

Electricity and Magnetism Technical Topics Worksheets

for A Level OCR A Physics

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Teacher's Introduction

This activity pack is designed to help your student develop skills both during class and in an extracurricular setting. The content covers a range of topics across the A Level course, specifically including content from **Module 4 – Electrons, waves and photons** and **Module 6 – Particles and medical physics**.

Remember!

Always check the exam board website for new information, including changes to the specification and sample assessment material.

Although each worksheet does cover a specific section of the A Level resource, the exercises are primarily skills-focused. These skills are all relevant to the A Level OCR A examinations. The competencies which are developed and tested by this resource include:

- Understanding and analysing resistance changes with current, temperature and other factors for a variety of components
- Calculating resistance and resistivity of components, including components with changing dimensions
- Analysing circuits with multiple resistors in series and parallel
- Understanding and calculating how power is dissipated in components and circuits
- Analysing circuits with multiple power sources
- Analysing and design potential divider circuits
- Understanding how electric fields in dielectrics produces capacitance
- Analysing circuits with multiple capacitors
- Understanding how capacitors charge and discharge over time, including analysing graphs
- Calculating potential differences and currents in transformers, including cases where the transformers are not 100% efficient
- Calculating the e.m.f. induced in a coil rotating in a magnetic field

The resource opens with a student introduction followed by a refresher for GCSE Electrics topics. This is followed by 12 worksheets, each of which covers one or more skills outlined above.

Each worksheet contains a short section of background information, followed by worked examples and then in-depth questions to test students' knowledge (an answers section can be found at the end of the resource). The questions are split into three levels of increasing difficulty, illustrated using the headings *Bright spark*, *Charging up* and *Shocking!*.

The pack also includes three full exam-style questions, which closely replicate the style of question found in an examination paper.

We hope this resource will be useful to your teaching, and help your students to tackle an area of physics which many find challenging, so that each student gains a deeper, holistic understanding of the subject.

T Brown, December 2017

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* resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers

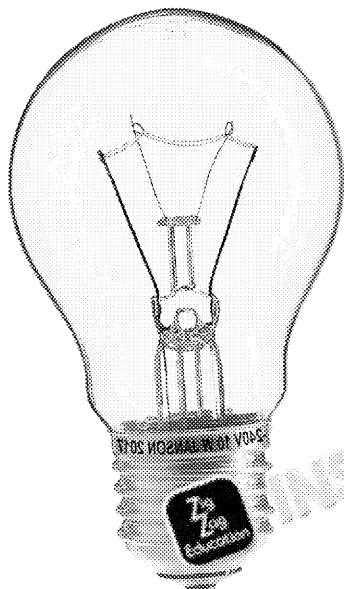
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SKILLS AND SPECIFICATION LINKS

This table is for either teachers' or students' use. It should help you to quickly see which topics are covered by this pack and which parts of the specification they relate to. As a result, you can identify areas which require improvement and use the appropriate worksheets.

Topic No.	Page Title	Page No.	Skills Covered	Specification Link
1	I–V characteristics	13	<ul style="list-style-type: none"> Drawing and interpreting I–V graphs for a range of components 	4.2.3
2	Resistivity	16	<ul style="list-style-type: none"> Performing calculations using resistivity, including for non-regular objects 	4.2.4
3	Power	19	<ul style="list-style-type: none"> Performing calculations involving power and energy usage in circuits 	4.2.5
4	Resistors in series and parallel	21	<ul style="list-style-type: none"> Performing calculations for combinations of resistors in series and parallel 	4.3.1
5	Multiple power sources	23	<ul style="list-style-type: none"> Analysing circuits with more than one power source Performing calculations using circuits with more than one power source 	4.3.1
6	Potential dividers	26	<ul style="list-style-type: none"> Interpreting and designing potential divider circuits using thermistors and LDRs Performing calculations to find resistances and potential differences in potential divider circuits 	4.3.3
7	Dielectrics	29	<ul style="list-style-type: none"> Understanding how capacitors use potentials to store charge Perform calculations involving capacitance and dielectrics 	6.2.3
8	Capacitors in series and parallel	32	<ul style="list-style-type: none"> Perform calculations involving multiple capacitors in series and parallel 	6.1.1
9	Charging and discharging capacitors	34	<ul style="list-style-type: none"> Analysing capacitor charge and discharge graphs and using them in calculations 	6.1.3
10	Transformers	38	<ul style="list-style-type: none"> Performing calculations using transformers 	6.3.3
11	Electricity generation	41	<ul style="list-style-type: none"> Performing calculations relating to electricity generation 	6.3.3
12	Electromagnetic phenomena	45	<ul style="list-style-type: none"> Analysing and interpreting situations involving Lenz's and Faraday's laws of electromagnetic induction 	6.3.3

STUDENT INTRODUCT



The effects of electricity and magnetism have been known for thousands of years, from lightning in the sky, static charge causing objects to attract, and fish such as the electric eel which can generate electricity. It wasn't until the eighteenth century, with the work of Benjamin Franklin, that the connection between all of these phenomena was established.

In the nineteenth century, inventions such as Alessandro Volta's battery and scientists to generate electricity, while Michael Faraday's experiments with electromagnetism and Edison's filament bulb harnessed electricity for practical use. From scientists such as Galileo Ferraris, Ányos Jedlik, and Nikola Tesla, Westinghouse allowed electricity to be reliably generated and transmitted over large distances for use in industries, public spaces, and homes.

Today, the applications of electricity and magnetism are vast. Power cables which transfer power across countries to homes and businesses, and highly complex circuits which use, store and process information, display videos of cats on smartphones.

Electricity and magnetism come up in a few different sections of your A Level course, in different forms and applications. Electricity and magnetism topics include a lot of theory, which is tricky to use, and require a strong understanding of the principles involved. When you study on electricity and magnetism, you'll have to be able to rearrange equations, and recognise how the different equations can be used together.

This pack will help you develop several core skills related to electricity, magnetism, and electromagnetism:

- understanding and analysing how resistance changes with current, temperature, and a variety of components
- calculating resistance and resistivity of components, including component combinations
- analysing circuits with multiple resistors in series and in parallel
- understanding and calculating how power is dissipated in components and circuits
- analysing circuits with multiple power sources
- analysing and designing potential divider circuits
- understanding how electric fields in dielectrics produces capacitance
- analysing circuits with multiple capacitors
- understanding how capacitors charge and discharge over time, including RC circuits
- analysing AC signals for information on circuits
- calculating potential differences and currents in transformers, including efficiency (they are not 100 % efficient)
- calculating the e.m.f. induced in a coil rotating in a magnetic field



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BACKGROUND – GCSE EL

If you studied GCSE Physics, you'll probably be familiar with some aspects of electricity. You may have seen circuit diagrams with the associated symbols, and designed and built simple circuits. You may have come across energy transfers, and calculated power dissipated in circuits. You'll probably have learnt about resistance and how resistors affect the current and potential difference in circuits. You may also have come across how magnetism are connected in the motor effect and generator effect, and how transformers and the national grid.

The next few pages are a quick summary of what you may have already covered in your memory and consolidate your knowledge before moving on to the more advanced topics. If you're not feeling confident, you should go slowly and spend some time building up your knowledge and understanding before moving on to these topics with the A Level.

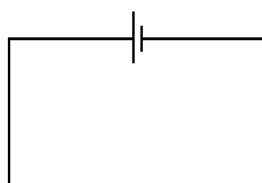
THE BASICS OF ELECTRICITY

Electricity is the **movement of charges** through a material, which transfers energy.

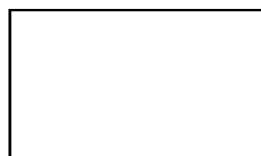
Materials which electricity can flow through are called **conductors**. Conductors include metals or electrolytes.

- In a **metal** such as copper, negative electrons move round from negative to positive.
- In an **electrolyte** such as salt water, ions flow from one electrode to another.
- In **insulators** such as rubber, electrons are tightly bound to individual atoms and cannot move around freely.

A **current** will only flow through a circuit if there is a **source of potential difference** and a **closed loop**.



Circuit A



Circuit B

A current will only flow through Circuit A.

Circuit B is a closed loop, but there is **no power source**, so current will not flow.

Circuit C has a power source but is **not a closed loop** – there is a gap in the bottom wire.

- **Current, I** , is the rate of flow of charge, **Q** , around a circuit.
- Current is measured in **amps**, given the symbol **I** .
- Charge is measured in **coulombs**, given the symbol **Q** .

The charge transferred in a circuit is given by



$$Q = It$$

So if a current of 5 A flows for 10 seconds, $Q = 5 \times 10 = 50$ C of charge will be transferred.

Current is the same at all points of a loop of a circuit.

Potential difference or **voltage, V** , is the energy transferred to a charge carrier. Potential difference is the driving force behind the current in a circuit.

Potential difference is measured in **volts**, given the symbol **V** .




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COMPONENTS

Each electrical component has a specific set of properties, and, therefore, can have a specific effect.

Each component has its own symbol, so that circuit diagrams can be drawn simply.

Component	Symbol	
Cell		Power source
Bulb		Load
 Voltmeter		Measurement
Ammeter		Measurement
Ohmmeter		Measurement
Resistor		Load
Switch		Control
Diode		Allows current to flow in one direction only
Light-emitting diode (LED)		Gives out light
 Thermistor		Resistance changes with temperature
 LDR (light-dependent resistor)		Resistance changes with light
Variable resistor		Resistance can be changed

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RESISTANCE

Resistance is the tendency of a component to oppose current.

Resistance, R , is measured in **ohms**, given the symbol Ω .

For a given potential difference, a component with **higher resistance** will have less current flowing through it.

The relationship between current, potential difference and resistance is

$$V = IR$$

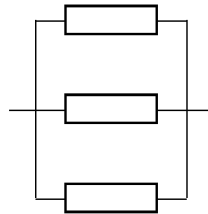
So a component with a resistance of $3\ \Omega$ and a current of $4\ \text{A}$ flowing through it will have a potential difference of $3\ \Omega \times 4 = 12\ \text{V}$ across it.

SERIES AND PARALLEL

Resistors and other components can be put into a number of different arrangements. If components are placed in **the same loop** of a circuit, they are **in series** with each other.



When components are placed into **different loops** of the same circuit, they are **in parallel**.



When resistors are placed in series, the total resistance of the arrangement is the sum of the individual resistances. The current through a series arrangement is the same at all points, and the potential difference is shared between the components.

For a series circuit, the total resistance is given by

$$R_{\text{total}} = R_1 + R_2 + R_3 + \dots$$

When resistors are placed in parallel, the total resistance of the arrangement is less than the largest individual resistance. The current has more paths to flow through, so each path needs to carry less current. The potential difference needed to push the same current is the same for each branch. Potential difference is the same across each parallel component, but the current is split across each branch.

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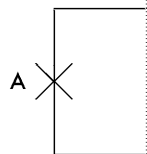
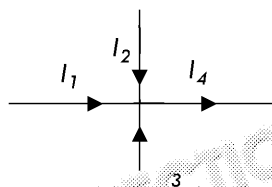


KIRCHHOFF'S LAWS

All circuits obey Kirchhoff's first and second laws. You may not have come across these laws by name, but you'll probably have used them before.

Kirchhoff's first law states that the current into any point of the circuit is equal to the current out of that point.

This means that charge is conserved throughout the circuit.



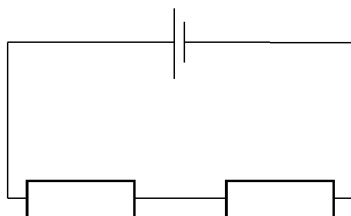
The current at a point must equal the current out of the point.

In the circuit on the left, $I_4 = I_1 + I_2 + I_3$.

In the circuit on the right, the current at point A = the current at point B.

Kirchhoff's second law states that the potential difference lost by charges is equal to the potential difference gained by the charges in the power source.

This means that energy is conserved throughout the circuit.



In this circuit, the potential difference supplied by the cell exactly equals the potential difference across both resistors.

Each resistor has a proportion of the potential difference across it – if the resistors are identical, the potential difference across each would be half the potential difference supplied by the cell.

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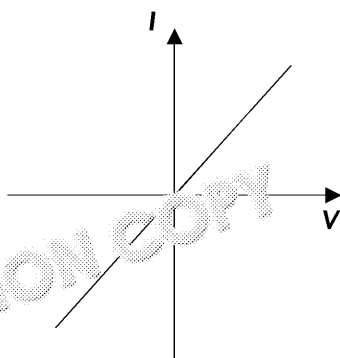
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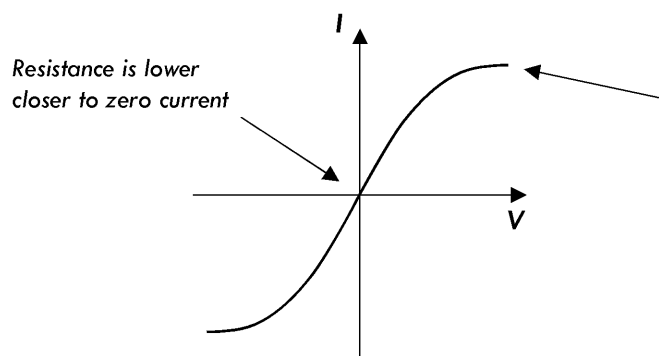
I-V CHARACTERISTICS

Components respond to increasing current and potential difference in different ways.

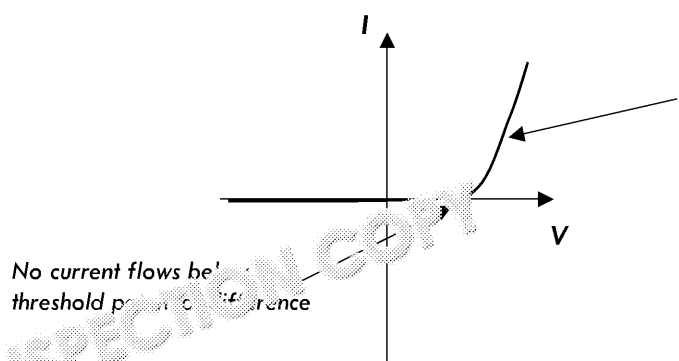
An **ohmic conductor** is one that has a **linear** relationship between potential difference and current. The resistance of an ohmic conductor is **constant**, no matter the current through it or the potential difference across it.



A **filament lamp** has increasing resistance with increased current. This is because the resistance of the filament increases as the current increases and a higher temperature is reached.



A **diode** only allows current to flow through in a single direction above a threshold potential difference.



Thermistors and Light-dependent resistors (LDRs) are components that change their resistance in response to changes in their environment.

The resistance of a thermistor **decreases with increasing temperature**. This can be used to measure temperature, or build a circuit that responds to changes in temperature.

The resistance of an LDR **decreases with increasing light levels**. This means it can be used to measure light intensity, or build a circuit that responds to changes in light levels, such as a light meter or a night light.

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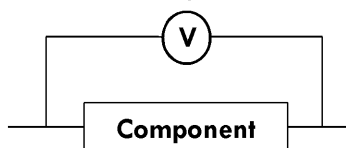
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MEASURING CURRENT, POTENTIAL DIFFERENCE

The potential difference across a component or section of a circuit can be measured using a **voltmeter**.

A voltmeter should be placed **in parallel** with the component or circuit section being investigated.



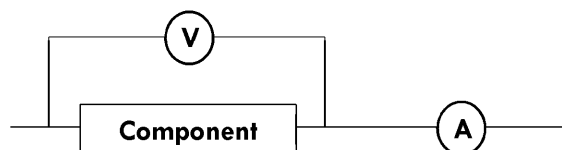
The current through a component can be measured using an **ammeter**.

An ammeter should be placed **in series** with the component being investigated.



The resistance of a component can be measured with **both an ammeter and a voltmeter**.

The ammeter is placed in series and the voltmeter in parallel with the component. The resistance is then calculated using the equation $R = \frac{V}{I}$.



A device called an **ohmmeter** can measure resistance directly. An ohmmeter is a digital device that uses the same principles as using a voltmeter and ammeter. An ohmmeter is placed in parallel with the component.

Power

As a current passes through a component, energy is transferred. This transfer of energy can be into many different forms – for example, as light, heat, or kinetic energy.

The amount of power, P , transferred by a component is given by

$$P = IV = I^2R$$

Power is energy per second. The energy, E , transferred by a component in time t is given by

$$E = Pt = QV$$

A component with a resistance of 8Ω and a current of 2 A through it will transfer a power of $P = 2^2 \times 8 = 32 \text{ W}$.

Over one minute, this component will have transferred $E = 32 \times 60 = 1920 \text{ J}$.

Not all power transferred by a component is useful. Some energy will be transferred that is considered 'waste' energy. For example, a bulb will produce light but also produce heat and sound. Components with a current running through them will produce heat.

Efficiency is the ratio of energy that is transferred by a component or appliance to the total energy provided to the component or appliance.

$$\text{efficiency} = \frac{\text{useful energy transferred}}{\text{total energy used}} \times 100 \%$$

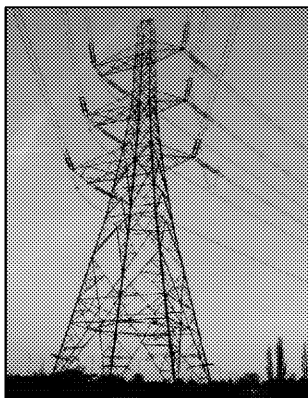
A bulb that uses 50 W of electricity to produce 40 W of light would have an efficiency of

$$\frac{40}{50} \times 100 \% = 80 \%$$

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TRANSMITTING ELECTRICITY



All electrical circuits need a source of potential difference. Batteries are sufficient for small portable devices, but for large-scale power, an external source of power.

Electrical energy production is one of the largest industries. Electricity is transmitted over the **National Grid**. The wires and transformers which transfer electricity from power stations to businesses.

Electricity is transmitted as an **alternating current** (AC). The direction of the current changes direction repeatedly.

Alternating current is different to **direct current** (DC). This means that the current supplied is a single, constant direction.

The electricity supplied to homes in the UK is AC, with a **potential difference of 230 V**.

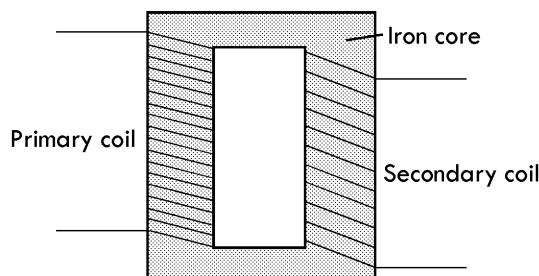
Transformers

When transmitting electricity, the current needs to be as low as possible to reduce power loss. For a given power, low current means high voltage (because $P = IV$ so $V = \frac{P}{I}$).

For this reason, electricity in power cables is transmitted at 400 kV, whereas it reaches homes at 230 V, or large businesses at 33 kV.

This change in potential differences is achieved through the use of **transformers**.

A transformer consists of two coils of wire linked by an iron core.



An alternating current in the primary coil induces an alternating current in the secondary coil. The ratio of the induced potential difference to the primary potential difference is equal to the ratio of the number of turns in the secondary coil to the number of turns in the primary coil.

The equation linking the number of turns in a transformer to the potential difference is:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

N_p = number of turns in primary coil
 N_s = number of turns in secondary coil
 V_p = potential difference across primary coil
 V_s = potential difference across secondary coil

THE GENERATOR EFFECT

When a coil of wire moves through a magnetic field, a current is produced in the conductor.

This is called the **generator effect** and is how most electricity is generated. In electrical generators, steam is heated up, creating high pressures and fast flows which are used to rapidly rotate a coil of wire in a magnetic field, generating a current in the coil.

The generator effect is also used in microphones, and other appliances where motion needs to be turned into a signal.

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POP QUIZ

You can use this quiz to test how solid your knowledge of GCSE electricity is. If you know the content well, you'll have a much better shot at getting top marks in your A Level.

- Fill in the gaps.

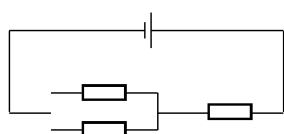
Current is the rate of flow of _____.

_____ is the energy transferred to a charge.

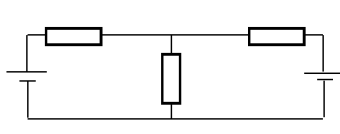
For a given potential difference, a lower-resistance component will have a _____ current flow through it.

- 23 C of charge flows through a bulb over the course of one minute. Calculate the current flowing through the bulb.

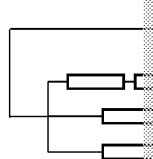
- For each of the circuits below, state whether or not a current will flow, and why.



Circuit A



Circuit B

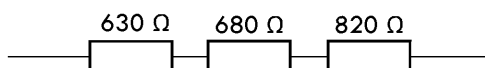


- Match each component with its symbol and use.

Component	Symbol
Thermistor	
LDR (light-dependent resistor)	
Voltmeter	
Variable resistor	
Ohmmeter	
Cell	
Switch	
Diode	
Resistor	
Capacitor	
Alloy	

- 850 mA flows through a resistor with a potential difference of 52 V across it. Calculate the resistance of the resistor.

- a) Calculate the total resistance of the resistors below.



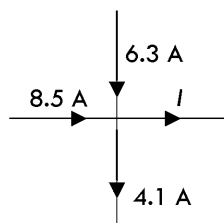
- What would the effect be of putting the resistors in parallel? Explain.

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7. Calculate the value of I in the section of circuit seen below.



8. Fill in the gaps to describe Kirchhoff's laws.

Kirchhoff's first law states that the current into a junction equals _____

Kirchhoff's second law states that the potential difference provided by a p

_____.

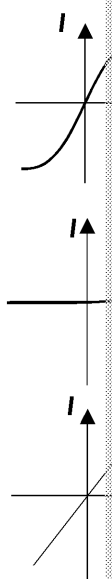
9. Match the components with their I - V graphs.



Ohmic conductor

Filament lamp

Diode



10. Draw a circuit which could be used to measure the resistance of a component.
11. a) A $220\ \Omega$ resistor has $190\ \text{mA}$ flowing through it.
Calculate the power transferred to the component.
- b) $370\ \text{J}$ of energy is transferred to $8.3\ \text{C}$.
Calculate the potential difference which transfers the energy.
12. Describe how and why transformers are used in the National Grid.
13. State one of the generator effect.



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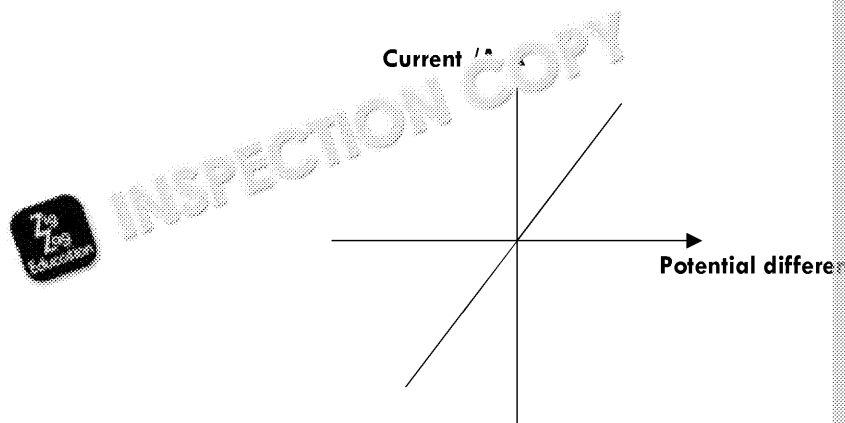
1. I-V CHARACTERIS

4.2.3: RESISTANCE

BACKGROUND

Different components respond to high and low currents differently, which can be shown on an I - V graph.

The I - V graph of an ohmic conductor is seen below.

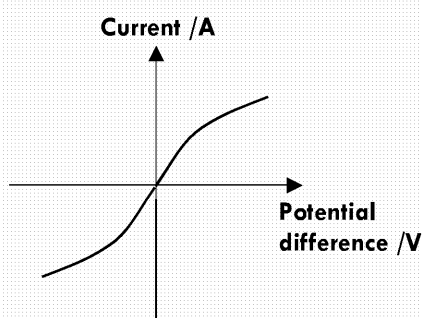


Resistance is given by $R = \frac{V}{I}$, and for an I - V graph gradient = $\frac{I}{V}$, so $R = \frac{1}{\text{gradient}}$.

You may encounter I - V graphs with potential difference on the y -axis and current on the x -axis. This is called a V - I diagram. For a V - I graph, $R = \text{gradient}$.

Example 1

The I - V graph of a filament lamp is shown below.



Describe the resistance of a filament lamp at different currents.

EXTENSION

Explain the properties of a filament lamp in terms of charge carriers.

For an I - V graph, gradient = $\frac{I}{V} = \frac{1}{R}$. A steeper gradient means a lower resistance, and a shallower gradient means a higher resistance.

The gradient is shallower at more potentials and potentials than near zero, so the resistance of the lamp increases with current and potential difference.

EXTENSION:

At higher currents, more power is dissipated and the lamp heats up.

As the filament heats up, atoms vibrate more vigorously and increasing resistance.

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EXAM TIP
Remember to check out which axis is current and which is potential difference.

Example 2

Current only passes through a semiconductor diode above a positive potential difference threshold, V_0 .

Above this threshold, the resistance of the diode decreases with increased current.

Current will flow at very high negative potential differences.

Draw the $V-I$ graph you would expect for a semiconductor diode.



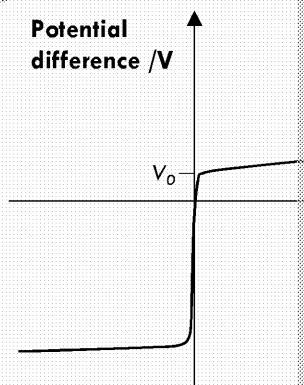
A $V-I$ graph has potential difference, V , on the y-axis and current, I , on the x-axis.

A current will only flow above V_0 , so below this value on the y-axis, $I = 0$.

Above V_0 , resistance decreases. $R = \frac{V}{I}$ so $R \propto \text{gradient}$, so the gradient will decrease.

At high negative potential differences, I is very large.

This means the $V-I$ graph for a semiconductor diode is:



QUESTIONS

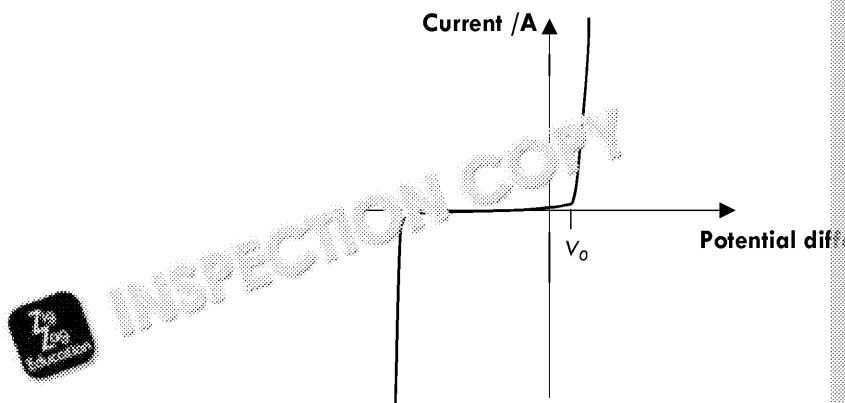
Bright spark

- The circuit to the right is used to measure the resistance of a metal resistor with different potential differences across the resistor.

As the current through the resistor increases, it heats up. Sketch the $I-V$ graph you would expect for the resistor.



- Explain the features of the graph seen below.



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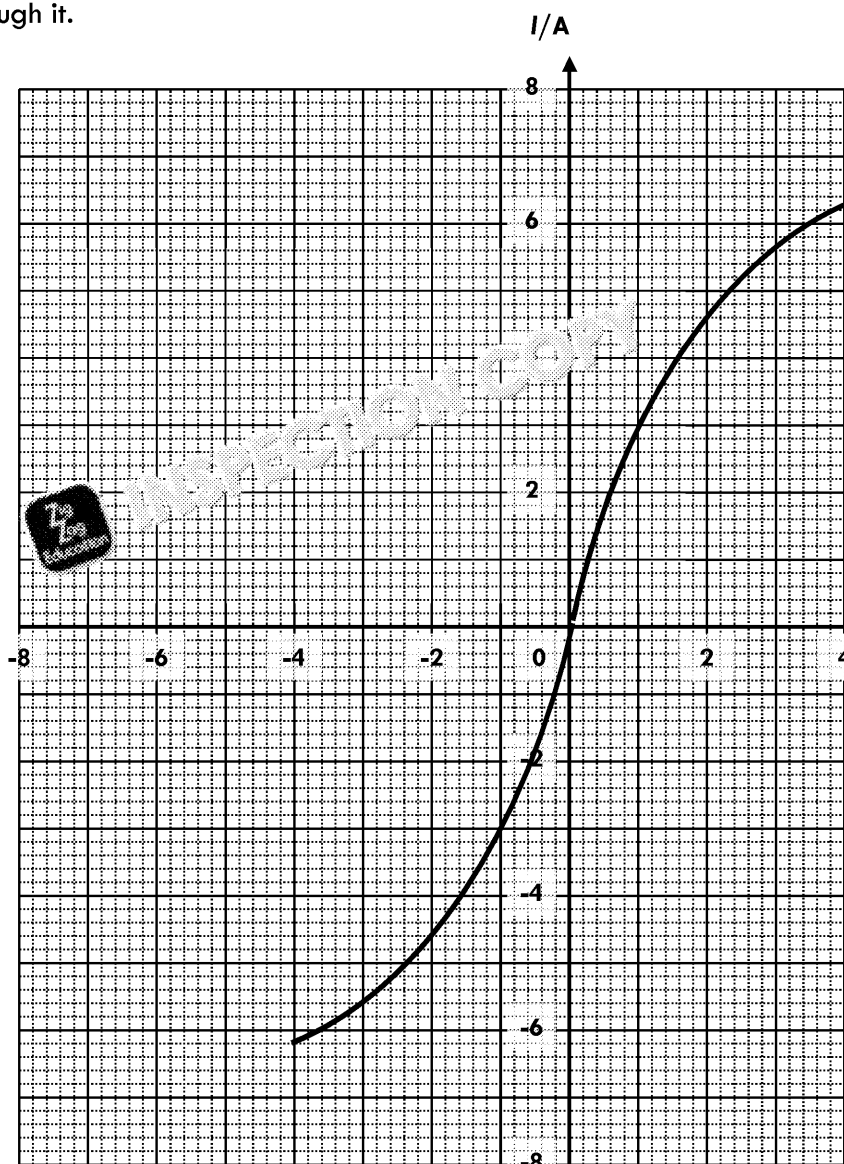


Charging up

- The current through and potential across a negative temperature coefficient thermistor measured at a number of currents and temperatures.

Sketch the $I-V$ graphs you would expect for the thermistor at different temperatures. Label the features of the graph.

- Describe an experiment you could perform to investigate the I - V characteristic of a component.
- Calculate the resistance when the component shown by the graph below has a current of 2.0 A flowing through it.



Shocking!

- A component consists of a negative region with an excess of electrons and a positive region with a lack of electrons.

Electrons can flow easily from the negative region to the positive region, as the electrons are attracted to the positive region.

As the current increases, more electrons are liberated from atoms in the negative region, and the resistance decreases.

Predict the shape of the I - V graph of this component.

- A memristor is a component which can record its previous resistance, and change its resistance accordingly.

At high currents, the memristor always has the same resistance. At low currents, it has two different resistances, depending on the previously 'remembered' resistance.

A memristor switches between high and low resistances at high currents, but is smooth at low currents.

If a memristor is currently at its higher resistance, it will remain at its higher resistance at high currents.

Sketch an I - V graph for a memristor.

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2. RESISTIVITY

4.2.4: RESISTIVITY

BACKGROUND

The resistance of an object – how well it can stop or conduct a current – depends on a couple of different factors.

- **Material:** A conductor, such as metal, will have a lower resistance than an insulator, such as wood or rubber, because it's easier for a current to pass through it.
- **Dimensions:** A longer wire will have a higher resistance because there's more current. A narrower wire will have a higher resistance as well because there's more current to pass through.

The resistance of an object is given by the following equation:

$$R = \frac{\rho L}{A}$$

or by rearranging it as follows:

$$\rho = \frac{RA}{L}$$

Example 1

Copper is used in electrical wires. 1.55 km of copper has a resistance of 20.7 mΩ.

The cross-section of the copper wire is a circle with a radius of 2.00 cm.

Calculate the resistivity of the copper wire.

EXTENSION:

An additional 380 m of wire is added to the copper wire.

Calculate the new resistance of the wire.

We have $R = 20.7 \times 10^{-3} \Omega$, $L_1 = 1.55 \text{ km}$
 $r_1 = 2.00 \text{ cm} = 2.00 \times 10^{-2} \text{ m}$.

First, we need the cross-sectional area of the wire.

$$A = \pi r^2$$

$$A = \pi \times (2.00 \times 10^{-2})^2$$

$$A = 1.257 \times 10^{-3} \text{ m}^2$$

Now we can calculate the resistivity of the wire.

$$\rho = \frac{RA}{L}$$

$$\rho = \frac{20.7 \times 10^{-3} \times 1.257 \times 10^{-3}}{1.55 \times 10^3}$$

$$\rho = 1.68 \times 10^{-8} \Omega \text{ m}$$

EXTENSION:

The resistivity of the copper is constant and hasn't been changed.

The new length of the copper wire is 1550 m.

$$R = \frac{\rho L}{A}$$

$$R = \frac{1.68 \times 10^{-8} \times 1930}{1.257 \times 10^{-3}}$$

$$R = 25.7 \text{ m}\Omega$$

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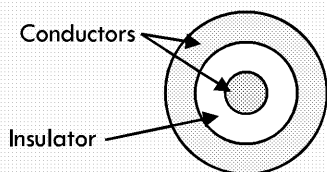


EXAM
Remember
use SI
units
In this
conversion

Example 2

A cable consists of a central wire, with a conductive ring around the central wire. An insulating ring lies between the wire and outer conductive ring.

The cross-section of the cable is seen here.



- In the cable, the inner wire is made of copper, with a resistivity of $1.68 \times 10^{-8} \Omega \text{ m}$. The cross-sectional area of the inner wire is $2.80 \times 10^{-5} \text{ m}^2$.
- The outer ring is made of iron, which has a resistivity of $9.71 \times 10^{-8} \Omega \text{ m}$. The cross-sectional area of the outer ring is $1.60 \times 10^{-4} \text{ m}^2$.
- A potential difference is applied across the cable.

Calculate the total resistance of 1.00 m of the cable.

The resistance of an object

$$R = \frac{\rho L}{A}$$

So the resistance of the inner

$$R_{\text{inner}} = \frac{1.68 \times 10^{-8} \times 1.00}{2.80 \times 10^{-5}}$$

$$R_{\text{inner}} = 6.000 \times 10^{-4} \Omega$$

And the resistance of the outer

$$R_{\text{outer}} = \frac{9.71 \times 10^{-8} \times 1.00}{1.60 \times 10^{-4}}$$

$$R_{\text{outer}} = 6.069 \times 10^{-4} \Omega$$

The wire and ring are in parallel, so the resistance is given by

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_{\text{inner}}} + \frac{1}{R_{\text{outer}}}$$

$$R_{\text{total}} = \frac{1}{1/R_{\text{inner}} + 1/R_{\text{outer}}}$$

$$R_{\text{total}} = \frac{1}{1/(6.000 \times 10^{-4}) + 1/(6.069 \times 10^{-4})}$$

$$R_{\text{total}} = 3.02 \times 10^{-4} \Omega$$

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QUESTIONS

Bright spark

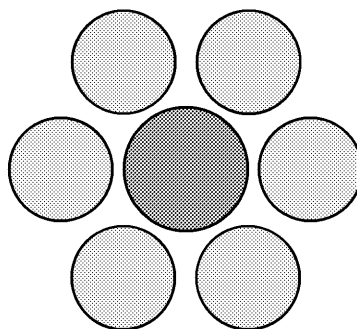
1. An iron bar has a length of 37.1 cm, and its cross-section is a square with side 1.25 cm.
The resistivity of iron is $9.71 \times 10^{-8} \Omega \text{ m}$.
Calculate the resistance of the iron bar.
2. A wire has a resistivity of $6.72 \times 10^{-7} \Omega \text{ m}$, and a resistance of 92.3 m Ω .
The cross-sectional area of the wire is 7.07 mm².
Calculate the length of the wire.

Charging up

3. A 3.00 cm diameter wire is cut from 2.40 m to 1.90 m.
The resistance of the wire decreases by 83.4 Ω .
Calculate the resistivity of the wire.
4. The resistivity of lead is $2.20 \times 10^{-7} \Omega \text{ m}$. A lead pipe has an inner radius of 8.30 cm for a length of 1.60 m. The pipe then widens so that it has an inner radius of 10.0 cm and an outer radius of 9.31 cm for 23.0 cm.
Calculate the resistance of the lead pipe.

Shocking!

5. A cable consists of a carbon wire, with a resistivity of $4.56 \times 10^{-6} \Omega \text{ m}$, surrounded by six aluminium wires, with a resistivity of $2.82 \times 10^{-8} \Omega \text{ m}$.



The radius of the carbon wire is 8.50 mm. The radius of each aluminium wire is 1.25 mm.

Calculate the power dissipated by 200 m of the cable carrying 320 mA.

6. A lump of conductive putty is moulded into a cylinder. The length of the cylinder is 10.0 cm and the radius is 0.662 cm.
The resistance of the putty cylinder is 31.8 Ω .
The same volume of putty is remoulded into a cylinder with a length of 5.00 cm.
Calculate the resistance of the new cylinder.

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3. POWER

4.2.5: POWER

BACKGROUND

A potential difference across a wire causes a transfer of energy. This can either be useful energy, such as causing a motor to turn, or wasted energy, such as a wire heating when a current flows through it.

The power transferred in a component is given by:

which can be written as:

or:

$$P = VI$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

Example 1

There is a potential difference of 15 V across a 40 Ω resistor.

Calculate the energy transferred across the resistor over one hour.

We have $V = 15 \text{ V}$ and $R = 40 \Omega$.

The equation $P = \frac{V^2}{R}$ will give the power transferred. We want the total energy.

Power is defined as the energy transferred per unit time, so

$$E = Pt = \frac{V^2}{R} t$$

In this case, $t = 1 \text{ hour} = 60 \times 60 = 3600 \text{ s}$

Substituting in values gives

$$E = \frac{15^2}{40} \times 3600$$

$$E = 20250 \text{ J}$$

Example 2

A kettle has a power rating of 80 W. The element of the kettle has a resistance of 600 Ω . Calculate the charge transferred to the element of the kettle in 2 minutes.

We have $P = 80 \text{ W}$ and $R = 600 \Omega$ and want charge.

Current is defined as the charge through a component per unit time, $Q = It$.

In this example, $t = 2 \text{ minutes} = 2 \times 60 = 120 \text{ s}$

To find current, we can use

$$P = I^2 R$$

which rearranges to

$$I = \sqrt{\frac{P}{R}} \quad \text{so} \quad Q = \sqrt{\frac{P}{R}} t$$

Substituting in values gives

$$Q = \sqrt{\frac{80}{600}} \times 120$$

$$Q = 43.8 \text{ C}$$

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Example 3

A power station produces 2.0 MW of power and transmits a potential difference 275 kV through wires with a resistance of 650 Ω .

Calculate the power dissipated by the cables.

There are two powers in this situation – the power produced by the power station, P , and the power dissipated by the cables, $P_{\text{dissipated}}$. It's important not to get these two mixed up.

The power dissipated by the cables is given by

$$P_{\text{dissipated}} = I^2 R$$

Substituting $I = \frac{P}{V}$ gives

$$I^2 R = \frac{P^2 R}{V^2}$$

Substituting in values gives

$$I^2 R = \frac{P^2 R}{V^2}$$

$$I^2 R = \frac{(2.0 \times 10^6)^2 \times 650}{(275 \times 10^3)^2}$$

$$I^2 R = 34 \text{ kW}$$



QUESTIONS

Bright spark

1. A bulb gives out a power of 40 W. It is connected to mains electricity, with a potential difference of 230 V.

Calculate the resistance of the filament in the bulb.

2. A resistor has a resistance of 38.6 k Ω and a current of 77.1 mA running through it. Calculate the energy dissipated by the resistor.

Charging up

3. A crane is powered by a cable which has a resistance of 855 Ω and a potential difference of 100 V across it.

Over five minutes, the crane lifts a 3150 kg crate by 9.39 m.

Calculate the efficiency of the crane.

4. 146 J of energy is transferred to a heating element over one minute. The resistance of the element is 95.4 k Ω .

Calculate the charge that passes through the heating element.

Shocking!

5. While being used, the cables that power magnets in a particle accelerator dissipate 1.5 MW of power. The cables have a potential difference of 30.9 MV across them.

The cables have a resistance of 408 k Ω .

Calculate the efficiency of the cables which supply the magnets used in the accelerator.

6. A magnetic propulsion system consists of a track which accelerates 1035 kg of a train from 0 to 68.6 m s⁻¹ in 5.36 s.

The tracks dissipate a power of 721 kW, and have a resistance of 907 Ω over a length of 25.6 kV across them.

Calculate the efficiency of the magnetic propulsion system.

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4. RESISTORS IN SERIES AND IN PARALLEL

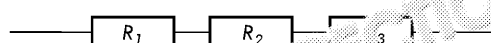
4.3.1: SERIES AND PARALLEL CIRCUITS

BACKGROUND

Current and potential difference act very differently in series and in parallel.

	Current	Potential difference
In series...	Current is the same at all points	Potential difference is split across each component
In parallel...	Current is split across each 'branch' of the circuit	Potential difference is the same across each component

The total resistance of components in series is:



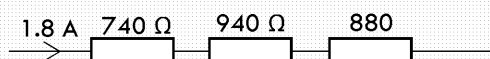
$$R_{total} = R_1 + R_2 + R_3 + \dots$$

The total resistance of components in parallel is:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Example 1

Calculate the potential difference across the section of circuit below.



The current is the same at each point in a series circuit, so we can treat all three resistors as one.

These components are in **series**, so:

$$R_{total} = R_1 + R_2 + R_3$$

Substituting in values gives

$$R_{total} = 740 + 940 + 880$$

$$R_{total} = 2560 \, \Omega$$

Use $V = IR$

and substitute in values to give

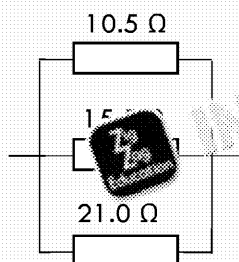
$$V = 1.8 \times 2560$$

$$V = 4.6 \, \text{kV}$$

Another method would be to find the potential difference across each resistor and then add them together.

Example 2

Calculate the total resistance of the section of circuit below.



These components are in **parallel**, so we need to use the reciprocal rule:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

We first need to rearrange for R_{total}

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

then substitute in values

$$R_{total} = \frac{1}{\frac{1}{10.5} + \frac{1}{15.5} + \frac{1}{21.0}}$$

$$R_{total} = 4.82 \, \Omega$$

Note: for components in parallel, the total resistance is always less than the resistance of any one component!

You might find it easier to first find

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{ (in this case } 0.2074 \text{) and then divide 1 by this value.}$$

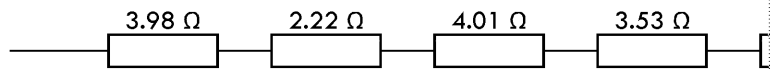
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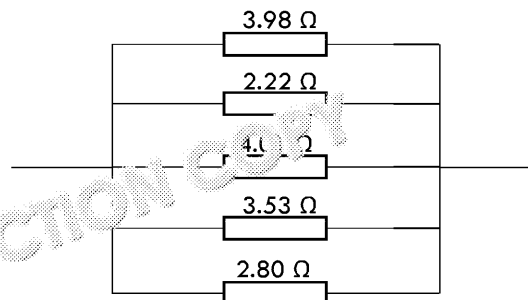
QUESTIONS

Bright spark

1. Calculate the combined resistance of the components below.

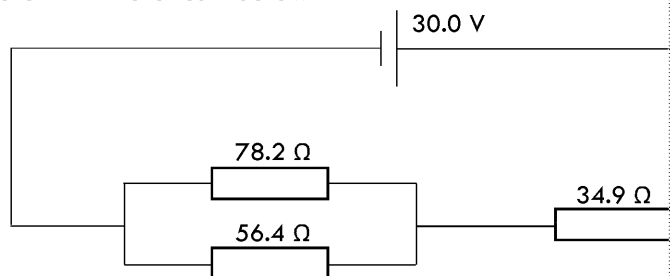


2. Calculate the combined resistance of the components below.

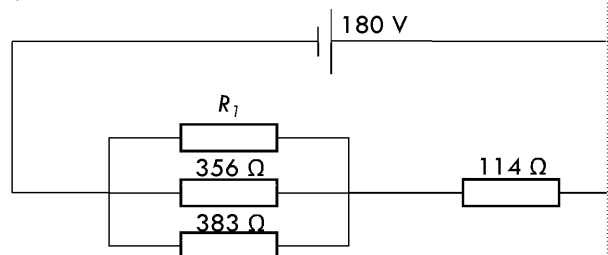


Charging up

3. Calculate the value of I in the circuit below.

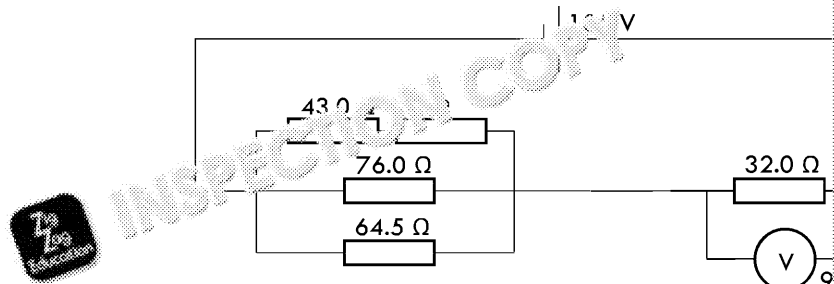


4. Calculate the value of R_1 in the circuit below.



Shocking!

5. Calculate the value of R in the circuit below.



6. A circuit is set up with a power source, an ammeter, a $12\ \Omega$ resistor and R_1 . The ammeter records a reading of 5 A .

The resistor, R_1 , is replaced with another resistor, R_2 , and the reading on the ammeter is 2 A .

A new set up was then arranged with a new power source, which supplies 26 V , an ammeter, and the two resistors, R_1 and R_2 , in series. The ammeter records a reading of 1 A .

Calculate R_1 and R_2 .

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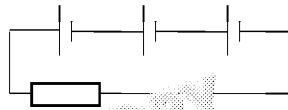
5. MULTIPLE POWER SOURCES

4.3.1: SERIES AND PARALLEL CIRCUITS

BACKGROUND

Power sources can be placed in series or in parallel.

Series

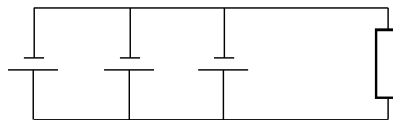


For a series circuit, the e.m.f. delivered to the circuit is the sum of the e.m.f.s of the cells.

$$\mathcal{E}_{\text{total}} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \dots$$

For power sources in series, the current can actually decrease – adding cells also adds internal resistance, so the total resistance of the circuit increases.

Parallel



Only cells providing the same e.m.f. can be put into parallel, and the e.m.f. is equal to the e.m.f. of a single cell.

$$\mathcal{E}_{\text{total}} = \mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_3 = \dots$$

When putting power sources in parallel, the current tends to increase compared to putting the cells in series. Putting the cells in parallel decreases the combined internal resistance, so the current increases.

Internal resistance

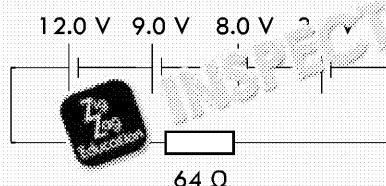
For a cell with a non-negligible internal resistance, the e.m.f. of the cell is given by:

$$\mathcal{E} = I(R + r)$$

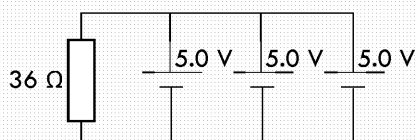
Example 1

Calculate the currents through the resistors in the circuits below.

Circuit 1



Circuit 2



Circuit 1

For a series circuit

$$\mathcal{E}_{\text{total}} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \mathcal{E}_4$$

Substituting in values gives

$$\mathcal{E}_{\text{total}} = 12.0 + 9.0 + 8.0 + 3.0$$

$$\mathcal{E}_{\text{total}} = 32 \text{ V}$$

which means there's a potential difference of 32 V across the resistor.

$$V = IR \rightarrow I = \frac{V}{R}$$

Substituting gives

$$I = \frac{32}{64}$$

$$I = 0.50 \text{ A}$$

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Example 2

In an experiment, the following components must be used:

$$\begin{aligned} 8 \times & \text{---|---|---} \quad \varepsilon = 28 \text{ V} \\ & \quad \quad \quad r = 0.80 \, \Omega \\ 1 \times & \text{---[]---} \quad R = 140 \, \Omega \end{aligned}$$

Determine whether the cells should be placed in series or parallel to reduce the dissipation of energy in the circuit.

The power dissipated by a resistor is given by

For cells in series

$$\varepsilon_{\text{series}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \dots$$

so

$$\varepsilon_{\text{series}} = 8 \times 28$$

$$\varepsilon_{\text{series}} = 224 \text{ V}$$

For cells in parallel

$$\varepsilon_{\text{parallel}} = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \dots$$

$$\varepsilon_{\text{parallel}} = 28 \text{ V}$$

for a cell with negligible internal resistance

$$\varepsilon = I r$$

which rearranges to

$$I = \frac{\varepsilon}{R - r}$$

However, we can't use $r = 0.80 \, \Omega$ as this is for a single cell. We have to use the effective internal resistance

For cells in series

$$r_{\text{series}} = r_1 + r_2 + r_3 + \dots$$

$$r_{\text{series}} = 8 \times 0.80$$

$$r_{\text{series}} = 6.4 \, \Omega$$

$$I_{\text{series}} = \frac{\varepsilon_{\text{series}}}{R - r_{\text{series}}}$$

$$I_{\text{series}} = \frac{224}{140 - 6.4}$$

$$I_{\text{series}} = 1.68 \text{ A}$$

For cells in parallel

$$\frac{1}{r_{\text{parallel}}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots$$

For n power sources in parallel

$$\frac{1}{r_{\text{parallel}}} = \frac{n}{r}$$

$$r_{\text{parallel}} = \frac{r}{n}$$

$$r_{\text{parallel}} = \frac{0.80}{8}$$

$$r_{\text{parallel}} = 0.10 \, \Omega$$

$$I_{\text{parallel}} = \frac{\varepsilon_{\text{parallel}}}{R - r_{\text{parallel}}}$$

$$I_{\text{parallel}} = \frac{28}{140 - 0.10}$$

$$I_{\text{parallel}} = 0.20 \text{ A}$$

The parallel arrangement draws less current than the series arrangement, so this dissipates less power in the resistor placed in parallel.

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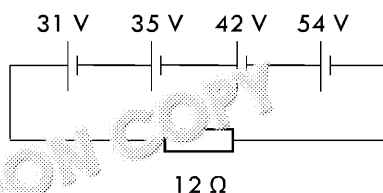


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QUESTIONS

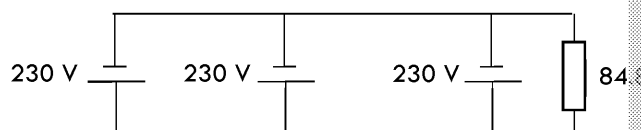
Bright spark

1. A circuit consists of 54 separate 18 V cells in parallel.
These cells are connected to a $13\ \Omega$ resistor.
Calculate the current through the resistor and a single cell.
2. Calculate the current in the circuit below.

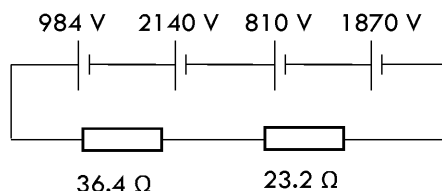


Charging up

3. Calculate the power dissipated by the circuit below.

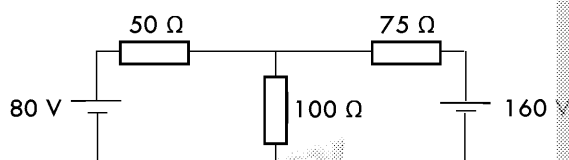


4. Calculate the power dissipated by the circuit below.

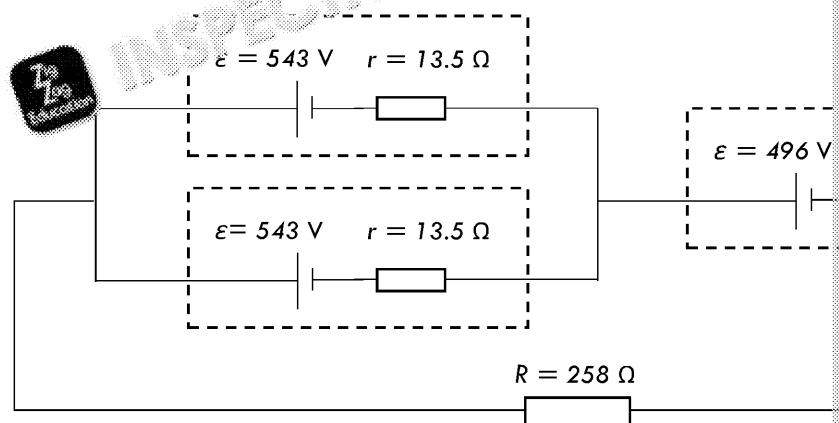


Shocking!

5. Calculate the current through each resistor in the circuit below.



6. Calculate the total power dissipated in the circuit below.



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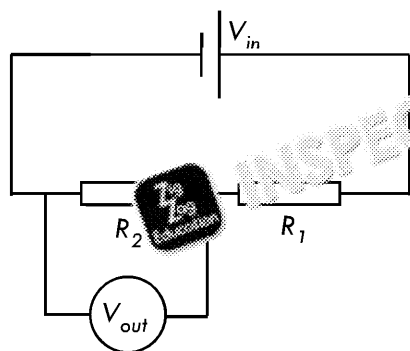
6. POTENTIAL DIVIDER

4.3.3: POTENTIAL DIVIDERS

BACKGROUND

A potential divider is used to create an output potential difference that is a specific fraction of the input potential difference.

The layout of a simple potential divider can be seen below.



The equation for the output voltage of a potential divider is

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

or

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_{total}}$$

The derivation

This can be derived by considering the current through the entire circuit.

$$V_{in} = IR_{total} = I(R_1 + R_2)$$

$$I = \frac{V_{in}}{R_1 + R_2}$$

$V = IR$ so

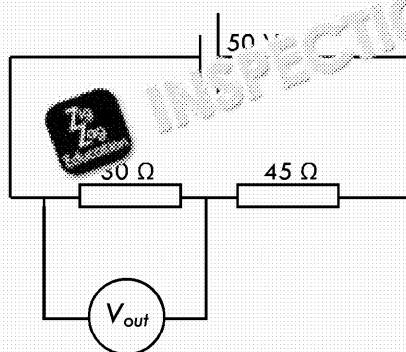
$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

These equations are **not** given in the data booklet, so make sure you learn them!

Example 1

A circuit is shown below.

Calculate the value of V_{out} .



We need to use

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

With $V_{in} = 50 \text{ V}$, $R_2 = 30 \Omega$

We want V_{out} , so there's no problem with the equation.

Putting the numbers in gives

$$V_{out} = 50 \times \frac{30}{30 + 45}$$

$$V_{out} = 20 \text{ V}$$

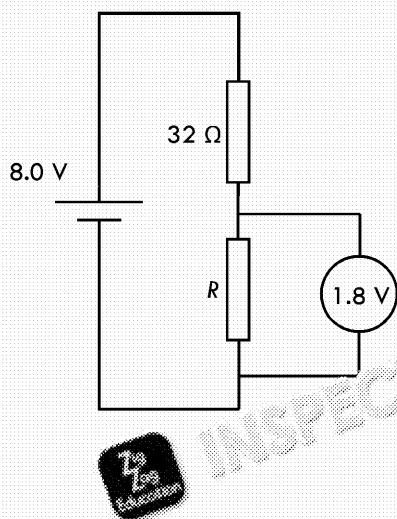
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Example 2

A circuit is shown below.
Calculate the value of R .



We need to use

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

With $V_{out} = 1.8 \text{ V}$, $V_{in} = 8.0 \text{ V}$ and

We want $R = R_2$, so we'll need to

$$V_{out}(R_1 + R_2) = V_{in}R_2 \quad [\text{multiplied}]$$

$$V_{in}R_2 - V_{out}R_2 = V_{out}R_1 \quad [\text{multiplied}]$$

$$R_2(V_{in} - V_{out}) = V_{out}R_1 \quad [\text{taken}]$$

$$R_2 = \frac{V_{out}R_1}{V_{in} - V_{out}} \quad [\text{divided}]$$

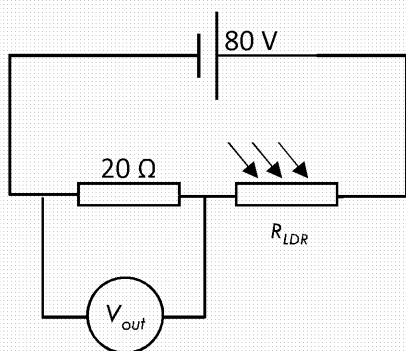
and substitute values in

$$R = \frac{1.8 \times 32}{8.0 - 1.8}$$

$$R = 9.3 \Omega$$

Example 3

A light-sensing circuit is seen below.



At $I = 30 \text{ W m}^{-2}$, V_{out} is 40 V.

Calculate R_{LDR} at $I = 90 \text{ W m}^{-2}$.

For this circuit, $R_{LDR} \propto I$

We first need to find R_{LDR} at 30 W m^{-2}

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

Rearrange for R_1

$$V_{out}(R_1 + R_2) = V_{in}R_2 \quad [\text{multiplied}]$$

$$V_{out}R_1 = V_{in}R_2 - V_{out}R_2 \quad [\text{multiplied from both sides}]$$

$$R_1 = \frac{V_{in}R_2 - V_{out}R_2}{V_{out}} \quad [\text{divided}]$$

and substitute values in

$$R_{LDR} = \frac{80 \times 20 - 40 \times 20}{40}$$

$$R_{LDR} = 20 \Omega$$

We know that $R_{LDR} \propto I$

so

$$\frac{R_{LDR 30}}{R_{LDR 90}} = \frac{I_{30}}{I_{90}}$$

Rearrange for $R_{LDR 90}$

$$R_{LDR 90} = \frac{I_{90}}{I_{30}} \times R_{LDR 30} \quad [\text{multiplied}]$$

both sides

Substitute values in

$$R_{LDR 90} = \frac{90}{30} \times 20$$

$$R_{LDR 90} = 60 \Omega$$

EXAM TIP

This question has a lot of variables.
Variables at a light intensity of $x \text{ W m}^{-2}$.
So the resistance of the LDR at 30 W m^{-2} is 20 Ω .

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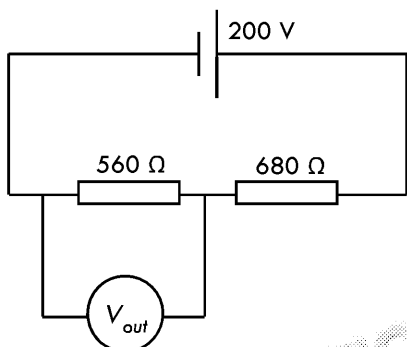
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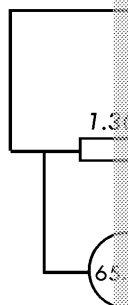
QUESTIONS

Bright spark

1. Calculate the value of V_{out} in the circuit below.

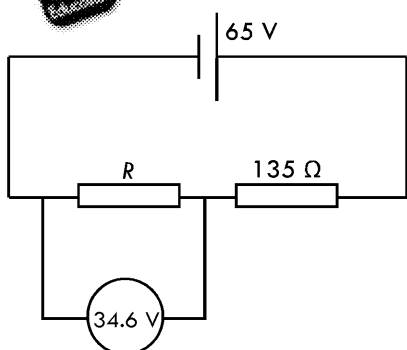


2. Calculate V_{in} in the circuit below.

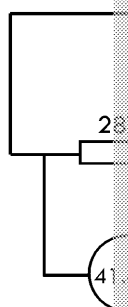


Charging up

3. Calculate the value of R in the circuit below.



4. Calculate R in the circuit below.

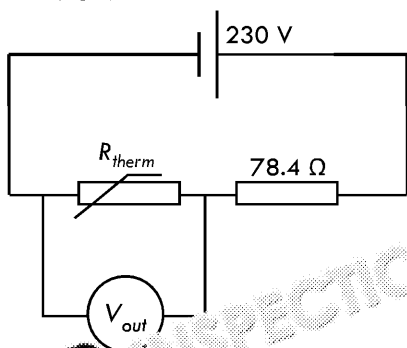


Shocking!

5. At 200 K, R_{therm} is 147 Ω.

The response of the thermistor is $R_{therm} \propto \frac{1}{T}$.

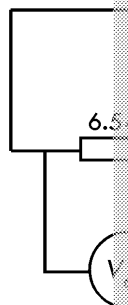
Calculate the temperature at which $V_{out} = 170$ V.



6. At 30.0 W m⁻², R_{therm} is 147 Ω.

The response of the thermistor is $R_{therm} \propto \frac{1}{T}$.

Calculate the temperature at which $V_{out} = 54.1$ V.



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7. DIELECTRICS

6.2.3: UNIFORM ELECTRIC FIELD

BACKGROUND

A capacitor is a component which stores charge, and dissipates that charge over time.

A capacitor consists of a dielectric (an insulating material) between two conducting plates.



The capacitance of a capacitor is given by

$$C = \frac{Q}{V}$$

For a parallel plate capacitor

$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

The energy stored by a capacitor is

$$E = \frac{1}{2} QV$$

C = capacitance
Q = charge
V = potential difference
A = area of plate
 ϵ_0 = permittivity of free space
 ϵ_r = relative permittivity of dielectric
d = distance between plates
E = energy

Example 1

A capacitor has a capacitance of $840 \mu\text{F}$ and stores a charge of 32 mC .

Calculate the energy stored by the capacitor.

Use

$$E = \frac{1}{2} QV$$

but we need it in a form with charge and capacitance difference.

$$V = \frac{Q}{C}$$

so

$$E = \frac{1}{2} Q \times \frac{Q}{C} = \frac{1}{2} \frac{Q^2}{C}$$

Substituting in values gives

$$E = \frac{1}{2} \times \frac{(32 \times 10^{-3})^2}{840 \times 10^{-6}}$$

$$E = 0.61 \text{ J}$$

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Example 2

A conductive sheet has an area of 78 cm². Placed 6.0 cm above the sheet is another conductive sheet with an area of 32 cm².

A dielectric is placed between the two sheets, such that the capacitance of the set-up is 12 nF.

Calculate the relative permittivity of the dielectric.

For the equation $C = \frac{A\epsilon_0\epsilon_r}{d}$, A refers to the area of the two plates.

This means that for this arrangement the size of the smaller sheet, 32 cm², is used.

Rearrange for ϵ_r

$$\epsilon_r = \frac{Cd}{A\epsilon_0}$$

Substitute in values

$$\epsilon_r = \frac{12 \times 10^{-9}}{32 \times 10^{-4}} \times \frac{6 \times 10^{-2}}{8.85 \times 10^{-12}}$$

$$\epsilon_r = 2.54 \times 10^{10}$$



EXAM TIP

The potential of an electric field at a distance r from a charge Q is given by:

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

This only applies in a vacuum and to conductors.

$$V = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{Q}{r}$$

This is the general form of the equation. For a vacuum, $\epsilon_r = 1$. This equation lets us find the potential in a capacitor.

Example 3

Calculate the capacitance of a metal sphere the size of Earth.

To do this we can use $C = \frac{Q}{V}$, but first we have to find the potential at a distance r away from a charge Q .

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Putting this into the equation for capacitance gives

$$C = \frac{Q}{V} = \left[\frac{Q}{\frac{Q}{4\pi\epsilon_0 r}} \right]$$

Rearranging, this gives

$$C = \frac{Q}{Q} \times 4\pi\epsilon_0 r = 4\pi\epsilon_0 r$$

For a sphere the size of Earth, with $r = 6.37 \times 10^6$ m

$$C = 4\pi \times 8.85 \times 10^{-12} \times 6.37 \times 10^6$$

$$C = 7.08 \times 10^{-4} \text{ F}$$



EXAM TIP

ϵ_r is used for relative permittivity when talking about capacitors. Make sure you use the correct value.

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QUESTIONS

Bright spark

1. The plates of a parallel plate capacitor have an area of 1.44 cm^2 and the capacitance of the capacitor is $963 \text{ }\mu\text{F}$.
Calculate the relative permittivity of the dielectric in the capacitor.
2. A capacitor has a capacitance of $185 \text{ }\mu\text{F}$ and stores a charge of 81.4 mC .
Calculate the potential difference across the capacitor and the energy stored.

Charging up

3. A capacitor has a capacitance of $24 \text{ }\mu\text{F}$. The capacitor has a resistance of $41.6 \text{ }\Omega$ and a current of 41.6 mA flowing through it.
Calculate the energy stored in the capacitor in J.
4. A capacitor consists of two circular parallel plates with a radius of 7.24 cm apart, and the dielectric constant between them has a relative permittivity of 3.40 .
Calculate the energy stored in the capacitor in kJ when there is a potential difference of 100 V across the two plates.

Shocking!

5. A storm cloud covers an area of 134 km^2 .
A lightning strike represents a potential difference of $6.14 \times 10^8 \text{ V}$ and a charge of 25 C is transferred.
Calculate the height of the storm cloud above the ground.
6. A spherical capacitor consists of a small metal sphere at the centre of a larger, hollow metal sphere, with a dielectric between, as shown.
 $R_1 = \text{radius of internal sphere} = 2.43 \text{ cm}$
 $R_2 = \text{radius of external sphere} = 6.09 \text{ cm}$
 $\epsilon_r = 3.40 \times 10^{10}$
Calculate the capacitance of the spherical capacitor.

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8. CAPACITORS IN SERIES AND

6.1.1: CAPACITORS

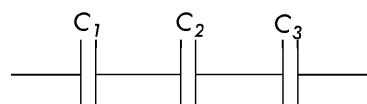
BACKGROUND

Capacitors are often only available in specific values of capacitance, but a range of higher or lower capacitance than those available. Like resistors, capacitors can be used in different arrangements, allowing a wide range of capacitance from few capacitors with

KEY SKILLS

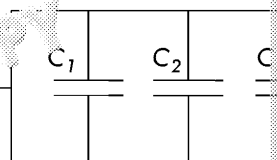
Perform calculations involving capacitors in series and parallel

For capacitors in series



$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

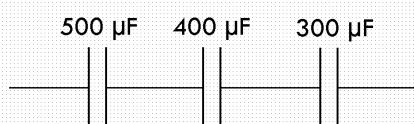
For capacitors in parallel



$$C_{total} = C_1 + C_2 + C_3$$

Example 1

Calculate the total capacitance of the arrangement of capacitors below.



For capacitors in series

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Substituting in values gives

$$\frac{1}{C_{total}} = \frac{1}{500 \times 10^{-6}} + \frac{1}{400 \times 10^{-6}}$$

$$\frac{1}{C_{total}} = 7833$$

Rearranging for C_{total} gives

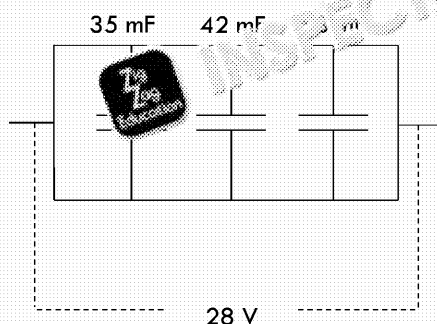
$$C_{total} = 1.28 \times 10^{-4} \text{ F}$$

$$C_{total} = 128 \mu\text{F}$$

As with resistors in parallel, you might prefer to rearrange and then substitute all values. Alternatively, you might prefer to start by substituting in values for the first two capacitors to get an answer, but both methods need you to use your calculator!

Example 2

Calculate the total energy of the arrangement of capacitors below.



For capacitors in parallel

$$C_{total} = C_1 + C_2 + C_3$$

Substituting in values

$$C_{total} = 35 \times 10^{-3} + 42 \times 10^{-3} + 6 \times 10^{-3}$$

$$C_{total} = 145 \text{ mF}$$

The capacitors act like one single capacitor, storing the same total energy as the arrangement of capacitors.

$$E = \frac{1}{2} C_{total} V^2$$

Substituting in values gives

$$E = \frac{1}{2} \times 145 \times 10^{-3} \times 28^2$$

$$E = 57 \text{ J}$$

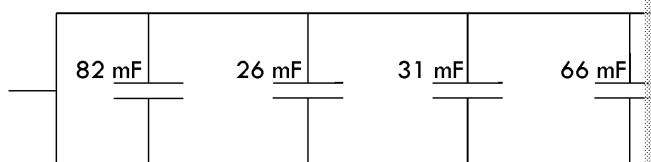
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QUESTIONS

Bright spark

1. Calculate the combined capacitance of the arrangement of capacitors below.



2. Calculate the combined capacitance in F of the arrangement of capacitors below.

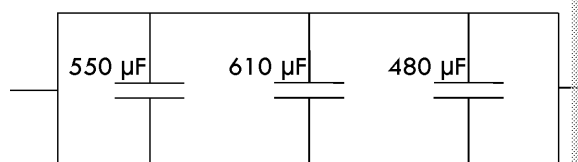


Charging up

3. The capacitors below can have a maximum potential difference of 500 V. Calculate the maximum total charge stored in the capacitors.



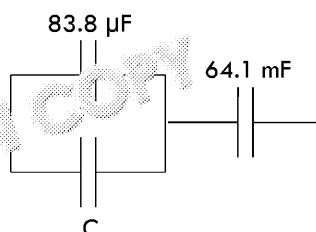
4. Calculate the potential difference across all of the capacitors below when the whole arrangement is 6.36 J.



Shocking!

5. The energy stored in the capacitors below is 65.0 J. 382 mC of charge is total.

Calculate the value of C.



6. The total capacitance of an arrangement of five capacitors is C_{total} .

A capacitor with a capacitance of C_1 is in series with a capacitor with a capacitance of C_2 . Capacitors with capacitances of C_3 and C_4 , which are in parallel with each other.

A capacitor with a capacitance of C_5 is in parallel with all of the other capacitors.

Find an expression for C_1 in terms of the other capacitances in the circuit.

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9. CHARGING AND DISCHARGING CAPACITORS

6.1.3: CHARGING AND DISCHARGING CAPACITORS

BACKGROUND

Capacitors charge and discharge exponentially.

	Discharging	Charging
Voltage	$V = V_0 e^{-\frac{t}{RC}}$	$V = V_0 (1 - e^{-\frac{t}{RC}})$
Charge	$Q = Q_0 e^{-\frac{t}{RC}}$	$Q = Q_0 (1 - e^{-\frac{t}{RC}})$
Current	$I = I_0 e^{-\frac{t}{RC}}$	$I = I_0 e^{-\frac{t}{RC}}$

I_0 , Q_0 and V_0 each give the initial value of I , Q and V .

When charging, the initial value of Q and V is zero. In this case, Q_0 and V_0 are the maximum charge stored by the capacitor and the potential difference across the capacitor.

RC is known as the time constant of the capacitor, and is the time it takes for the capacitor to discharge to $e^{-1} = 0.37$ of its initial charge.

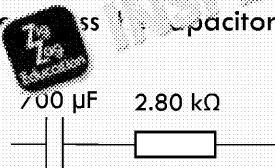
The half-life of the capacitor and coupled resistor – the time taken for the charge to fall to half its initial value – is given by $T_{1/2} = 0.69 RC$.

Understanding and controlling the time taken for a capacitor to discharge can be useful for timing signals or store energy for use later.

Example 1

A capacitor is discharging through a resistor, as shown below.

Calculate the time taken for the potential difference across the capacitor to reach 10.0 V if the initial potential difference across the capacitor is 120 V.



We need to use an equation for potential difference

$$V = V_0 e^{-\frac{t}{RC}}$$

Rearrange for t

$$\frac{V}{V_0} = e^{-\frac{t}{RC}}$$

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC}$$

$$t = -RC \times \ln\left(\frac{V}{V_0}\right)$$

Substitute in values

$$t = -2.80 \times 10^3 \times 700 \times 10^{-6}$$

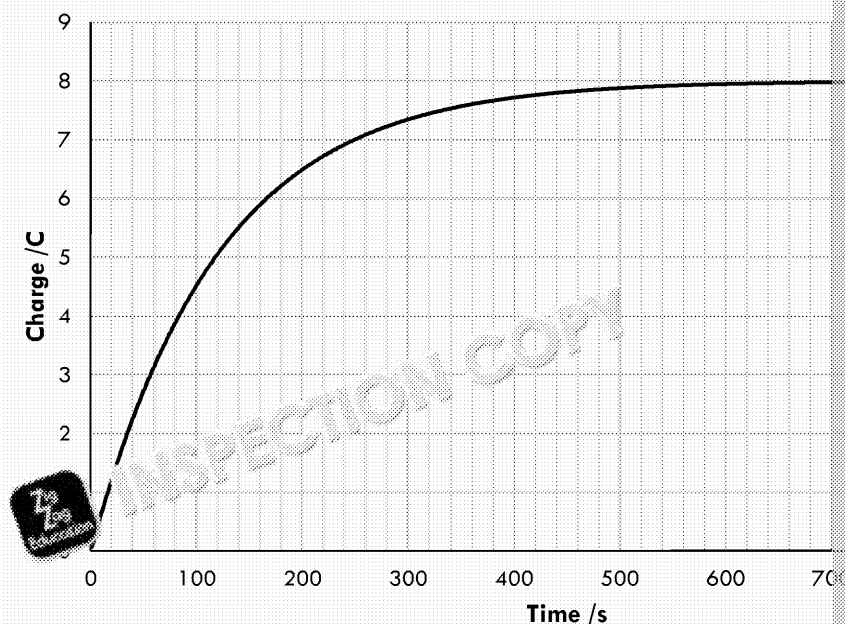
$$t = 4.87 \text{ s}$$

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Example 2

A capacitor charges through a resistor with a resistance of $30 \text{ k}\Omega$, producing



Determine the capacitance of the capacitor.

This graph is approaching 8.0 C , which means this is Q_0 .

We can either look for the half-life, $T_{1/2}$, which, here, is the time taken to charge to $0.5Q_0$.

We know that $T_{1/2} = 0.69RC$, so

$$RC = \frac{T_{1/2}}{0.69} = \frac{80}{0.69}$$

$$RC = 116 \text{ s}$$

Alternatively, we can look directly for RC , which is the time taken for a capacitor to charge to $0.37 Q_0$, or charge to $(1 - 0.37)Q_0 = 0.63Q_0$

$$0.63Q_0 = 0.63 \times 8.0 = 5.04 \text{ C}$$

This again gives us an RC of about 115 s !

We then use our value for RC to find C

$$C = \frac{RC}{R} = \frac{115}{30 \times 10^3}$$

$$C = 0.0038 \text{ F} = 3.8 \text{ mF}$$

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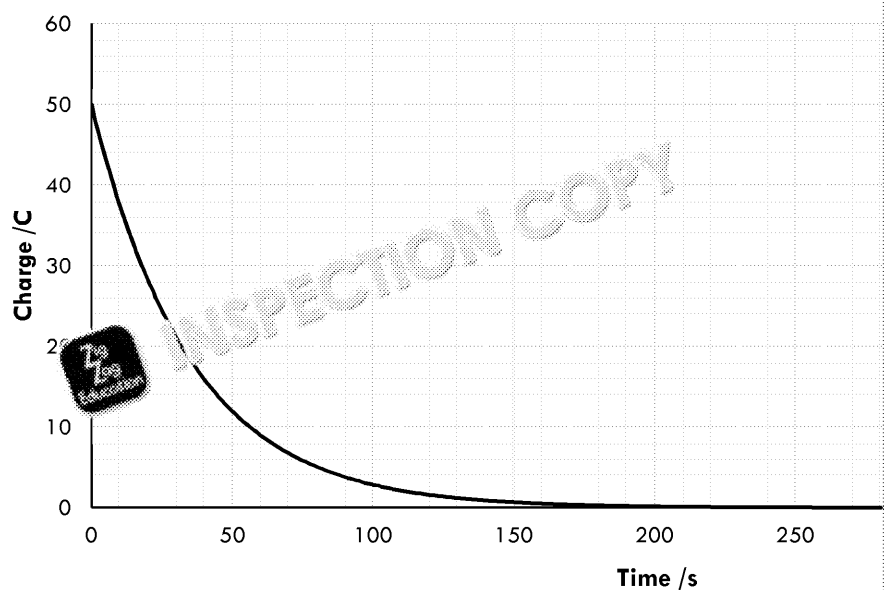
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QUESTIONS

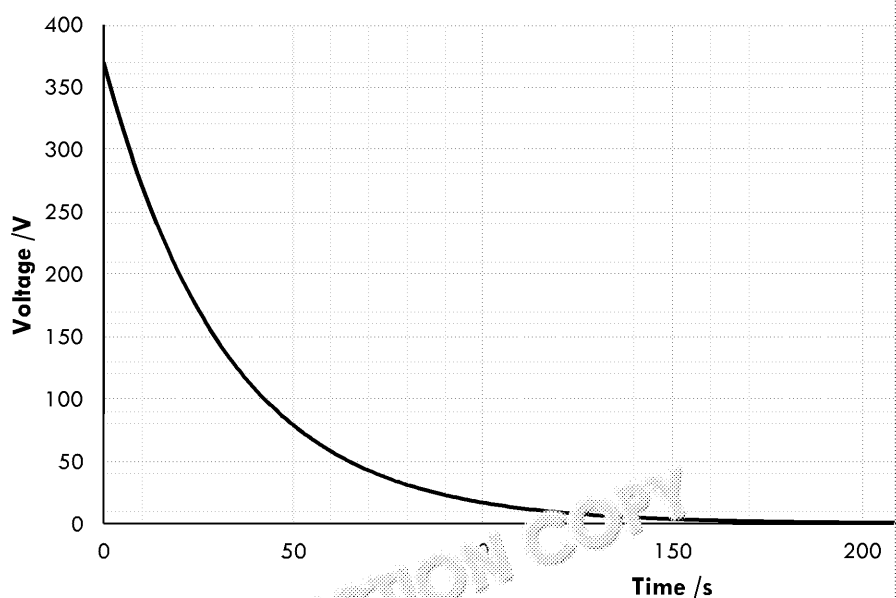
Bright spark

1. A capacitor discharges with a time constant of 38 s with an initial current of 5.0 A. Calculate the current out of the capacitor after one minute.
2. Calculate the time constant and half-life of the capacitor shown in the graph.



Charging up

3. A 450 μF capacitor discharges through a resistor, producing the graph.



Calculate the resistance of the resistor in $\text{k}\Omega$.

4. A 400 μF capacitor discharges through a resistor from an initial potential difference of 300 V. It takes 108 s to reach 100 V.

Calculate the resistance of the resistor in $\text{k}\Omega$.

5. A capacitor is charged through a 24.5 $\text{k}\Omega$ resistor.

After 64.3 s, the charge stored in the capacitor has reached 81.9 % of the maximum charge.

Calculate the capacitance of the capacitor.

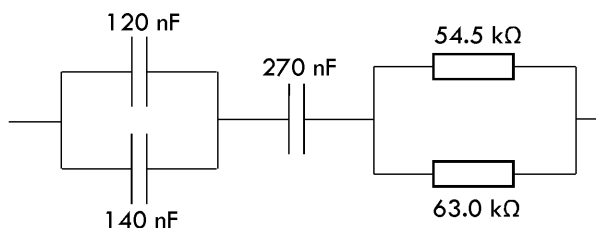
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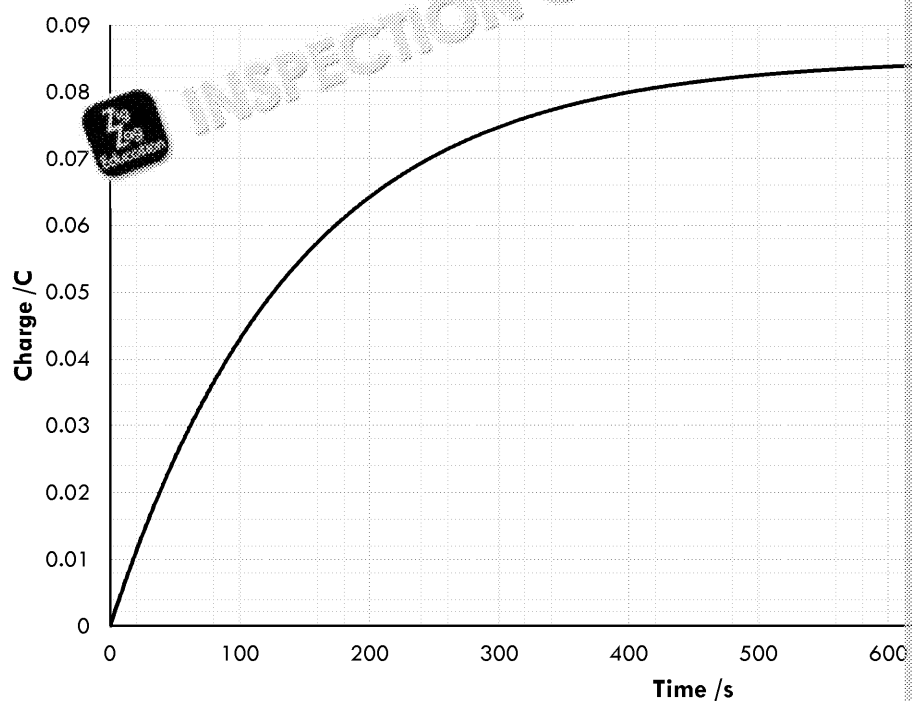


Shocking!

6. Calculate the time taken for the charge in the capacitors below to reach 37.8 C.



7. The graph below shows a capacitor charging through a resistor with a resistance of 100 Ω.



- a) After what time will 1.81 J of energy be stored?
b) Calculate the percentage of the potential difference across the capacitor compared to its initial value.

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10. TRANSFORMER

6.3.3: ELECTROMAGNETISM

BACKGROUND

A transformer consists of two coils of wires around an iron core.

A transformer can be used to increase or decrease a potential difference; this is because transmitting power at low currents reduces power loss, and appliances may require a specific potential difference to function.

The relationship between the number of turns in the coils of a transformer and the potential difference across the coils is



$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

The efficiency of a transformer is given by

$$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$$

N_p = num
 N_s = num
 V_p = po
 V_s = po
 I_p = cur
 I_s = cur

EXAM TIP

Efficiencies
percentage
decimal in

For exam
equivalent

Example 1

Electricity is transmitted at a potential difference of 11 kV.

Electrical substations convert this to the 230 V used in homes and businesses.

The secondary coil of one such substation has 500 turns.

Calculate the number of turns in the primary coil.

Use $\frac{N_s}{N_p} = \frac{V_s}{V_p}$

with $N_s = 500$, $V_s = 230$ V and $V_p = 11 \times 10^3$ V

We need to rearrange for N_p

$$N_p = N_s \frac{V_p}{V_s} \quad [\text{multiply both sides}]$$

Then substitute in values

$$N_p = 500 \times \frac{11 \times 10^3}{230}$$

$$N_p = 23900 \text{ turns}$$

Example 2

A transformer is 80 % efficient.

The primary coil of a transformer is connected to a 50 V AC power supply, and has 80 A flowing through it.

The secondary coil provides 90 V.

Calculate current through the secondary coil.

We need to use

$$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$$

with efficiency = 80 % = 0.8, $V_p = 50$ V

$I_p = 80$ A and $V_s = 90$ V.

Rearrange for I_s

$$I_s = \text{efficiency} \times \frac{I_p V_p}{V_s}$$

$$I_s = 0.8 \times \frac{80 \times 50}{90}$$

$$I_s = 36 \text{ A}$$

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Example 3

A transformer is 75 % efficient. The potential difference across the primary coil is 300 V. The resistance of the primary coil is 100 Ω .

The primary coil has 80 turns and the secondary coil has 15.

Calculate the current in the secondary coil.

We need to use

$$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$$

but we don't have anything in the correct units
 $\text{efficiency} = 75 \% = 0.75$, $V_p = 300 \text{ V}$,
 $N_s = 15$.

IV actually gives us the power transferred, which can be expressed as $I^2 R$ or $\frac{V^2}{R}$.

We can write

$$\text{efficiency} = \frac{I_s V_s}{I_p V_p} = \frac{I_s V_s}{\frac{V_p^2}{R_p}}$$

which we can rearrange for I_s

$$I_s = \text{efficiency} \times \frac{V_p^2}{R_p V_s}$$

But we don't have V_s ! Instead, we need

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

which rearranges for V_s

$$V_s = \frac{V_p N_s}{N_p}$$

Substituting in values gives

$$V_s = 300 \times \frac{15}{80}$$

$$V_s = 56.25 \text{ V}$$

and substituting values into our earlier equation

$$I_s = \text{efficiency} \times \frac{V_p^2}{R_p V_s}$$

$$I_s = 0.75 \times \frac{300^2}{100 \times 56.25}$$

$$I_s = 12 \text{ A}$$

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QUESTIONS

Bright spark

1. A transformer has 350 V across its primary coil, which has 240 turns.
The secondary coil has 140 turns.
Calculate the potential difference across the secondary coil.
2. A transformer has a current of 130 mA running through its primary coil and a current of 1.3 A running through its secondary coil.
The potential difference across the primary coil is 7.5 kV and across the secondary coil is 150 V.
Calculate the efficiency of the transformer.

Charging up

3. A transformer is 94.2 % efficient. The potential difference across the secondary coil is 240 V and the current in the secondary coil is 1.5 A.
The power supplied to the primary coil is 6.50 kW.
Calculate the current in the primary coil.
4. The number of turns in the primary coil of a transformer is 400 and the number of turns in the secondary coil is 700.
The current through the primary coil is 84.3 A and the current through the secondary coil is 48.2 A.
Calculate the efficiency of the transformer.
5. The power supplied to the primary coil of a transformer is 850 W.
The efficiency of the transformer is 90.0 %.
The resistance of the secondary coil is 77.3 Ω .
Calculate the current through the secondary coil.

Shocking!

6. There is a potential difference of 230 V across the primary coil of a transformer and a current of 34.0 A running through it. The primary coil has 180 turns.
The secondary coil of the transformer has 66.7 % more turns than the primary coil and the resistance of the secondary coil is 77.8 Ω .
Calculate the efficiency of the transformer.
7. In a step-down transformer, one of the coils has 1.5 times as many coils as the other.
The primary coil of a transformer has a resistance of 640 Ω . The primary current is 0.5 A and the secondary current is 1.5 A.
The transformer is 72.0 % efficient.
Calculate the current in the secondary transformer.

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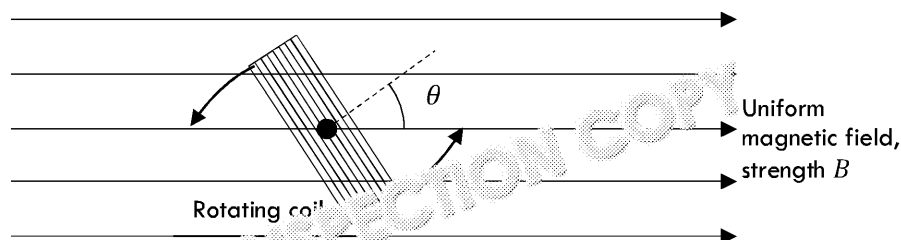


11. ELECTRICITY GENERATION

6.3.3: ELECTROMAGNETISM

BACKGROUND

Electromagnetic induction is the production of a potential difference across a conductor when it is moved through a magnetic field. Power stations rotate coils of wire in magnetic fields to generate electricity.



Faraday's law states that the magnitude of the e.m.f. induced in a coil is

$$\varepsilon = N \frac{\Delta \Phi}{\Delta t}$$

This equation states that the potential created in a coil is equal to the change in magnetic flux linkage through the coil ($N\Phi$) over time, where

$$N\Phi = BAN \cos \theta$$

This means that the magnetic flux linkage is related to the strength of the magnetic field, the area of the coil and the number of turns in the coil.

Lenz's law states that any e.m.f. induced will oppose the change that induces the e.m.f.

A coil rotating uniformly in a magnetic field will have an induced e.m.f. of

$$\varepsilon = BAN\omega \sin \omega t$$

This relates the change in magnetic flux linkage of the coil to the rate the coil is rotating at.

Example 1

A coil has 80 turns and a cross-sectional area of 0.85 m^2 .

The coil is in a magnetic field with magnetic field strength 36 mT .

The coil's cross-section initially makes an angle of 30° with the magnetic field.

The coil then rotates over 15 s so that its cross-section makes an angle of 65° to the magnetic field.

Calculate the e.m.f. induced in the coil.

Use $\varepsilon = N \frac{\Delta \Phi}{\Delta t}$

but instead of being given $\Delta \Phi$,

$$N\Phi = BAN \cos \theta$$

$\Delta \Phi$ is the difference in flux between

$$\Delta \Phi = BA(\cos \theta_2 - \cos \theta_1)$$

$$\Delta \Phi = 36 \times 10^{-3} \times 0.85 \times (\cos 65^\circ - \cos 30^\circ)$$

$$\Delta \Phi = -0.0158 \text{ T m}^2$$

Putting this into our original equation

$$\varepsilon = N \frac{\Delta \Phi}{\Delta t}$$

$$\varepsilon = 80 \times \frac{-0.0158}{15}$$

$$\varepsilon = -0.084 \text{ V}$$

EXAM TIP

A negative e.m.f. indicates the current flows in the opposite direction to what was assumed.

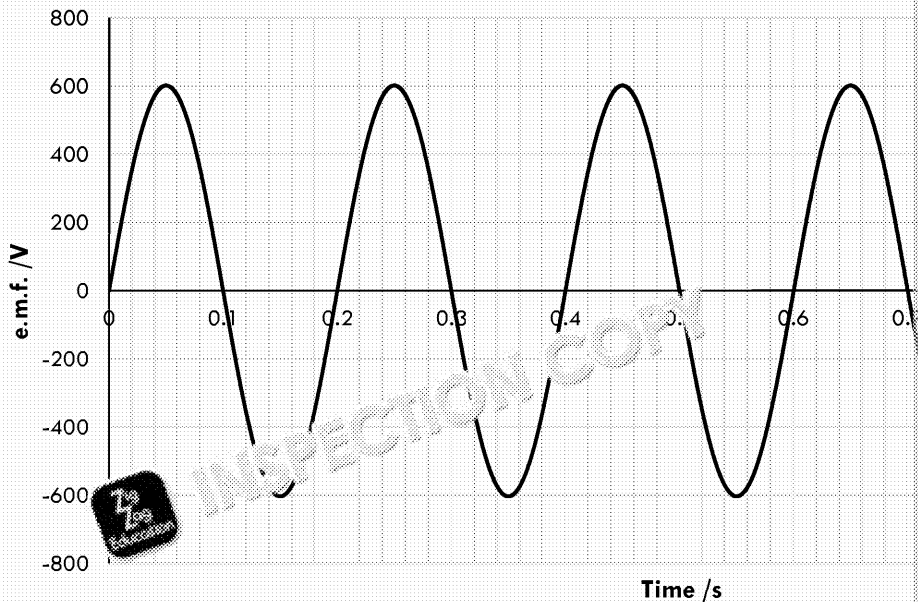
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Example 2

A coil rotates in a magnetic field, producing the graph below.



The flux of the coil, Φ , is 0.0128 T m^2 .

Calculate the number of turns in the coil.

From the graph, peak e.m.f. = 615 V and the period of the e.m.f. is 0.2 s .

The period of a sine wave is 2π radians.

Here we have $\sin \omega t$, so $\omega t = 2\pi$

$$\omega = \frac{2\pi}{t} = \frac{2\pi}{0.2}$$

$$\omega = 31.4 \text{ rad s}^{-1}$$

The peak of the e.m.f. at $\theta = 0$ or $\omega t = \frac{\pi}{2}$ is $\epsilon = BAN\omega$

$BA = \Phi$, which is given, and we found ω in our last step

So rearranging for N gives

$$N = \frac{\epsilon}{\Phi\omega}$$

Substituting in values gives

$$N = \frac{615}{0.0128 \times 31.4}$$

$$N = 1530 \text{ turns}$$

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QUESTIONS

Bright spark

1. A coil of wire with a cross-sectional area of $3.6 \times 10^{-3} \text{ m}^2$ and 200 turns of magnetic field strength $19 \mu\text{T}$.

The cross-section of the wire makes an angle of 0.40 rad to the magnetic field.

Calculate the magnetic flux linkage in the coil.

2. A coil with 38 turns and a cross-sectional area of 12.6 cm^2 is perpendicular to a magnetic field. The magnetic field increases from 14.1 mT to 93.7 mT over 60.0 ms .

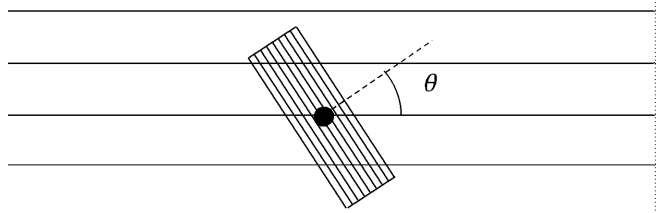
Calculate the e.m.f. induced in the coil.

3. A coil has 120 turns and a cross-sectional area of 728 cm^2 . The coil rotates in a magnetic field of strength 107 mT at a rate of 170 revolutions per minute.

Calculate the e.m.f. generated in the coil one minute after it starts rotating with its cross-section perpendicular to the magnetic field.

Charging up

4. The diagram below shows a coil of wire in a uniform magnetic field. The

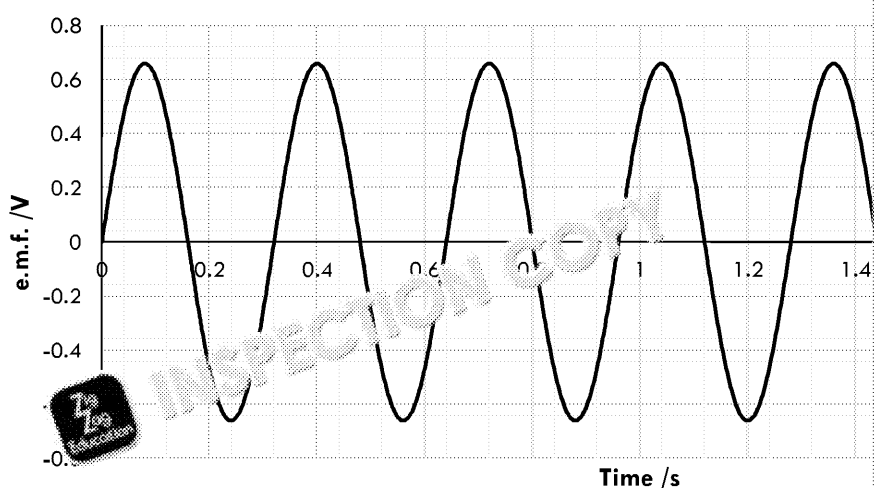


At $t = 0$, $\theta = 0^\circ$. At $t = 2.50 \text{ s}$, $\theta = 36^\circ$.

The e.m.f. induced in the coil is 130 mV .

Calculate the cross-sectional area of the coil.

5. The graph below shows the e.m.f. of a coil rotating in a magnetic field.



Calculate the angular frequency of the coil.

6. A coil has 50 turns and a cross-sectional area of 300 mm^2 and is placed perpendicular to a magnetic field of magnetic field strength 20 mT .

The coil is pulled 15 cm up and out of the magnetic field, inducing an e.m.f. of 1.2 V .

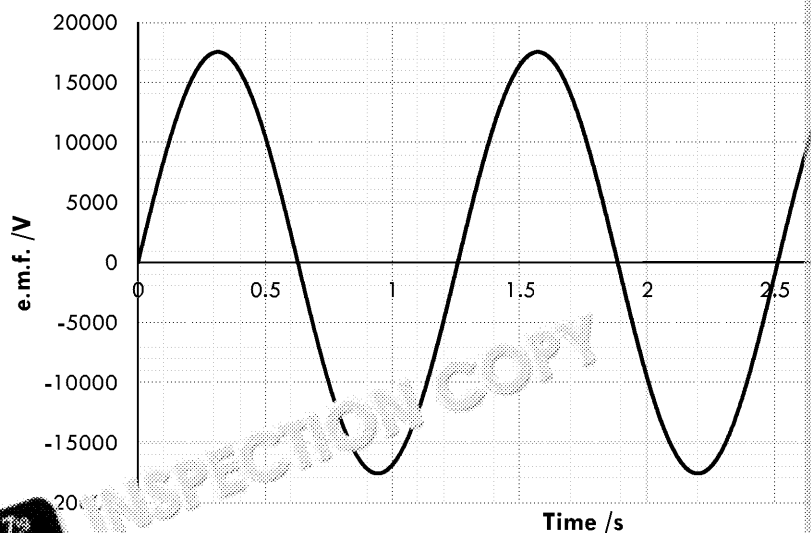
Calculate the speed at which the coil is moved.

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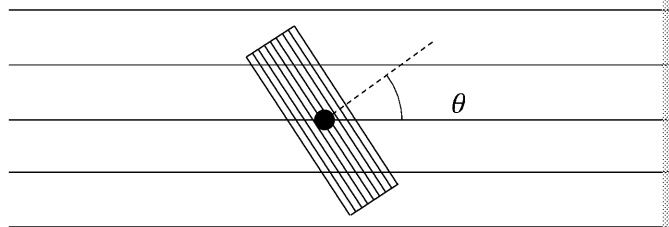
Shocking!

7. The graph below shows the e.m.f. of a coil rotating in a magnetic field.



The number of turns in the coil is 3000 and the cross-sectional area of the coil is 0.02 m^2 . Calculate the magnetic field strength.

8.



The coil shown above rotates at $\omega = 2.88 \text{ rad s}^{-1}$ and the number of turns in the coil is 3000. The e.m.f. induced in the coil is 55.6 mV as the coil moves from maximum linkage.

Calculate the effective area of the coil presented to the magnetic field.

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12. ELECTROMAGNETIC PH

6.3.3: ELECTROMAGNETISM

BACKGROUND

Electricity and magnetism have a strange connection – an electric current can generate magnetic fields, and magnetic fields can generate currents. Meanwhile, an electric current – or another moving charge – in a magnetic fi

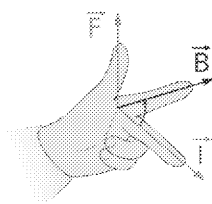
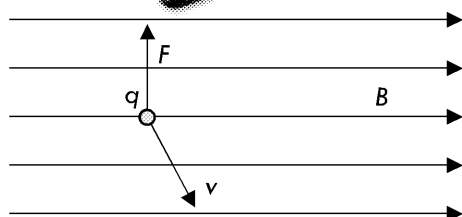
Lorentz force

The Lorentz force is the force felt by a moving charge in a magnetic field.

The force felt by a moving charge is given by

$$F = Bqv$$

Fleming's left hand rule gives the direction of the force felt by a moving



Hold your left hand

- Your thumb represents the **force felt**.
- Your first finger represents the **magnetic field**.
- Your second finger represents the **direction of positive current**.

Magnetic fields and moving charges

A moving charge, or a current, **generates a magnetic field**.

The direction of the magnetic field can be determined by the **right hand grip rule**: if you hold your right thumb out and curl your fingers around; your thumb is the direction of the current and your fingers point in the direction of the generated magnetic field.

Faraday's law

Faraday's law describes how an **e.m.f. is generated across a conductor** as it moves through a magnetic field, or as the magnetic field changes around the conductor.

The e.m.f. induced in the conductor is proportional to the rate of change of magnetic flux in the conductor – the magnetic field passing through a cross-section of the conductor.

Lenz's law

Lenz's law states that any change in e.m.f. or magnetic field will oppose the source of that change.

Example 1

A copper ball is pushed through a copper tube.

It takes significantly longer for the ball to pass through the tube than it would do had it just been dropped in free space.

Explain this effect.

This effect isn't to do with friction, air resistance in the tube, as you might expect, but rather the fact that the ball and tube are made of copper.

As the copper ball passes through the tube, the magnetic field lines are cut, and a magnetic field is induced.

The copper ball falls through the magnetic field, so that it experiences a **Lorentz force**.

Lenz's law states that the change brought about by the magnetic field will oppose the change, so that the

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Example 2

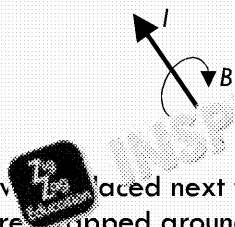
A solenoid consists of a wire wrapped into a coil.

A current flows through the solenoid, generating a magnetic field.

Draw the magnetic field around a solenoid, explaining its shape.

In the diagram, current is travelling right to left across the front edge of the solenoid, and left to right on the back edge.

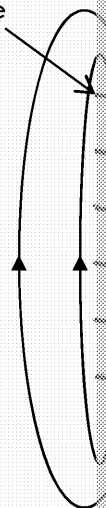
A single straight wire produces a magnetic field around it as shown below.



Multiple wires placed next to each other (or a single wire wrapped around itself) would produce a single magnetic field that was the sum of each individually produced magnetic field.

Using the right hand grip rule, the magnetic field travels up on the outside of the solenoid and down on the inside of the solenoid.

Coil of current-carrying wire



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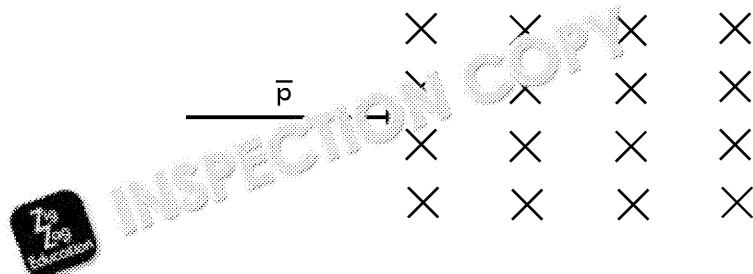
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QUESTIONS

Bright spark

- Two wires lie next to each other, with currents facing in the same direction.
Draw the magnetic field generated around both wires, and explain its strength.
- An antiproton travels through a magnetic field, as shown below.
The magnetic field faces **into the page**.
Complete the diagram to show the path of the antiproton as it travels through the field.

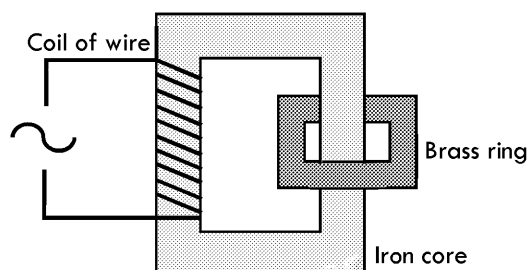


Charging up

- As a plane flies, it passes through a magnetic field, which points directly downwards.
Using Fleming's left hand rule, explain why the plane experiences a force that acts backwards due to Earth's magnetic field.
- Two identical copper balls are dropped through two metal tubes.
Both tubes are identical, except one is made of copper and one is made of steel.
Copper is a better conductor than steel.
Describe and explain the effect this has on the balls as they fall through the tubes.

Shocking!

- A set-up is shown below.

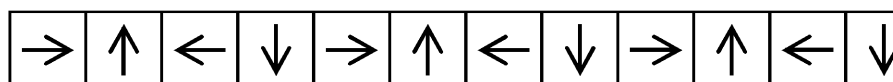


Describe and explain the effect of an AC current being applied through the coil.

- A Halbach array is an arrangement of magnets that generates a strong magnetic field on one side of the array, and zero magnetic field on the other side.

A Halbach array is shown below, with the arrows showing the direction of the magnetic field from each magnet.

Predict which side of the array will produce a strong magnetic field, explain your answer.

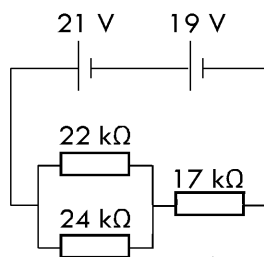


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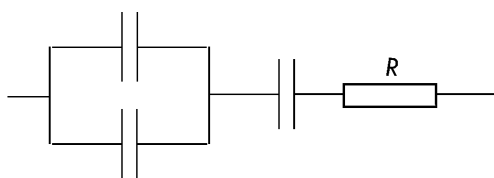


EXAM-STYLE QUESTIONS

1. Fig. 1.1 shows a circuit.



- Calculate the combined resistance of the three resistors.
 - The 21 V power source has an internal resistance of $0.68 \, \Omega$.
The 19 V power source has an internal resistance of $0.34 \, \Omega$.
Calculate the current through the power sources to an appropriate number of significant figures.
 - Calculate the power dissipated by the total circuit.
2. A block of tungsten has a square cross section with a side length of $8.3 \, \text{cm}$, and a length of $1.2 \, \text{m}$. The resistance of the block is $1.1 \times 10^{-4} \, \Omega$.
- Calculate the resistivity of tungsten.
 - The tungsten is stretched out to a new length of $1.3 \, \text{m}$.
Calculate the resistance of the new tungsten wire.
 - The wire transmits a power of $45 \, \text{W}$, with a potential difference of $92 \, \text{V}$ across it.
Calculate the power dissipated by the wire.
3. Three identical parallel-plate capacitors and one resistor are placed in the arrangement shown in Fig. 3.1.



- Each of the capacitors has a capacitance of $850 \, \mu\text{F}$.
Each plate of the capacitors is a circle with a radius of $4.1 \, \text{mm}$, and the plates are separated by a dielectric.
Calculate the relative permittivity of the dielectric between the plates.
- Calculate the total capacitance of the arrangement seen in Fig. 3.1.

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- (c) The capacitors discharge through the resistor, producing the graph seen in Fig.

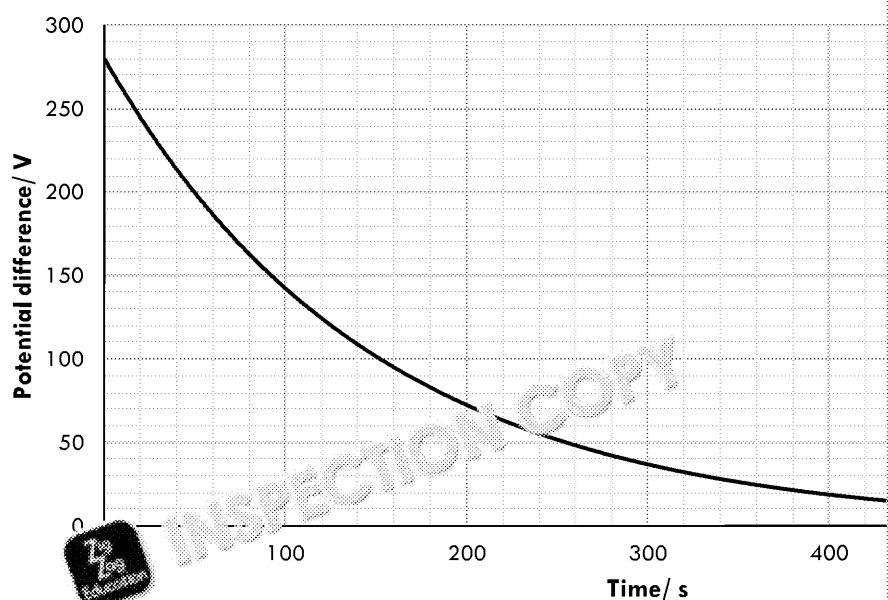


Fig. 3.2

Calculate the value of R .

4. A coil with a cross-sectional area of 0.16 m^2 and 320 turns rotates in a magnetic field of strength B , producing the graph seen in Fig. 4.1.

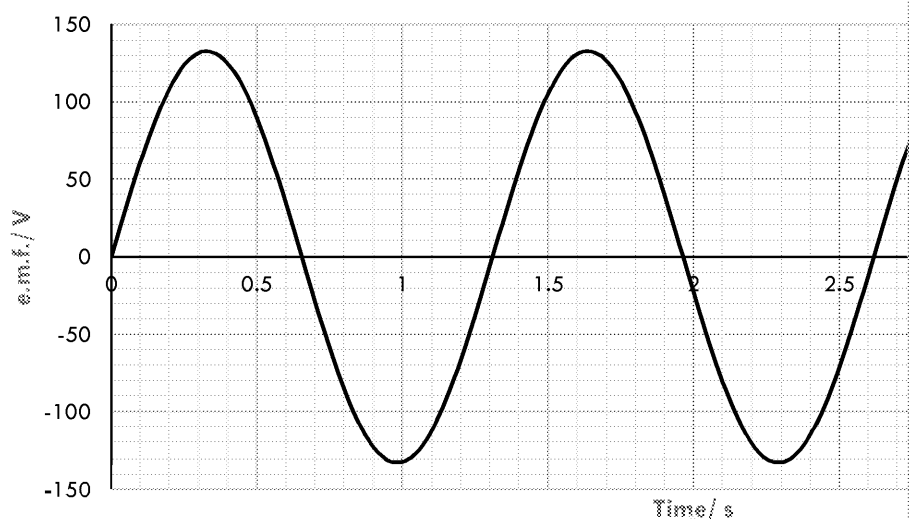


Fig. 4.1

- Calculate the angular frequency of the coil.
- Calculate the magnetic field strength B .
- The e.m.f. produced is put through a transformer with 75 turns on the primary and 150 turns on the secondary coil.
The current through the primary coil of the transformer is 210 mA, and the potential difference across the primary coil is 12 V.
Calculate the current through the secondary coil of the transformer.
- The potential difference across the secondary coil of the transformer is displayed on an oscilloscope. The trace has a vertical height of 4.0 divisions and a horizontal width of 12 divisions.

Describe how the oscilloscope should be set up to show a single wavelength.

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ANSWERS

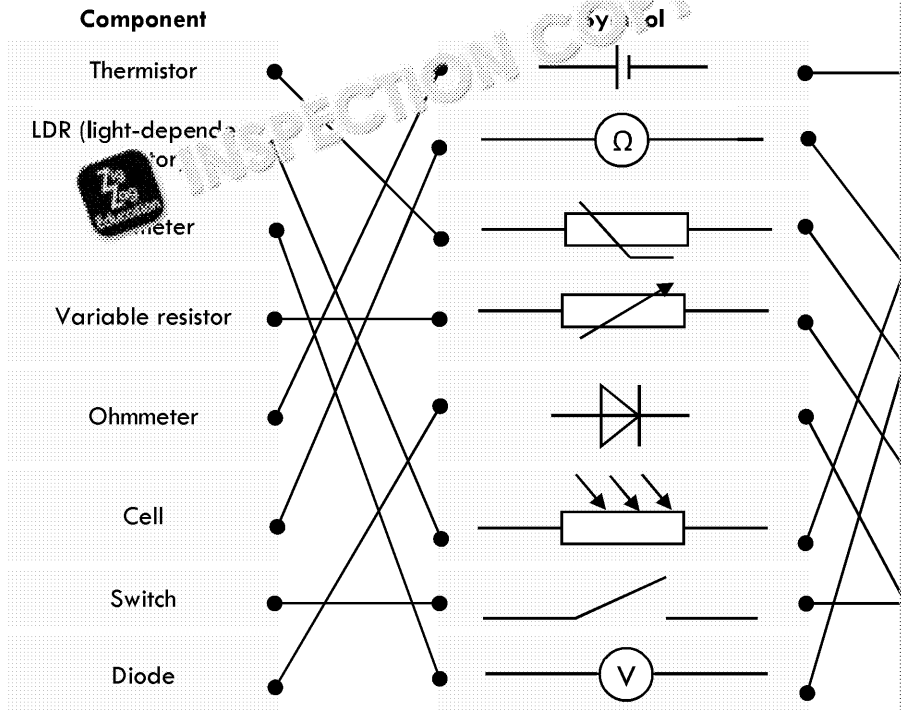
GCSE POP QUIZ

- Current is the rate of flow of **charge**.
Potential difference is the energy transferred to a charge.
For a given potential difference, a lower resistance component will have a **higher** current flowing through it.
- Circuit A – no, the circuit is not complete.
Circuit B – yes, the circuit is complete.
Circuit C – no, there is no potential difference source and the circuit is not complete.

2. $Q = It$

$$I = \frac{Q}{t} = \frac{23}{60} = 0.38 \text{ A}$$

4. Component



5. $V = IR$

$$R = \frac{V}{I} = \frac{52}{850 \times 10^{-3}} = 61 \Omega$$

- a) $R_{total} = R_1 + R_2 + R_3 = 630 + 680 + 820 = 2130 \Omega$
b) The resistance would decrease as each branch of the circuit would have less current, requiring less potential difference to push the current through each branch.

7. $I_{in} = I_{out}$

$$8.5 + 6.3 = 4.1 + I$$

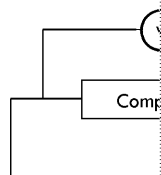
$$I = 8.5 + 6.3 - 4.1 = 10.7 \text{ A}$$

- Kirchhoff's first law states that the current into a junction equals the current out of the junction.
Kirchhoff's second law states that the potential difference gained from a source is equal to the **potential difference lost in the circuit**.
- 9.

Ohmic conductor

Filament lamp

Diode



11. a) $P = I^2 R = (190 \times 10^{-3})^2 \times 220 = 7.9 \text{ W}$

b) $E = QV$

$$V = \frac{E}{Q} = \frac{370}{8.3} = 45 \text{ V}$$

- Transformers are used to increase or reduce power loss in cables. They step up the voltage of electricity for use in the National Grid.

- Generate electricity for the National Grid by converting motion to electricity for the National Grid. They convert vibrations to signals.

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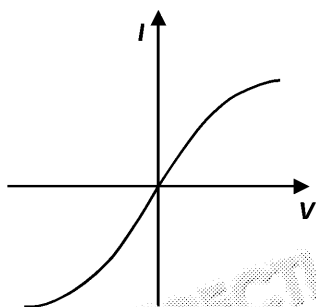
1. I-V CHARACTERISTICS

Bright spark

1. As metal heats up, its resistance increases.

$$R = \frac{V}{I} \text{ so the gradient of an } I\text{-}V \text{ graph is } \frac{1}{R}.$$

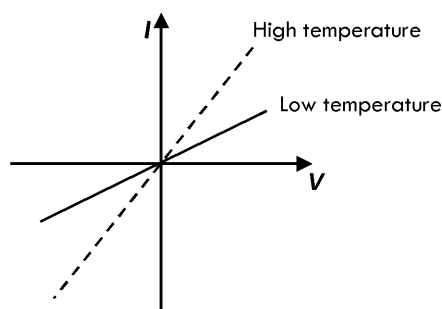
At higher currents (and temperatures), resistance increases and the gradient of an I - V graph decreases.



2. Current flows above a threshold potential difference (V_0) or at high negative potential differences. Resistance decreases with increasing current.

Charging up

3.



A thermistor's resistance decreases with temperature. For an I - V graph, $\text{gradient} \propto \frac{1}{R}$ so a lower resistance means a higher gradient.

4. • Place ammeter in series
• Place voltmeter in parallel
• Take readings of V and I
• Repeat with increasing V

5. $R \propto \frac{1}{\text{gradient}}$

Gradient at tangent

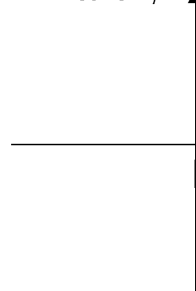
$$\text{Gradient} = 0.76$$

$$R = 1.3 \, \Omega$$

Locking!

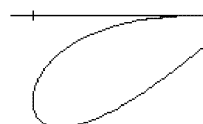
6. The component described is a diode.
This means that the current only flows in one direction below a threshold potential difference.

Current / A



7.

Current / A



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2. RESISTIVITY

Bright spark

$$1. \quad R = \frac{\rho L}{A}$$

$$R = \frac{9.71 \times 10^{-8} \times 37.1 \times 10^{-2}}{(8.43 \times 10^{-3})^2}$$

$$R = 5.07 \times 10^{-4} \Omega$$

$$2. \quad R = \frac{\rho L}{A}$$

$$L = \frac{RA}{\rho}$$

$$L = \frac{92.3 \times 10^{-3} \times 7.07 \times 10^{-6}}{6.72 \times 10^{-7}}$$

$$L = 0.971 \text{ m}$$

Charging up

$$3. \quad R = \frac{\rho L}{A}$$

$$\Delta R = \frac{\rho}{A} (L_2 - L_1)$$

$$\rho = \frac{\Delta R A}{L_2 - L_1}$$

$$\rho = \frac{-83.4 \times (1.50 \times 10^{-2})^2}{1.90 - 2.40}$$

$$\rho = 0.118 \Omega \text{ m}$$

4. Section 1

$$R_1 = \frac{\rho L}{A}$$

$$R_1 = \frac{2.20 \times 10^{-7} \times 1.60}{\pi \times (8.30 \times 10^{-2})^2 - \pi \times (5.06 \times 10^{-2})^2}$$

$$R_1 = 2.588 \times 10^{-5} \Omega$$

Section 2

$$R_2 = \frac{\rho L}{A}$$

$$R_2 = \frac{2.20 \times 10^{-7} \times 0.230}{\pi \times (9.31 \times 10^{-2})^2 - \pi \times (6.64 \times 10^{-2})^2}$$

$$R_2 = 3.782 \times 10^{-6} \Omega$$

Total

$$R = R_1 + R_2$$

$$R = 2.97 \times 10^{-5} \Omega$$

Shocking!

$$5. \quad R = \frac{\rho L}{A}$$

$$R_{\text{carbon}} = \frac{4.56 \times 10^{-6} \times \pi (8.50 \times 10^{-3})^2}{\pi (8.50 \times 10^{-3})^2}$$

$$R_{\text{carbon}} = 4.018 \Omega$$

$$R_{\text{aluminium}} = \frac{2.82 \times 10^{-3}}{\pi (6.00 \times 10^{-3})^2}$$

$$R_{\text{aluminium}} = 49.87 \text{ m}\Omega$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_{\text{aluminium}}} + \frac{1}{R_{\text{carbon}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{49.87 \times 10^{-3}} + \frac{1}{4.018}$$

$$\frac{1}{R_{\text{total}}} = 120.6$$

$$R_{\text{total}} = 8.295 \text{ m}\Omega$$

$$P = I^2 R$$

$$P = (320 \times 10^{-3})^2 \times 8.295 \times 10^{-3}$$

$$P = 8.49 \times 10^{-4} \text{ W}$$

$$6. \quad A = \pi r^2$$

$$A_1 = \pi \times (0.661 \times 10^{-2})^2$$

$$A_1 = 1.373 \times 10^{-4} \text{ m}^2$$

$$\rho = \frac{RA}{L}$$

$$\rho = \frac{31.8 \times 1.373 \times 10^{-4}}{8.51 \times 10^{-2}}$$

$$\rho = 51.29 \text{ m}\Omega \text{ m}$$

Cross-section of new

$$V = L_1 A_1$$

$$V = 8.51 \times 10^{-2} \times 1.373 \times 10^{-4}$$

$$V = 1.168 \times 10^{-5} \text{ m}^3$$

$$A_2 = \frac{V}{L_2}$$

$$A_2 = \frac{1.168 \times 10^{-5}}{5.49 \times 10^{-2}}$$

$$A_2 = 2.128 \times 10^{-4} \text{ m}^2$$

Resistance of new cyl

$$\rho = \frac{RA}{L}$$

$$R_2 = \frac{\rho L_2}{A_2}$$

$$R_2 = \frac{51.29 \times 10^{-3} \times 5.49 \times 10^{-2}}{2.128 \times 10^{-4}}$$

$$R_2 = 13.2 \Omega$$

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3. POWER

Bright spark

$$1. \quad P = \frac{V^2}{R}$$

$$R = \frac{V^2}{P}$$

$$R = \frac{230^2}{40}$$

$$R = 1300 \, \Omega$$

$$2. \quad P = I^2 R$$

$$E = Pt$$

$$E = I^2 R t$$

$$E = (77.1 \times 10^{-3})^2 \times 38.6 \times 10^3 \times 2.7$$

$$E = 63.8 \, \text{kJ}$$

Charging up

$$3. \quad P = \frac{V^2}{R}$$

$$E = Pt$$

$$E = \frac{V^2}{R} t$$

$$E = \frac{1090^2}{855} \times 5 \times 60$$

$$E = 416.9 \times 10^3 \, \text{J}$$

$$E_{\text{grav}} = mgh$$

$$E_{\text{grav}} = 3150 \times 9.81 \times 9.39$$

$$E_{\text{grav}} = 290.2 \times 10^3 \, \text{J}$$

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}}$$

$$\text{efficiency} = \frac{E_{\text{grav}}}{E}$$

$$\text{efficiency} = \frac{290.2 \times 10^3}{416.9 \times 10^3}$$

$$\text{efficiency} = 0.696 \text{ or } 69.6 \%$$

$$4. \quad P = I^2 R$$

$$E = Pt$$

$$E = I^2 R t$$

$$I = \frac{Q}{t}$$

$$E = \frac{Q^2 R}{t}$$

$$Q = \sqrt{\frac{Et}{R}}$$

$$Q = \sqrt{\frac{1}{R}}$$

$$Q = 0.30 \, \text{C}$$

Shocking!

$$5. \quad I^2 R = \frac{P^2 R}{V^2}$$

$$I^2 R = \frac{(24.8 \times 10^6)^2 \times 40}{(30.9 \times 10^6)^2}$$

$$I^2 R = 262.8 \, \text{kW}$$

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}}$$

$$\text{efficiency} = \frac{P - I^2 R}{P}$$

$$\text{efficiency} = \frac{30.9 \times 10^6 - 262.8 \times 10^3}{30.9 \times 10^6}$$

$$\text{efficiency} = 0.991 \text{ or } 99.1 \%$$

$$6. \quad E_k = \frac{1}{2} mv^2$$

$$E_k = \frac{1}{2} \times 1035 \times 68^2$$

$$E_k = 2.435 \times 10^6 \, \text{J}$$

$$I^2 R = \frac{P^2 R}{V^2}$$

$$P = \sqrt{\frac{V^2 I^2 R}{R}}$$

$$P = \sqrt{\frac{(25.6 \times 10^3)^2 \times 7.2}{907}}$$

$$P = 721.8 \, \text{kW}$$

$$E = Pt$$

$$E = 721.8 \times 10^3 \times 2.7$$

$$E = 3.869 \times 10^6 \, \text{J}$$

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}}$$

$$\text{efficiency} = \frac{E_k}{E}$$

$$\text{efficiency} = \frac{2.435 \times 10^6}{3.869 \times 10^6}$$

$$\text{efficiency} = 0.629 \text{ or } 62.9 \%$$

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4. RESISTORS IN SERIES AND IN PARALLEL

Bright spark

$$1. \quad R_{total} = R_1 + R_2 + R_3 + R_4 + R_5$$

$$R_{total} = 3.98 + 2.22 + 4.01 + 3.53 + 2.80$$

$$R_{total} = 16.5 \, \Omega$$

$$2. \quad \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}}$$

$$R_{total} = \frac{1}{\frac{1}{3.98} + \frac{1}{2.22} + \frac{1}{4.01} + \frac{1}{3.53} + \frac{1}{2.80}}$$

$$R_{total} = 0.628 \, \Omega$$

Charging up

$$3. \quad R_{total} = R_1 + R_{parallel}$$

$$\frac{1}{R_{parallel}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{parallel} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{total} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{total} = 34.9 + \frac{1}{\frac{1}{78.2} + \frac{1}{56.4}}$$

$$R_{total} = 67.67 \, \Omega$$

$$V = IR_{total}$$

$$I = \frac{V}{R_{total}}$$

$$I = \frac{30.0}{67.67}$$

$$I = 443 \, \text{mA}$$

$$4. \quad V = IR_{total}$$

$$R_{total} = \frac{V}{I}$$

$$R_{total} = \frac{180}{780 \times 10^{-3}}$$

$$R_{total} = 230.8 \, \Omega$$

$$R_{total} = R_{series} + R_{parallel}$$

$$R_{parallel} = R_{total} - R_{series}$$

$$R_{parallel} = 230.8 - 114$$

$$R_{parallel} = 116.8 \, \Omega$$

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_1 = \frac{1}{\frac{1}{R_{parallel}} - \frac{1}{R_2} - \frac{1}{R_3}}$$

$$R_1 = \frac{1}{\frac{1}{116.8} - \frac{1}{356} - \frac{1}{383}}$$

$$R_1 = 318 \, \Omega$$

Shocking!

$$5. \quad V_{total} = V_1 + V_2$$

$$V_1 = V_{total} - V_2$$

$$V_1 = 185 - 96.5$$

$$V_1 = 88.5 \, \text{V}$$

$$V = IR$$

$$I = \frac{V}{R}$$

$$I = \frac{96.5}{32.0}$$

$$I = 3.016 \, \text{A}$$

$$V_1 = IR_{total}$$

$$R_{total} = \frac{V_1}{I}$$

$$R_{total} = \frac{88.5}{3.016}$$

$$R_{total} = 29.34 \, \Omega$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4 + R_5}$$

$$\frac{1}{R_4 + R_5} = \frac{1}{R_{total}} - \frac{1}{R_1} - \frac{1}{R_2}$$

$$R_4 = \frac{1}{\frac{1}{R_{total}} - \frac{1}{R_1} - \frac{1}{R_2}}$$

$$R = \frac{1}{\frac{1}{29.34} - \frac{1}{76.0} - \frac{1}{100}}$$

$$R = 141 \, \Omega$$

$$6. \quad V = IR$$

$$V_{original} = 5 \times (12 + R_1)$$

$$V_{original} = 20 \times (12 + R_2)$$

$$26 = 0.25 \times (R_1 + R_2)$$

$$5 \times (12 + R_1) = 20 \times (12 + R_2)$$

$$60 + 5R_1 = 240 + 20R_2$$

$$R_1 = 36 + 4R_2$$

$$26 = 0.25 (36 + 4R_2)$$

$$26 = 9 + 1.25R_2$$

$$R_2 = 13.6 \, \Omega$$

$$R_1 = 36 + 4R_2$$

$$R_1 = 90.4 \, \Omega$$

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6. POTENTIAL DIVIDERS

Bright spark

$$1. \quad V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

$$V_{out} = 200 \times \frac{560}{560 + 680}$$

$$V_{out} = 90.3 \text{ V}$$

$$2. \quad V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

$$V_{in} = V_{out} \frac{R_1 + R_2}{R_2}$$

$$V_{in} = 65.0 \times \frac{1.30 \times 10^3 + 2.95 \times 10^3}{1.30 \times 10^3}$$

$$V_{in} = 213 \text{ V}$$

Charging up

$$3. \quad V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

$$R = \frac{V_{in} R_2}{V_{out} - V_{in}}$$

$$R = \frac{34.6 \times 10^3}{65 - 34.6}$$

$$R = 154 \text{ } \Omega$$

$$4. \quad V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

$$R_1 = \frac{V_{in} R_2}{V_{out}} - R_2$$

$$R = \frac{120 \times 287}{41.6} - 287$$

$$R = 541 \text{ } \Omega$$

Shocking!

$$5. \quad \text{For } V_{out} = 170 \text{ V}$$

$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

$$R_2 = \frac{V_{out} R_1}{V_{in} - V_{out}}$$

$$R_{therm \text{ T}} = \frac{170 \times 78.4}{230 - 170}$$

$$R_{therm \text{ T}} = 222.1 \text{ } \Omega$$

$$R_{therm} \propto \frac{1}{T}$$

$$\frac{R_{therm \text{ 200 K}}}{R_{therm \text{ T}}} = \frac{T}{200}$$

$$T = 200 \times \frac{R_{therm \text{ 200 K}}}{R_{therm \text{ T}}}$$

$$T = 200 \times \frac{147}{222.1}$$

$$T = 132 \text{ K}$$

$$6. \quad V_{out} = V_{in} \frac{R_{LDR}}{R_1 + R_{LDR}}$$

$$\text{At } 30 \text{ W m}^{-2}$$

$$R_{LDR} = \frac{(V_{in} - V_{out}) R_2}{V_{out}}$$

$$R_{LDR} = \frac{(85.0 - 30.0) \times 10^3}{30.0}$$

$$R_{LDR} = 11\,990 \text{ } \Omega$$

$$\text{For } V_{out} = 54.1 \text{ V}$$

$$R_{LDR} = \frac{(V_{in} - V_{out}) R_2}{V_{out}}$$

$$R_{LDR} = \frac{(85.0 - 54.1) \times 10^3}{54.1}$$

$$R_{LDR} = 3735 \text{ k}\Omega$$

$$R_{LDR} \propto I^3$$

$$R_{LDR} \text{ decreases by } \frac{37}{119.9}$$

$$I \text{ changes by a factor of } \frac{119.9}{37}$$

$$0.6779 \times 30.0 = 20.3$$

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7. DIELECTRICS

Bright spark

$$1. \quad C = \frac{A\epsilon_0\epsilon_r}{d}$$

$$\epsilon_r = \frac{Cd}{A\epsilon_0}$$

$$\epsilon_r = \frac{963 \times 10^{-6} \times 1.98 \times 10^{-3}}{1.44 \times 10^{-4} \times 8.85 \times 10^{-12}}$$

$$\epsilon_r = 1.50 \times 10^9$$

$$2. \quad C = \frac{Q}{V}$$

$$V = \frac{Q}{C}$$

$$V = \frac{81.4 \times 10^{-3}}{185 \times 10^{-6}}$$

$$V = 440 \text{ V}$$

$$E = \frac{1}{2} QV$$

$$E = \frac{1}{2} \frac{Q^2}{C}$$

$$E = \frac{1}{2} \frac{(81.4 \times 10^{-3})^2}{185 \times 10^{-6}}$$

$$E = 17.9 \text{ J}$$

Charging up

$$3. \quad E = \frac{1}{2} QV$$

$$E = \frac{1}{2} CV^2$$

$$E = \frac{1}{2} C(IR)^2$$

$$E = \frac{1}{2} \times 249 \times 10^{-6} \times (41.6 \times 10^{-3} \times 341)^2$$

$$E = 25.1 \times 10^{-3} \text{ J}$$

$$4. \quad E = \frac{1}{2} QV$$

$$E = \frac{1}{2} CV^2$$

$$C = \frac{A\epsilon_0\epsilon_r}{d}$$

$$C = \frac{\pi r^2 \epsilon_0 \epsilon_r}{d}$$

$$C = \frac{\pi \times (7.24 \times 10^{-2})^2 \times 8.85 \times 10^{-12} \times 6.84 \times 10^{11}}{14.1 \times 10^{-2}}$$

$$C = 0.7070 \text{ F}$$

$$E = \frac{1}{2} \times 0.7070 \times 85.0^2$$

$$E = 2550 \text{ J}$$

$$E = 2.5$$

Shocking!

$$5. \quad C = \frac{A\epsilon_0\epsilon_r}{d}$$

$$d = \frac{A\epsilon_0\epsilon_r}{C}$$

$$E = \frac{1}{2} CV^2$$

$$C = \frac{2E}{V^2}$$

$$C = \frac{2 \times 5.51 \times 10^9}{(6.14 \times 10^8)^2}$$

$$C = 23 \times 10^{-8} \text{ F}$$

$$d = \frac{134 \times 10^6 \times 8.85 \times 10^{-12} \times 1}{2.293 \times 10^{-8}}$$

$$d = 40.6 \text{ km}$$

$$6. \quad V = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{Q}{r} = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{Q}{R_2 - R_1}$$

$$Q = CV = C \times \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{Q}{R_2 - R_1}$$

$$C = 4\pi\epsilon_0\epsilon_r(R_2 - R_1)$$

$$C = 4\pi \times 8.85 \times 10^{-12} \times 3$$

$$C = 0.138 \text{ F}$$

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8. CAPACITORS IN SERIES AND IN PARALLEL

Bright spark

- $C_{total} = C_1 + C_2 + C_3 + C_4$
 $C_{total} = 82 \times 10^{-3} + 26 \times 10^{-3} + 31 \times 10^{-3} + 66 \times 10^{-3}$
 $C_{total} = 205 \text{ mF}$
- $$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$\frac{1}{C_{total}} = \frac{1}{255 \times 10^{-12}} + \frac{1}{310 \times 10^{-12}} + \frac{1}{415 \times 10^{-12}} + \frac{1}{220 \times 10^{-12}}$$

$$\frac{1}{C_{total}} = 1.410 \times 10^{10}$$
 $C_{total} = 70.9 \text{ pF}$
 $[p = 10^{-12}]$
 $C_{total} = 7.09 \times 10^{-11} \text{ F}$

Charging up

- $$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{total}} = \frac{1}{35.7 \times 10^{-9}} + \frac{1}{18.2 \times 10^{-9}} + \frac{1}{60.3 \times 10^{-9}}$$

$$\frac{1}{C_{total}} = 99.54 \times 10^6$$
 $C_{total} = 10.05 \text{ nF}$

$$C = \frac{Q}{V}$$

$$Q = CV$$

$$Q = 10.05 \times 10^{-9} \times 500 \times 3$$

$$Q = 1.51 \times 10^{-5} \text{ C}$$

- $C_{total} = C_1 + C_2 + C_3$
 $C_{total} = 550 \times 10^{-6} + 610 \times 10^{-6} + 480 \times 10^{-6}$
 $C_{total} = 1640 \text{ }\mu\text{F}$

$$E = \frac{1}{2} CV^2$$

$$V = \sqrt{\frac{2E}{C}}$$

$$V = \sqrt{\frac{2 \times 6.36}{1640 \times 10^{-6}}}$$

$$V = 88.1 \text{ V}$$

Shocking

- $$E = \frac{1}{2} \frac{Q^2}{C_{total}}$$

$$C_{total} = \frac{1}{2} \frac{Q^2}{E}$$

$$C_{total} = \frac{1}{2} \frac{(382 \times 10^{-3})^2}{65.0}$$

$$C_{total} = 1.122 \times 10^{-3}$$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2 + C}$$

$$\frac{1}{C_2 + C} = \frac{1}{C_{total}} - \frac{1}{C_1}$$

$$\frac{1}{C_2 + C} = \frac{1}{1.122 \times 10^{-3}}$$

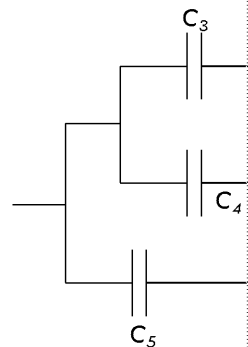
$$\frac{1}{C_2 + C} = 875.7$$

$$C_2 + C = 1.142 \times 10^{-3}$$

$$C = 1.142 \times 10^{-3} - C_2$$

$$C = 1.06 \text{ mF}$$

- The circuit described



$$C_{total} = C_{top} + C_5$$

$$\frac{1}{C_{top}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_{top} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$\frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3 + C_4}} = C_{total}$$

$$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3 + C_4} = \frac{1}{C_{total}}$$

$$\frac{1}{C_1} = \frac{1}{C_{total}} - \frac{1}{C_2} - \frac{1}{C_3 + C_4}$$

$$C_1 = \frac{1}{\frac{1}{C_{total}} - \frac{1}{C_2} - \frac{1}{C_3 + C_4}}$$

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9. CHARGING AND DISCHARGING CAPACITORS

Bright spark

$$1. \quad I = I_0 e^{-\frac{t}{RC}}$$

$$I = 72 \times 10^{-3} \times e^{-\frac{60}{38}}$$

$$I = 15 \text{ mA}$$

$$2. \quad Q_0 = 50 \text{ C}$$

$$T_{1/2} = 25 \text{ s}$$

$$RC = \frac{T_{1/2}}{0.69} = 36 \text{ s}$$

Charging up

$$3. \quad V_0 = 370 \text{ V}$$

$$V_{RC} = 129.5 \text{ V}$$

$$RC = 32.5 \text{ s}$$

$$R = \frac{32.5}{450}$$

$$R = 72.2 \text{ } \Omega$$

$$4. \quad V = V_0 e^{-\frac{t}{RC}}$$

$$R = -\frac{t}{C \ln \frac{V}{V_0}}$$

$$R = -\frac{108}{400 \times 10^{-6} \times \ln \frac{100}{250}}$$

$$R = 295 \text{ k}\Omega$$

$$5. \quad Q = Q_0(1 - e^{-\frac{t}{RC}})$$

$$\frac{Q}{Q_0} = 1 - e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = 1 - \frac{Q}{Q_0}$$

$$-\frac{t}{RC} = \ln(1 - \frac{Q}{Q_0})$$

$$C = -\frac{t}{R \times \ln(1 - \frac{Q}{Q_0})}$$

$$C = -\frac{64.3}{24.5 \times 10^{-3} \times \ln(1 - 0.819)}$$

$$C = 1.54 \times 10^{-3} \text{ F}$$

Shocking!

$$6. \quad \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{total}} = \frac{1}{54.5 \times 10^3} + \frac{1}{62.5 \times 10^3}$$

$$R_{total} = 29.22 \text{ k}\Omega$$

$$C_{parallel} = 120 \times 10^{-6} \text{ F}$$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{260 \times 10^{-6}}$$

$$C_{total} = 132.5 \text{ nF}$$

$$Q = Q_0 e^{-\frac{t}{RC}}$$

$$t = -RC \ln(\frac{Q}{Q_0})$$

$$t = -29.22 \times 10^3 \times \ln(\frac{0.085}{0.140})$$

$$t = 0.685 \text{ s}$$

$$7. \quad a) \quad Q_0 = 0.085 \text{ C}$$

$$Q_{RC} = 0.085 \times e^{-\frac{140}{93.0 \times 10^3}}$$

$$RC = 140 \text{ s}$$

$$C = \frac{140}{93.0 \times 10^3}$$

$$C = 1.51 \times 10^{-3} \text{ F}$$

For $E = 1.81 \text{ J}$

$$E = \frac{1}{2} \frac{Q^2}{C}$$

$$Q = \sqrt{2EC} = \sqrt{2 \times 1.81 \times 1.51 \times 10^{-3}}$$

$$Q = 0.0739 \text{ C}$$

This value of Q is less than Q_0

$$b) \quad V = V_0(1 - e^{-\frac{t}{RC}})$$

$$\frac{V}{V_0} = 1 - e^{-\frac{t}{RC}}$$

$$e^{-\frac{t}{RC}} = 1 - \frac{V}{V_0}$$

$$-\frac{t}{RC} = \ln(1 - \frac{V}{V_0})$$

$$\frac{V}{V_0} = 0.865 = e^{-\frac{0.685}{RC}}$$

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10. TRANSFORMERS

Bright spark

$$1. \quad \frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$$V_s = \frac{V_p N_s}{N_p}$$

$$V_s = \frac{350 \times 140}{240}$$

$$V_s = 204 \text{ V}$$

$$2. \quad \text{efficiency} = \frac{I_s V_s}{I_p V_p}$$

$$\text{efficiency} = \frac{670 \times 10^{-3} \times 450}{130 \times 10^{-3} \times 7.5 \times 10^3}$$

$$\text{efficiency} = 0.31 \text{ or } 31 \%$$

Charging up

$$3. \quad \text{efficiency} = \frac{I_s V_s}{I_p V_p}$$

$$I_s = \text{efficiency} \times \frac{I_p V_p}{V_s}$$

$$I_s = 0.942 \times \frac{6.50 \times 10^3}{192}$$

$$I_s = 31.9 \text{ A}$$

$$4. \quad \frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$$\text{efficiency} = \frac{I_s V_s}{I_p V_p} = \frac{I_s N_s}{I_p N_p}$$

$$\text{efficiency} = \frac{37.6 \times 700}{84.3 \times 400}$$

$$\text{efficiency} = 0.781 \text{ or } 78.1 \%$$

$$5. \quad \text{efficiency} = \frac{I_s V_s}{I_p V_p} = \frac{P_s}{P_p}$$

$$P = I^2 R$$

$$\text{efficiency} = \frac{I_s^2 R_s}{I_p^2 R_p}$$

$$I_s = \sqrt{\text{efficiency} \times \frac{P_p}{R_s}}$$

$$I_s = \sqrt{0.900 \times \frac{850}{77.3}}$$

$$I_s = 3.15 \text{ A}$$

Shocking!

$$6. \quad \text{efficiency} = \frac{I_s V_s}{I_p V_p}$$

$$\text{efficiency} = \frac{V_s^2}{R_s} \times \frac{1}{I_p V_p}$$

$$N_s = 1.67 \times N_p = 1$$

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$$V_s = \frac{V_p N_s}{N_p}$$

$$V_s = \frac{230 \times 300}{180}$$

$$V_s = 383.3 \text{ V}$$

$$\text{efficiency} = \frac{383.3^2}{77.8} \times \frac{1}{I_p V_p}$$

$$\text{efficiency} = 0.241 \text{ or } 24.1 \%$$

$$7. \quad P_p = \frac{V_p^2}{R_p}$$

$$V_p = \sqrt{P R}$$

$$V_p = \sqrt{985 \times 640}$$

$$V_p = 794.0 \text{ V}$$

$$V_p = I_p R_p$$

$$I_p = \frac{V_p}{R_p}$$

$$I_p = \frac{794.0}{640}$$

$$I_p = 1.241 \text{ A}$$

Step-down transformer

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

$$V_s = \frac{V_p N_s}{N_p}$$

$$V_s = \frac{794.0}{1.76}$$

$$V_s = 451.1 \text{ V}$$

$$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$$

$$I_s = \text{efficiency} \times \frac{I_p V_p}{V_s}$$

$$I_s = 0.720 \times \frac{1.241 \times 794.0}{451.1}$$

$$I_s = 1.57 \text{ A}$$

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11. ELECTRICITY GENERATION

Bright spark

1. $N\Phi = BAN \cos \theta$

$$N\Phi = 19 \times 10^{-6} \times 3.6 \times 10^{-3} \times 200 \cos 0.40$$

$$N\Phi = 1.26 \times 10^{-5} \text{ T m}^2$$

2. $\epsilon = N \frac{\Delta\Phi}{\Delta t}$

$$\epsilon = N \frac{\Delta B}{\Delta t}$$

$$\epsilon = 38 \times \frac{12.6 \times 10^{-4} \times (93.7 \times 10^{-3} - 14.1 \times 10^{-3})}{60.0}$$

$$\epsilon = 6.35 \times 10^{-5} \text{ V}$$

3. $\epsilon = BAN\omega \sin \omega t$

$$\omega = 170 \times \frac{2\pi}{60} = 17.80 \text{ rad s}^{-1}$$

$$\epsilon = 407 \times 10^{-3} \times 10^{-4} \times 120 \times 17.80 \times \sin(17.80 \times 60)$$

$$\epsilon = -1.3 \text{ V}$$

Charging up

4. $\epsilon = N \frac{\Delta\Phi}{\Delta t}$

$$N\Phi = BAN \cos \theta$$

$$\epsilon = \frac{BAN \Delta \cos \theta}{\Delta t}$$

$$A = \frac{\epsilon \Delta t}{BN \Delta \cos \theta}$$

$$A = \frac{130 \times 10^{-3} \times 2.50}{84 \times 10^{-3} \times 85 \times (\cos 36 - \cos 0)}$$

$$A = 0.238 \text{ m}^2$$

5. Period = 2π

$$T = 0.32 \text{ s}$$

$$\omega t = 2\pi$$

$$\omega = \frac{2\pi}{0.32}$$

$$\omega = 19.6 \text{ rad s}^{-1}$$

6. $\epsilon = N \frac{\Delta\Phi}{\Delta t}$

$$\Delta t = \frac{N \Delta\Phi}{\epsilon}$$

$$\Delta t = \frac{NBA}{\epsilon}$$

$$\Delta t = \frac{50 \times 20 \times 10^{-3} \times 300 \times 10^{-3}}{70 \times 10^{-5}}$$

$$\Delta t = 4.29 \text{ s}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{1.5 \times 10^{-2}}{4.29 \times 10^{-3}}$$

$$v = 35 \text{ m s}^{-1}$$

Shocking!

7. $T = 1.25 \text{ s}$

$$\omega = \frac{2\pi}{T}$$

$$\epsilon = BAN\omega$$

$$BAN\omega = 17.500$$

$$B = \frac{17.500}{AN\omega}$$

$$B = \frac{17.500}{1.80 \times 200 \times 2\pi}$$

$$B = 0.644 \text{ T}$$

8. $\epsilon = N \frac{\Delta\Phi}{\Delta t}$

$$N\Phi = BAN \cos \theta$$

$$\epsilon = \frac{BAN \Delta \cos \theta}{\Delta t}$$

$$\Delta t = \frac{\epsilon \Delta \cos \theta}{BAN}$$

$$\Delta t = 0.545 \text{ s}$$

$$A = \frac{\epsilon \Delta t}{BN \Delta \cos \theta}$$

$$\Delta \cos \theta = 1$$

$$A = \frac{55.6 \times 10^{-3} \times 0.545}{27.3 \times 10^{-3} \times 85}$$

$$A = 0.0222 \text{ m}^2$$

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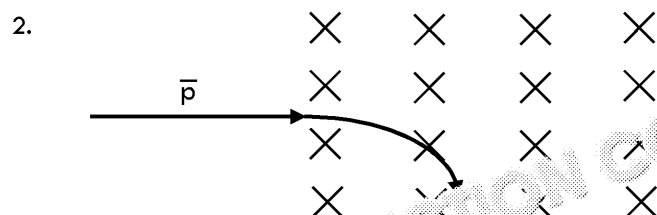
12. ELECTROMAGNETIC PHENOMENA

Bright spark

1. Directly between the wires, the two magnetic fields generated by each wire oppose each other, cancelling out.

Further from the wires, both magnetic fields have the same direction, adding to a large magnetic field.

The overall magnetic field is the same shape as for a single wire.



Charging up

3. The charge carriers (electrons) in the plane's metal frame are moving forwards with the simulation's magnetic field.

Using Fleming's left hand rule (with the magnetic field upwards and positive current to the right), the electrons experience a force to the left, creating a positive current to the right.

A current is now set up to the right. Using Fleming's left hand rule again for this new current and the magnetic field, a force is experienced to the back of the plane – opposing the plane's motion.

4. The balls dropped through the tubes create a magnetic field by the charges moving past each other.

The copper tube has more charge carriers than steel as it is a better conductor, so a stronger magnetic field is produced.

Because a stronger magnetic field is produced, a stronger force is felt by the ball passing through.

This force opposes the motion of the ball due to Lenz's law – the force opposes the change in magnetic flux. This means that the ball in the copper tube moves more slowly than the ball dropped through the steel tube.

Shocking!

5. As an alternating current is passed through the coil of wire, an e.m.f. is induced in the brass ring.

This e.m.f. changes direction with the change in direction of the AC power supply.

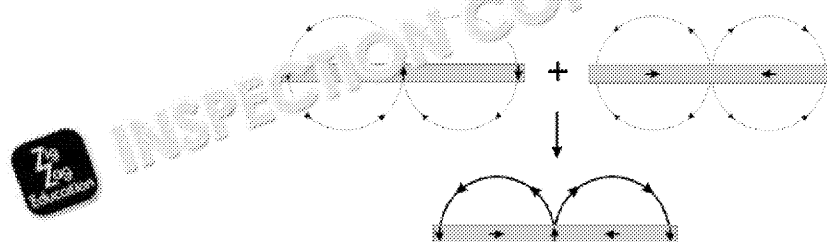
This induces a current in the brass ring.

The magnetic field produced by the changing current in the iron core interacts with the magnetic field of the AC power supply, pushing the ring upwards.

As the current or frequency of the AC power supply increases, the brass ring experiences a stronger force.

If the current or frequency of the AC power supply is high enough, the brass ring could be pushed into the air.

6. Taking a small section of the array:



For a downward-facing arrow, the arrows surrounding it add to the magnetic field above the array.

For an upward-facing arrow, the arrows surrounding it add to the magnetic field below the array.

Where the magnetic fields add, the magnetic field is much stronger. Where the magnetic fields cancel out, the field is weak, or zero.

This means that the magnetic field is much stronger above the array than below it.

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EXAM-STYLE QUESTIONS

1	a	$R_{\text{total}} = R_{\text{series}} + R_{\text{parallel}}$ $\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} \checkmark$ $\frac{1}{R_{\text{parallel}}} = \frac{1}{22 \times 10^3} + \frac{1}{24 \times 10^3}$ $\frac{1}{R_{\text{parallel}}} = 8.71 \times 10^{-5} \checkmark$ $R_{\text{parallel}} = 11500 \Omega$ $R_{\text{total}} = 11500 + 17 \times 10^3 \checkmark$ $R_{\text{total}} = 28.5 \times 10^3 \Omega \checkmark$
	b	$\mathcal{E}_{\text{total}} = \mathcal{E}_1 + \mathcal{E}_2$ $\mathcal{E}_{\text{total}} = 21 + 19$ $\mathcal{E}_{\text{total}} = 40 \text{ V} \checkmark$ $r_{\text{total}} = r_1 + r_2$ $r_{\text{total}} = 0.68 + 0.34$ $r_{\text{total}} = 1.02 \Omega \checkmark$ $I = \frac{\mathcal{E}}{R + r}$ $I = \frac{40}{28.5 \times 10^3 + 1.02} \checkmark$ $I = 1.4 \times 10^{-3} \text{ A} \checkmark$
	c	$P = I^2(R + r)$ $P = (1.4 \times 10^{-3})^2 \times (28.5 \times 10^3 + 102) \checkmark$ $P = 0.056 \text{ W} \checkmark$
2	a	$\rho = \frac{RA}{L}$ $\rho = \frac{1.1 \times 10^{-4} \times (8.3 \times 10^{-2})^2}{12 \times 10^{-2}} \checkmark$ $\rho = 6.31 \times 10^{-6} \Omega \text{ m} \checkmark$
	b	$V = A_1 L_1 = 12 \times 10^{-2} \times (8.3 \times 10^{-2})^2$ $V = 8.27 \times 10^{-4} \text{ m}^3 \checkmark$ $A_2 = \frac{V}{L_2} = \frac{8.27 \times 10^{-4}}{1.3}$ $A_2 = 6.36 \times 10^{-4} \text{ m}^2 \checkmark$ $R_2 = \frac{\rho L_2}{A_2}$ $R_2 = \frac{6.31 \times 10^{-6} \times 1.3}{6.36 \times 10^{-4}} \checkmark$ $R_2 = 1.29 \times 10^{-2} \Omega \checkmark$
	c	$P_{\text{dissipated}} = I^2 R$ $I = \frac{P}{V}$ $P_{\text{dissipated}} = \frac{P^2 R}{V^2} \checkmark$ $P_{\text{dissipated}} = \frac{45^2 \times 1.29 \times 10^{-2}}{92^2} \checkmark$ $P_{\text{dissipated}} = 3.1 \times 10^{-3} \text{ W} \checkmark$

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3	a	$C = \frac{A\epsilon_r\epsilon_0}{d}$ $\epsilon_r = \frac{dC}{A\epsilon_0} \checkmark$ $\epsilon_r = \frac{6.5 \times 10^{-3} \times 850 \times 10^{-6}}{\pi \times (4.1 \times 10^{-3})^2 \times 8.85 \times 10^{-12}} \checkmark$ $\epsilon_r = 1.18 \times 10^{10} \checkmark$
	b	$C_{\text{parallel}} = C_1 + C_2$ $C_{\text{parallel}} = 2 \times 850 \times 10^{-6}$ $C_{\text{parallel}} = 1.70 \times 10^{-3} \text{ F} \checkmark$ $\frac{1}{C_{\text{total}}} = \frac{1}{C_{\text{series}}} + \frac{1}{C_{\text{parallel}}}$ $\frac{1}{C_{\text{total}}} = \frac{1}{850 \times 10^{-6}} + \frac{1}{1.70 \times 10^{-3}}$ $\frac{1}{C_{\text{total}}} = 1770 \checkmark$ $C_{\text{total}} = 5.65 \times 10^{-4}$
	c	 $\frac{V_0}{280} = 0.05 \text{ s} \checkmark$ $T_{1/2} = 0.69RC$ $R = \frac{T_{1/2}}{0.69 \times C} \checkmark$ $R = \frac{105}{0.69 \times 5.65 \times 10^{-4}} \checkmark$ $R = 270 \times 10^3 \Omega \checkmark$
4	a	$\omega T = 2\pi \checkmark$ $\omega = \frac{2\pi}{T}$ $\omega = \frac{2\pi}{1.3} \checkmark$ $\omega = 4.83 \text{ rad s}^{-1} \checkmark$
	b	$\epsilon = BAN\omega \sin \omega t$ $B = \frac{\epsilon}{AN\omega} \checkmark$ $B = \frac{135}{0.16 \times 320 \times 4.83} \checkmark$ $B = 0.55 \text{ T} \checkmark$
	c	$\text{efficiency} = \frac{I_s V_s}{I_p V_p}$ $\text{efficiency} = \frac{I_s N_s}{I_p N_p} \checkmark$ $I_s = \text{efficiency} \times \frac{I_p N_p}{N_s} \checkmark$ $I_s = 0.83 \times \frac{210 \times 10^{-3} \times 75}{110} \checkmark$ $I_s = 0.12 \text{ A} \checkmark$
	d	$\text{Horizontal divisions} = \frac{T}{\text{number of divisions}}$ $\text{Horizontal divisions} = \frac{1.3}{17} \checkmark$ $\text{Horizontal divisions} = 0.076 \checkmark$  $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ $V_s = 135 \times \frac{110}{75}$ $V_s = 198 \text{ V} \checkmark$ $\text{Vertical divisions} = \frac{V}{\text{number of divisions}}$ $\text{Vertical divisions} = \frac{198}{8}$ $\text{Vertical divisions} = 25 \text{ s div}^{-1} \checkmark$

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