED bulb

total energy input

total energy input

For the traditional

total energy input

total energy input

Difference in pow

Difference in pov

2. Gain in kinetic en
potential energy

$E_{k,\ bottom}-E_{k,top}=\Delta E$

$\frac{1}{2}m\left(v_{bottom}^{2}-v_{top}^{2}\right)=$

$v_{bottom}=\sqrt{2gh+v_{to}^{2}}$

$v_{bottom}=\sqrt{2\times9.81}$

$\boldsymbol{v_{bottom}=28}$ **m s^{-1}**

Speeding up

3. Loss in gravitation
energy

$efficiency\times m_{boulder}$

$$h_{2}=\frac{\frac{1}{2}m_{rock}v^{2}}{efficiency\times m_{boulder}}$$

$$h_{2}=\frac{\frac{1}{2}\times172\times7.14^{2}}{0.062\times263\times9.81}$$

$\boldsymbol{h_{2}=27.4}$ **m**

4. Gain in kinetic energy = energy provided by engine

$$\frac{1}{2}mv^2=Pt\times efficiency$$

$$v=\sqrt{\frac{2Pt}{m}\times efficiency}$$

$$v=\sqrt{\frac{2\times 4.13\times 10^3\times 164}{919}\times 0.551}$$

$$\mathbf{v=28.5\ m\ s^{-1}}$$

Top speed

5. $$efficiency=\frac{useful\ energy\ output}{total\ energy\ input}$$

$$efficiency=\frac{\Delta E_p+W}{E_{motor}}$$

$$efficiency=\frac{mgh+Fd}{E_{motor}}$$

$$d=\sqrt{0.22^2+0.84^2}$$

$$d=0.868\ m$$

$$efficien\text{[illegible]}3\times 9\text{[illegible]}\times\text{[illegible]}2+0.55\times 0.868 \over 0.94$$

$$\mathbf{efficiency\text{[illegible]}.54=54\ \%}$$

6. $$E_{hammer}=0.93E_{spring}$$

$$E_{ball}=0.82(E_{hammer}-\Delta mgh)$$

$$E_{hammer}=0.93\times\frac{1}{2}kx^2=0.93\times\frac{1}{2}\times 12\times 0.061^2$$

$$E_{hammer}=0.0208\ J$$

$$E_{ball}=0.82\times(0.0208-0.034\times 9.81\times 0.023)$$

$$E_{ball}=0.0108\ J$$

$$E_{ball}=\frac{1}{2}mv^2$$

$$v=\sqrt{\frac{2E_{ball}}{m}}=\sqrt{\frac{2\times 0.0108}{0.0093}}$$

$$\mathbf{v=1.52\ m\ s^{-1}}$$

7. $$efficiency=\frac{useful\ output\ energy}{total\ input\ energy}$$

input = kinetic energy of wind

$$input=\frac{1}{2}mv^2$$

$$m=wind\ rate\times time\times\pi r^2=8.75\times 60.0\times\pi\times 35.4^2=2.067\times 10^6$$ kg

$$v=wind\ rate\div density=8.75\div 1.225=7.143\ m\ s^{-1}$$

$$input=\frac{1}{2}\times 2.067\times 10^6\times 7.143^2$$

$$input=52.73\times 10^6\ J$$

useful output energy
=*input − heat − kinetic energy after*

$$heat=5\text{[illegible]}0^6\ \text{[illegible]}18=9.491\times 10^6\ J$$

Mass leaving the wind turbine is the same as the mass entering the turbine

$$v=5.59\div 1.225=4.563\ m\ s^{-1}$$

$$kinetic\ energy=\frac{1}{2}\times 2.067\times 10^6\times 4.563^2$$

$$kinetic\ energy=21.52\times 10^6\ J$$

useful output energy = (52.73 – 21.52 – 9.491) x 10^6 = 21.72 x 10^6 J

$$\mathbf{efficiency=\frac{21.72\times 10^6}{52.73\times 10^6}=0.412\ or\ 41.2\ \%}$$

8. DENSITY A

Setting off

1. $$\rho=\frac{m}{V}$$

$$V=\frac{m}{\rho}$$

$$\frac{4}{3}\pi r^3=\frac{m}{\rho}$$

$$r=\left(\frac{3\times m}{4\pi\rho}\right)^{\frac{1}{3}}=($$

$$\mathbf{r=0.03288\ m}$$

2. $$p=\frac{F}{A}$$

$$F=pA$$

$$mg-F_{up}=pA$$

$$m=\frac{pA+F_{up}}{g}=\frac{13}{}$$

$$\mathbf{m=0.0221\ kg}$$

3. $$upthrust=weig$$

$$V=\frac{upthrust}{\rho g}$$

$$Al=\frac{upthrust}{\rho g}$$

$$l=\frac{upthrust}{\rho gA}=\frac{}{1360}$$

$$\mathbf{l=0.971\ m}$$

Speeding up

4. Floating, so

$$mg=upthrust$$

$$V_{total}\rho_{balsam}g=$$

$$wlh\rho_{balsam}=wlx$$

$$x=\frac{h\rho_{balsam}}{\rho_{water}}=$$

$$\mathbf{x=0.114\ m}$$

5. $$F_{drag}=upthrust$$

$$6\pi r\eta v=\frac{4}{3}\pi r^3\rho$$

$$\eta=\frac{2}{9}\frac{r^2\rho g}{v}=\frac{2}{9}\frac{0.0}{}$$

$$\mathbf{\eta=0.794\ Pa\ s}$$

Top speed

6. $$ma=mg-upthr$$

(ρ_s= density of s

V_c= volume of c

$$\rho_s(V_s-V_c)a=\rho_s$$

$$\rho_s(V_s-V_c)(g-a$$

$$\rho_s=\frac{\rho_m V_s g}{(V_s-V_c)(g-a)}$$

$$\rho_s=\frac{656\times 2}{(2.04^3-0.883}$$

$$\mathbf{\rho_s=718\ kg\ m^{-3}}$$

7. $p=\frac{F_{total}}{A}$

$A=\frac{F_{total}}{p}=\frac{\rho glA+F}{p}$

$\pi r_{outer}^2-\pi r_{inner}^2=\frac{\rho gl(\pi r_{outer}^2-\pi r_{inner}^2)+F}{p}$

$p\pi r_{outer}^2-p\pi r_{inner}^2=\rho gl\pi r_{outer}^2-\rho gl\pi r_{inner}^2+F$

$p\pi r_{inner}^2-\rho gl\pi r_{inner}^2=p\pi r_{outer}^2-\rho g\ l\pi r_{outer}^2-F$

$r_{inner}^2\pi(p-\rho gl)=r_{outer}^2\pi(p-\rho gl)-F$

$r_{inner}=\sqrt{\frac{r_{outer}^2\pi(p-\rho gl)-F}{\pi(p-\rho gl)}}$

$r_{inner}=\sqrt{\frac{(4.10\times10^{-2})^2\times\pi\times(4.26\times10^3-11300\times\text{[illegible]})-11.5}{\pi\times(4.26\times10^3-\text{[illegible]}.8\times\text{[illegible]}.284)}}$

$\mathbf{r_{inner}=0.0426}$ **m**

9. MATERIALS

Setting off

1. $F=k\Delta L$

$\Delta L=\frac{F}{k}=\frac{23.1}{405}$

$\Delta L=0.0570$ m

2. $Young\ modulus=\frac{stress}{strain}=\frac{stress\times L}{\Delta L}$

$stress=\frac{Young\ modulus\times\Delta L}{L}=\frac{2.38\times10^9\times(38.3\times10^{-2}-38.1\times10^{-2})}{38.1\times10^{-2}}$

$stress=1.25\times10^7$ Pa

Speeding up

3. a) Work done = area under graph

By counting squares, or otherwise:

355 squares

1 square = 0.2 N × 1 mm = 2×10^{-4} J

Work done = 355 × 2 × 10^{-4} J

Work done = 0.071 J

b) Stiffness, $k=\frac{F}{\Delta L}=gradient=\frac{\Delta y}{\Delta x}$

By considering the linear portion of the graph and taking measurements:

$k=\frac{4.2}{8.0\times10^{-3}}$

$k=530$ N m^{-1}

4. Young mod[illegible]s$=\frac{Fl}{\text{[illegible]}}$

$F=\frac{Young\ \text{[illegible]}\ \Delta L}{\text{[illegible]}}=\frac{1380\times10^9\times\pi\times(4.14\times10^{-9})^2\times0.02\times10^{-2}}{8.00\times10^{-2}}$

$F=1.86\times10^{-7}$ N

Top speed

5. a) Young modu

Need to dete

kN mm^{-2} = –

Young modu

Young modu

b) $strain=\frac{\Delta L}{L}$

from graph,

$L=\frac{\Delta L}{strain}=\frac{3.0\times}{0.09}$

$L=0.31$ m

c) stress = $\frac{F}{A}$

from graph,

$A=\frac{F}{stress}=\frac{84}{1.2\times}$

$A=7\times10^{-7}$ m

6. Young modulus=

$F=k\Delta L$

$k=\frac{F}{\Delta L}$

Young modulus=

$k=\frac{Young\ modulus\times A}{L}$

$A=\pi r^2$

$c=2\pi r$

$A=\frac{c^2}{4\pi}=\frac{(0.888\times10}{4\pi}$

$A=6.275\times10^{-8}$

$k=\frac{Young\ modulus\times A}{L}=$

$k=4.77\times10^4$ N

10. MOMENTUM

Setting off

1. $p = mv$

   $p_{boat} = 4.5 \times 0.068$

   $p_{boat} = 0.306$ g m s^{-1}

   momentum conserved so $p_{boat} = p_{boat+penny}$

   $p = (m_{boat} + m_{penny})v$

   $v = \frac{p}{m_{boat} + m_{penny}}$

   $v = \frac{0.306}{4.5+2.5}$

   $v = 0.044$ m s^{-1}

2. $F = \frac{\Delta(mv)}{\Delta t}$

   $\Delta v = \frac{F\Delta t}{m}$

   $\Delta v = \frac{64.2 \text{[illegible]}}{20.3 \times 10^6}$

   $\Delta v = 427$ m s^{-1}

Speeding up

3. $m_1 v_1 = m_2 v_2$

   $m_2 = 31.5 – 0.280 \times 60 = 14.7$ kg

   $v_2 = \frac{m_1 v_1}{m_2}$

   $v_2 = \frac{31.5 \times 4.00}{14.7}$

   $v_2 = 8.57$ m s^{-1}

4. a) Initially

   $p = m_A u_A + m_B u_B$

   $p = 0.236 \times 2.10 - 0.318 \times 2.30$

   $p = -0.2358$ kg m s^{-1}

   After

   $p = m_A v_A + m_B v_B$

   $v_B = \frac{p - m_A v_A}{m_B}$

   $v_B = \frac{-0.2358 + 0.236 \times 0.810}{-0.318}$

   $v_B = -0.140$ m s^{-1}

   Answer is negative so ball B travels to the left after the collision

   b) In an inelastic collision, kinetic energy **before** collision ≠ kinetic energy **after** collisio

   $E_{before} = \frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2$

   $E_{before} = \frac{1}{2} \times 0.236 \times 2.10^2 + \frac{1}{2} \times 0.318 \times 0.230^2$

   $E_{before} = 1.36$ J

   $E_{after} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

   $E_{after} = \frac{1}{2} \times 0.236 \times 0.810^2 + \frac{1}{2} \times 0.318 \times 0.140^2$

   $E_{after} = 0.0805$ J

   $E_{before} \neq E_{after}$ so collision is inelastic

Top speed

5. $v^2=u^2+2as$

$u=0\ \text{m s}^{-1},\ a=g=9.81\ \text{m s}^{-2},\ s=1.25\ \text{m}$

$v_1=\sqrt{2\times9.81\times1.25}$

$v_1=-4.952\ \text{m s}^{-1}$

$v=0\ \text{m s}^{-1},\ a=g=9.81\ \text{m s}^{-2},\ s=1.08\ \text{m}$

$v_2=\sqrt{2\times9.81\times1.08}$

$v_2=4.603\ \text{m s}^{-1}$

$F=\frac{\Delta(mv)}{\Delta t}$

$F=\frac{0.0585\times(4.603+4.952)}{0.0680}$

$\boldsymbol{F=8.22}$ **N**

6. Before

Horizontally

$p_x=m_Au_{A,x}+m_Bu_{B,x}$

$p_x=19.2\times0.881\cos60.0+17.4\times0.625$

$p_x=19.33\ \text{kg m s}^{-1}$

Vertically

$p_y=m_Au_{A,\ y}$

$p_y=19.2\times0.881\sin 60$

$p_y=14.65\ \text{kg m s}^{-1}$

After

Horizontally

$v_{B,x}=\frac{p-m_Av_{A,x}}{m_B}$

$v_{B,x}=\frac{19.33-19.2\times0.790\cos28.0}{17.4}$

$v_{B,x}=0.3412\ \text{m s}^{-1}$

Vertically

$v_{B,y}=\frac{p,y-m_Av_{A,y}}{m_B}$

$v_{B,y}=\frac{14.65-19.2\times0.7}{17.4}$

$v_{B,y}=0.4327\ \text{m s}^{-1}$

$v_B=\sqrt{v_{B,x}^2+v_{B,y}^2}$

$v_B=\sqrt{0.3412^2+0.4327^2}$

$v_B=0.551\ \text{m s}^{-1}$

$\vartheta=\tan^{-1}\frac{v_{B,y}}{v_{B,x}}$

$\vartheta=\tan^{-1}\frac{0.4327}{0.3412}$

$\vartheta=51.7°$

7. $v^2=u^2+2as$

$v=0$ m [illegible] 55 m, $a=\frac{F}{m}=\frac{-6630}{1870+2350}=-1.571\ \text{m s}^{-2}$

$u=\sqrt{2\times1.571\times3.55}$

$u=3.340\ \text{m s}^{-1}$

$p_{before}=p_{after}$

$v_B=\frac{(m_A+m_B)u}{m_B}$

$v_B=7.54\ \text{m s}^{-1}$

11. CIRCULAR MOTION

Setting off

1. a) $\omega=\frac{2\pi}{T}$

$T=\frac{60}{45}=\frac{4}{3}$

$\omega=2\pi\times\frac{3}{4}$

$\boldsymbol{\omega=4.7}$ **rad** $\mathbf{s^{-1}}$ (4.71 to 3 significant figures)

b) $v=\omega r$

$v=4.71\times0.18$

$\boldsymbol{v=0.85}$ **m** $\mathbf{s^{-1}}$

2. $a=\omega^2 r$

$a=\left(\frac{2\pi}{T}\right)^2 r=\frac{4\pi^2 r}{\text{[illegible]}^2}$

$T=\sqrt{\frac{4\pi^2\text{[illegible]}}{a}}$

$T=\sqrt{\frac{4\times\pi^2\times5.5}{3\times9.81}}=2.72$ s

$number\ of\ rotations=\frac{18}{2.72}$

number of rotations=6.6 **rotations**

Speeding up

3. Centripetal force provided by the component of the normal force towards the centre

$F_{centripetal}=N\sin\vartheta=\frac{mv^2}{r}$

The vertical component of the normal force balances the car's weight

$mg=N\cos\vartheta$

$\frac{N\sin\vartheta}{N\cos\vartheta}=\tan\vartheta=\frac{mv^2}{rmg}$

$\vartheta=\tan^{-1}\frac{v^2}{rg}$

$v=88.5\times10^3\div(60\times60)=24.58$ m s^{-1}

$\vartheta=\tan^{-1}\frac{24.58^2}{78.3\times9.81}$

$\boldsymbol{\vartheta=38.2°}$

4. $m_{Europa}\omega^2 r=\frac{Gm_{Jupiter}m_{Europa}}{r^2}$

$\frac{4\pi^2}{T^2}=\frac{Gm_{Jupiter}}{r^3}$

$T=\sqrt{\frac{4\pi^2 r^3}{Gm_{\text{[illegible]}}}}$

$T=\sqrt{\frac{4\pi^{\text{[illegible]}}\times(671\times10^6)^3}{6.67\times10^{-11}\times1.90\times10^{27}}}$

$\boldsymbol{t=307\ 000}$ **s** (= 85.2 hours)

Top speed

5. $v = v_{centripetal} + v_{water}$

$F_{centripetal} = \frac{mv^2_{centripetal}}{r} = N \sin\vartheta$

$mg = N \cos\vartheta$

$\tan\vartheta = \frac{v^2_{centripetal}}{rg}$

$v_{centripetal} = \sqrt{rg \tan\vartheta}$

$P = F_{engine} v = F_{engine}(v_{water} + \sqrt{rg \tan\vartheta})$

$P = 13.5 \times 10^3 \times (24.5 + \sqrt{23.4 \times 9.81 \times \tan 62.1})$

$P = 612$ kW

6. For passenger not to fall out of seat

$mg = \frac{mv^2_{final}}{[illegible]}$

$v_{final} = \sqrt{[illegible]}$

Gain in gravitational potential energy = loss in kinetic energy

$mgh = \frac{1}{2} m (v^2_{final} - v^2_{initial})$

$h = 2r$

$2gr = \frac{1}{2}(gr - v^2_{initial})$

$v_{initial} = \sqrt{-3gr}$

$v_{initial} = \sqrt{-3 \times -9.81 \times 6.5}$

$v_{initial} = 14$ m s^{-1}

12. SIMPLE HARMONIC MOTION

Setting off

1. a) The minimum speed for all simple harmonic motion is $v_{min} = 0$ m s^{-1}. This occurs at th

b) Maximum speed = ωA

$\omega = \frac{\text{Maximum speed}}{A} = \frac{1.1}{58 \times 10^{-3}}$

$\omega = 19$ rad s^{-1}

2. a) $a = -\omega^2 x = -(2\pi f)^2 x$

$f = \sqrt{\frac{a}{4\pi^2 x}} = \sqrt{\frac{310}{4 \times \pi^2 \times 220 \times 10^{-9}}}$

$f = 6000$ Hz

b) Maximum acceleration = $\omega^2 A$

$A = \frac{\text{Maximum acceleration}}{\omega^2} = \frac{\text{Maximum acceleration}}{a/x} = \frac{410}{310/(220 \times 10^{-9})}$

$A = 2[illegible]$ m

Speeding up

3. $x=A\cos\omega t$

$$t=\frac{1}{\omega}\cos^{-1}\frac{x}{A}=\frac{1}{15.2}\cos^{-1}\frac{4.00\times10^{-3}}{6.20\times10^{-3}}$$

$t=3.28$ s

4. $v=\omega\sqrt{A^2-x^2}$

$$A=\pm\sqrt{\frac{v^2}{\omega^2}+x^2}=\pm\sqrt{{v^2}\Big/{-\frac{a}{x}}+x^2}=\pm\sqrt{\frac{5.67^2}{10.9\Big/3.55\times10^{-2}}+\left(-3.55\times10^{-2}\right)^2}$$

$A=\pm0.326$ m

Top speed

5. $a=-\omega^2 x$

$x=A\cos\omega t$

$a=-\omega^2$

$$t=\frac{1}{\omega}\cos^{-1}\frac{a}{-\omega^2 A}=\frac{1}{2\pi f}\cos^{-1}\frac{a}{-(2\pi f)^2 A}=\frac{1}{2\pi\times98.1}\times\cos^{-1}\frac{50.0}{-(2\pi\times98.1)^2\times281\times10^{-6}}$$

$t=0.0334$ s

6. $v=\pm\omega\sqrt{A^2-x^2}$

$x=A\cos\omega t$

$v=\pm\omega\sqrt{A^2-A^2\cos^2\omega t}$

$$\cos^2\omega t=1-\frac{v^2}{\omega^2A^2}$$

$$\cos\omega t=\sqrt{1-\frac{v^2}{\omega^2A^2}}$$

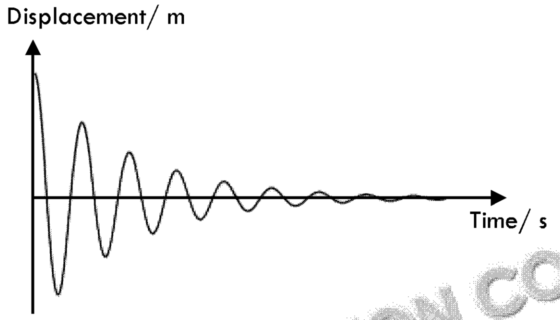
$$t=\frac{1}{\omega}\cos^{-1}\sqrt{1-\frac{v^2}{\omega^2A^2}}=\frac{T}{2\pi}\cos^{-1}\sqrt{1-\frac{v^2T^2}{4\pi^2A^2}}=\frac{0.0359}{2\pi}\cos^{-1}\sqrt{1-\frac{\left(1.28\times10^{-3}\right)^2\times0.0359^2}{4\pi^2\left(22.1\times10^{-6}\right)^2}}$$

$t=0.110$ s

13. SIMPLE HARMONIC SYSTEMS AND DAMPING

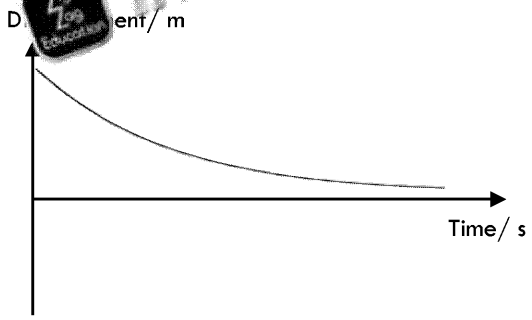
Setting off

1. a)



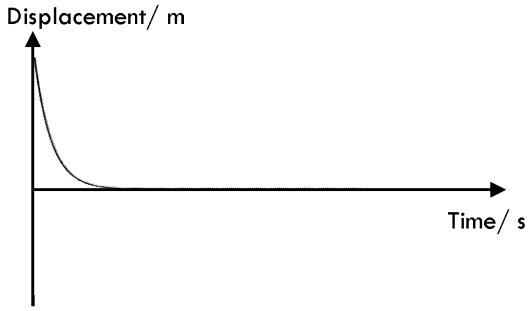
Application: church bell, bab[illegible] bungee cord – any application which requi
away slowly over time

b)



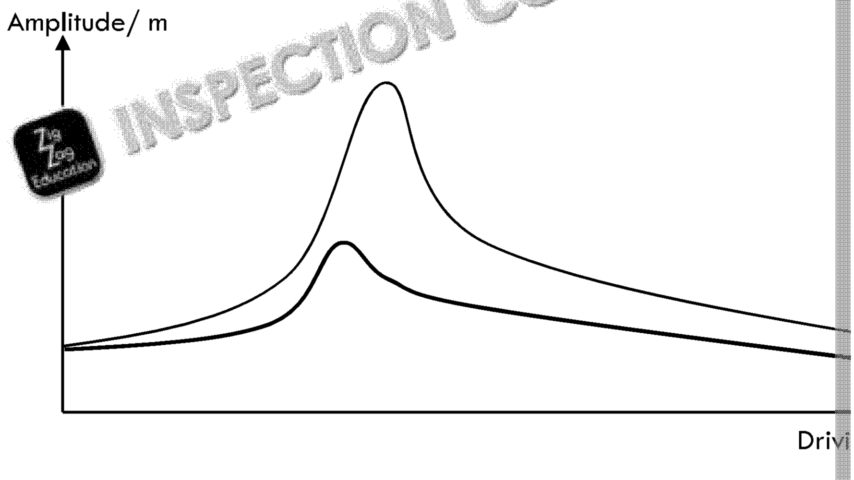
Application: stop a heavy door slamming, safety mats, slow automatic shut-off val
– any application which is used to try to slow down and reduce the amount of osci

c)



Application: car suspension, speedometer needle, recoil absorption in guns, comput
– any application that requires a fast reaction and no oscillation afterwards.

2.



Amplitude of oscillation reduced at all frequencies. Resonant frequency (peak) occurs

Speeding up

3. $T=2\pi\sqrt{\frac{l}{g}}$

$l=g\left(\frac{T}{2\pi}\right)^2=g\left(\frac{1}{2\pi f}\right)^2=9.81\times\left(\frac{1}{2\pi\times2.33}\right)^2$

$\boldsymbol{l=0.0458}$ **m**

4. $T=2\pi\sqrt{\frac{m}{k}}$

$F=kx$

$\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{F/x}}$

$m=\frac{F}{x}\frac{1}{\omega^2}=\frac{0.261}{0.994\times10^{-3}}\times\frac{1}{8.42^2}$

$\boldsymbol{m=3.70}$ **kg**

5. $2\pi\sqrt{\frac{l}{g}}=2$ [illegible] $\sqrt{\frac{m}{k}}$

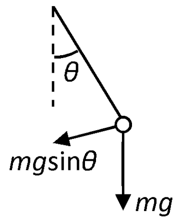
$\frac{l}{g}=\frac{m}{k}$

$k=\frac{mg}{l}=\frac{1.98\times9.81}{3.64}$

$\boldsymbol{k=5.34}$ **N m⁻¹**

Top speed

6. First consider the restoring force on the pendulum due to gravity:



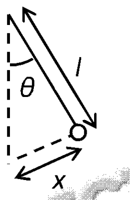
So $F=mg\sin\vartheta$

For $\vartheta<10°$, $\sin\vartheta\sim\vartheta$

so

$F=mg\vartheta$

Compare this to the displacement of the pendulum from its equilibrium position:



$x=l\tan\vartheta$

For $\vartheta<$ [illegible] $\vartheta\sim\vartheta$

so

$x=l\vartheta\rightarrow\vartheta=\frac{x}{l}$

so

$F=mg\frac{x}{l}$

Comparing this to $F=ma$

$ma=-mg\frac{x}{l}\longrightarrow a=-\frac{g}{l}x$

This is the form of the defining equation of simple harmonic motion.

$a = -\omega^2 x$

so

$\omega^2 = \frac{g}{l}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{l}$

$\boldsymbol{T = 2\pi\sqrt{\frac{l}{g}}}$

7. For simple harmonic motion:
total energy = kinetic energy + potential energy

For mass m and spring constant k:

kinetic energy $= \frac{1}{2}mv^2$

potential energy $= \frac{1}{2}kx^2$

At maximum amplitude total energy = potential energy

total energy $= \frac{1}{2}kA^2$

Total energy is constant, so this applies at all points of the system's motion

$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

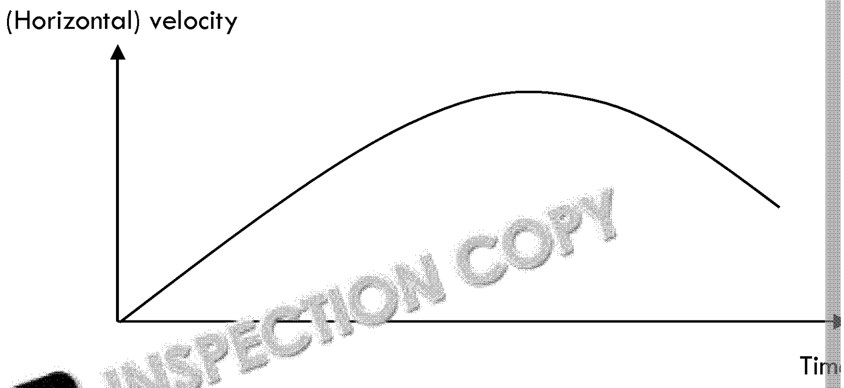
$v^2 = \frac{k}{m}\left(A^2 - x^2\right)$

$v = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$

$T = 2\pi\sqrt{\frac{m}{k}}$ so $\sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f$

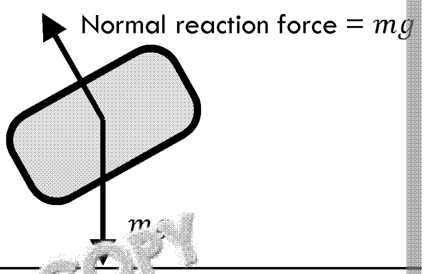
$\boldsymbol{v = \pm 2\pi f\sqrt{A^2 - x^2}}$

EXAM-STYLE QUESTIONS

Question 1	
1.1	Axes labelled correctly (units are not required) Increases from zero to a maximum Decreases from maximum to a positive value As below: 
1.2	Lo[illegible]vitational potential energy = gain in kinetic energy $mg\Delta h=\frac{\text{[illegible]}}{2}mv^2$ $v=\sqrt{2gh}$ $v=\sqrt{2\times9.81\times(0.510-0.240)}$ ($v=2.30$ m s^{-1})
1.3	Vertically $v_v=2.30\,\sin 32.3$ $v_v=1.23$ m s^{-1} ($v_v=1.229$ m s^{-1} to 4 significant figures) Horizontally $v_h=2.30\,\cos 32.3$ $v_h=1.94$ m s^{-1} ($v_h=1.944$ m s^{-1} to 4 significant figures)
1.4	To top of parabola $v=u+at$ $t=\frac{v-u}{a}$ $t=\frac{0-1.229}{-9.81}$ $t=0.1253$ s $s=ut+\frac{1}{2}at^2$ $s=1.229\times0.1253-\frac{1}{2}9.81\times0.1253^2$ $s=0.07698$ m To ground $s=ut+\frac{1}{2}at^2$ $t=\sqrt{\frac{2s}{a}}$ $t=\sqrt{\frac{2\times(0.24+0.07698)}{9.81}}$ $t=0.2542$ s $v=\frac{\Delta s}{\text{[illegible]}}$ Δs[illegible] $\Delta s=$[illegible]$44\times(0.2542+0.1253)$ $\Delta s=0.738$ m
Total	

Question 2	
2.1	$\Delta mv=F\Delta t$ $\Delta mv=163\times1.84$ $\Delta mv=$ 300 kg m s^{-1} (299.9 kg m s^{-1} to 4 significant figures)
2.2	momentum $_{\text{before}}$ = momentum $_{\text{after}}$ $p_{A\ before}+m_B v_{B\ before}=m_A v_{A\ after}+m_B v_{B\ after}$ $m_B=\frac{m_A v_{A\ after}-p_{A\ before}}{v_{B\ before}-v_{B\ after}}$ $m_B=\frac{-215\times0.333-299.9}{-1.41-0.536}$ $m_B=191$ kg
2.3	Safety features extend time over which momentum is changed Decreases the force experienced by passengers a[illegible] Δt
Total	

Question 3	
3.1	Cl[illegible]e e[illegible] spring in place Me[illegible]itial length Ha[illegible]sses from end of spring in regular increments (e.g. 100 g) Measure length of spring (Repeat for each mass added)
3.2	For linear section, Hooke's law is obeyed After limit of proportionality Bonds between atoms weaken/break and atoms can be more easily separated
3.3	Spring constant k= gradient $\left(=\frac{\Delta y}{\Delta x}\right)$ e.g. $k=\frac{5.0}{12.5\times10^{-2}}$ $k=40$ N m^{-1} (Accept 35 to 45 N m^{-1})
3.4	$E_e=\frac{1}{2}ke^2$ $E_e=\frac{1}{2}\times40\times0.0871^2$ $E_e=0.1517$ J $E_{in}=Pt$ $E_{in}=0.432\times3.00$ $E_{in}=1.296$ J $efficiency=\frac{E_e}{E_{in}}$
	$efficiency=\frac{0.1517}{1.296}$
	$efficiency=0.117$ (or $efficiency=11.7$ %)
Total	

Question 4	
4.1	Weight (or *mg*) labelled vertically downwards Normal reaction force labelled at right angles to track Normal reaction force labelled as $=mg\cos\vartheta$ (Penalise labelling of separate centripetal or centrifugal force) As below: 
4.2	$\frac{mv^2}{r}=N\cos\vartheta$ $\frac{mv^2}{r}=mg\cos\vartheta\cos\vartheta$ $v=\sqrt{[illegible]s\vartheta\ [illegible]}$ $v=\sqrt{[illegible]21.0\times\cos 40.3\cos 40.3}$ v=10.9 m s^{-1} (v= 10.95 m s^{-1} to 4 significant figures)
4.3	$P=Fv$ $P=5.51v^3$ $P=5.51\times10.95^3$ $P=7230$ W
Total	

GCSE POP QUIZ

1. a) Scalar
   b) Vector
   c) Vector
   d) Scalar
   e) Vector
   f) Scalar

2. $F_{resultant}=5+4-3$

   $F_{resultant}=6$ N

   to the right

3. $v=\frac{\Delta s}{\Delta t}=\frac{80}{140}$

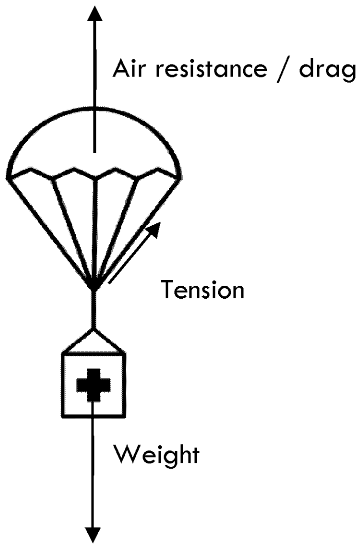
   $v=0.57$ m s^{-1}

4. $a=\frac{\Delta v}{\Delta t}$

   $\Delta t=\frac{v_2-v_1}{a}=\frac{5.0-1.0}{0.60}$

   $\Delta t=6.7$ s

5.



6. $W=mg$

   $\frac{W_{Jupiter}}{W_{Earth}}=\frac{g_{Jupiter}}{g_{Earth}}$

   $g_{Jupiter}=g_{Earth}\times\frac{W_{Jupiter}}{W_{Earth}}=9.8\times\frac{86}{\text{[illegible]}}$

   $g_{Jupiter}=25$ m s^{-2}

7. a) As the ball rolls down a hill, the resultant force acts downwards, so the ball acceler (Newton's second law). When the ball hits the wall, the weight is counteracted by opposite direction (Newton's third law) and the resultant force is zero so the ball is
   b) The engine causes a resultant force, causing the boat to accelerate (Newton's seco increase with the boat's speed as it exerts more force on the water (Newton's third balance with the force of the boat's engine and the boat stays at a constant speed

8. $F_{resultant}=ma$

$F-F_{resistive}=ma$

$F_{resistive}=F-ma=13-2.0\times6.2$

$F_{resistive}=0.6$ N

9. $moment=force\times distance$

$distance=\frac{moment}{force}=\frac{11}{29}$

$distance=0.38$ m

10. To conserve momentum, the student gains the same momentum as they impart to the b
Their change in velocity is determined using

$momentum=mass\times velocity$

$velocity=\frac{momentum}{mass}=\frac{m_{ball}v_{ball}}{m_{student}}=\frac{0.40\times}{}$ [illegible]

$velocity=$ [illegible] 3 m [illegible]

11. gain in gravitational potential energy = loss of kinetic energy

$mgh=\frac{1}{2}mv^2$

$h=\frac{v^2}{2g}=\frac{5.4^2}{2\times9.81}$

$h=1.5$ m

12. a) Chemical energy (in the battery) is transferred by a current doing work into kinet
energy, and heat energy.

b) $efficiency=\frac{useful\ output\ energy}{input\ energy}=\frac{6.8}{7.1}$

$efficiency=0.96$

13. $power=\frac{energy\ transferred}{time}$

$energy\ transferred=power\times time=40\times60\times60$

$energy\ transferred=144\ 000$ J

14. a) $F=ke=67\times0.081$

$F=5.4$ N

b) $E_e=\frac{1}{2}ke^2=\frac{1}{2}\times67\times0.081^2$

$E_e=0.22$ J