

Maths Skills

for GCSE Edexcel Physics

Also applicable for GCSE Combined Science (1SC0)

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Teacher's Introduction

This GCSE Maths Skills pack will support students studying the Edexcel GCSE Physics or Combined Science specification with the key mathematical skills needed in their course.

Mathematical skills pose a challenge for many students, with some finding it difficult to see how a skill learned in a Maths lesson is applied in a Physics lesson. This resource has been designed to support students in making this connection. It gives a gentle, conversational review of the skill, with worked examples, and offers students the opportunity to practise the skill in isolation and then also in the context of an examination-style question. By using this resource, students can ensure they have the skills they need for each section of the Physics course. They can work through the chapters proactively, or they can be directed to them as support for skills identified in class as in need of some improvement.

There are five chapters covering all the key maths skills needed for GCSE Physics. Each chapter contains the following elements:

- **Specification overview** – this provides an overview of the skills and explains what the exam board requires students to demonstrate in the exam with the skills.
- **Theoretical overview** – a brief summary recapping the skills and demonstrating how to apply the skills.
- **Worked examples** – shows one or more fully worked questions which use the relevant skill, to demonstrate how students should approach them.
- **Practice questions** – each skill is concluded with practice questions that increase in difficulty. All the physics knowledge needed to complete the question will be provided, and the question focuses on testing students' understanding of the maths skill itself.

There are two diagnostic tests for each chapter. The first is designed to be used before you work through each chapter and is provided at the start of the resource. The second is designed to be used after reviewing the chapter's content and is provided after the main content of the resource, just before the answers. The tests will allow you to identify areas for particular focus before undertaking the work, and then afterwards, should further focus on particular areas be necessary.

The chapters cover:

1. Arithmetic
2. Algebra
3. Handling data
4. Graphs
5. Trigonometry

I have covered some of the graphs content under algebra and handling data, since they naturally lend themselves to this.

October 2022

Students' Introduction

Mathematical skills pose a challenge for many students, with some finding it difficult to see how a Maths lesson is applied in a Physics lesson. This resource has been designed to bridge this connection. It gives a gentle, conversational review of the skill, with worked examples and an opportunity to practise the skill in isolation and then also in the context of an exam question. By using these resources, you can ensure you have the skills you need for each section of the course. You can work through the chapters proactively, or your teacher will direct you to the skills identified in class as in need of some improvement.

There are five chapters. There are also two sets of diagnostic tests to help identify areas for improvement. These are linked to the relevant chapters. Within each chapter, there are four elements:

- **Specification overview** – this provides an overview of the skills and explains how you can demonstrate in the exam with the skills.
- **Theoretical overview** – a brief summary recapping the skills and demonstrating how to use them.
- **Worked examples** – shows one or more fully worked questions which use the skills and explains how you should approach them.
- **Practice questions** – each skill is concluded with practice questions that increase your knowledge needed to complete the question will be provided, and the questions help to build your understanding of the maths skill itself.

The chapters cover:

1. Arithmetic
2. Algebra
3. Handling data
4. Graphs
5. Trigonometry

I have covered some of the graphs content under algebra and handling data, since this is relevant to this.

There are two diagnostic tests for each chapter. The first is designed to be used before starting a chapter. The second is designed to be used after reviewing the chapter's content to help identify areas for particular focus before undertaking the work, and then afterwards to see if any particular areas be necessary.

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DIAGNOSTIC TEST

A1 Arithmetic

1. Write $\frac{1}{4}$ as a decimal.
.....
2. Write 0.333333 as a fraction.
.....
3. Write 1024 in standard form.
.....
4. 1 GB is 1×10^9 bytes. Write this number as a decimal.
.....
5. An electric motor needs 12 V to operate normally. A cell provides 12.5 % of this. Write 12.5 % as a decimal.
.....
6. If I am making a string of 125 LED lamps and I have only 25 LEDs, what percentage of the string do I have?
.....
7. Estimate how many adult males you would need in order to have the same mass as 1000 kg of lead.
.....
.....

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B1 Algebra

1. Write the following expression in words: $x \gg y$

.....

2. Make variable t the subject of this equation: $v = u + at$

.....

3. A certain constant can be calculated using the equation

$$\text{constant} = \frac{\text{force} \times \text{distance}^2}{\text{mass}^2}$$

Given that the units of mass = kg, force = N and distance = m, what are the units of the constant?

.....

4. A car is travelling at 10 m/s and accelerates to 20 m/s in 1.5 seconds. Calculate the acceleration of the car. Use the equation in question 2, where $u = 10$ m/s, $v = 20$ m/s and $t = 1.5$ s.

.....

.....

5. If I start 10 m from my house and walk away at 3 m/s in a straight line, describe the relationship between my distance from the house and the time I walk for.

.....

6. Sand pours into a bucket at a rate of 150 g per second. If the bucket was empty, describe the relationship between the weight of sand in the bucket and the time for which the sand has been pouring.

.....

7. If the current, I , through a lamp increases from 0 A to 5 A in 0.002 s, calculate the resistance of the lamp.

.....

.....

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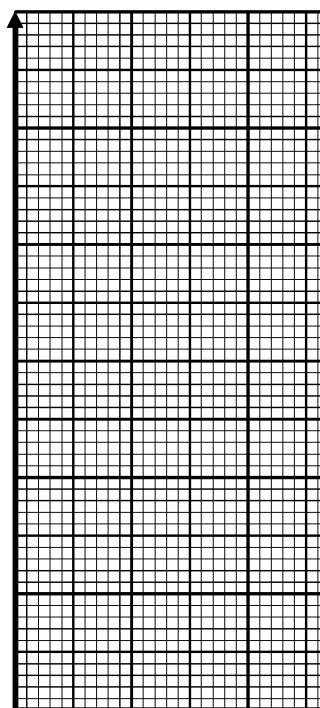
C1 Handling data

1. Write 12 323 to three significant figures.
.....
2. The finishing times in seconds for a group of athletes running a 100 m race were
10.01 10.01 10.28 9.97 10.52 10.72 10.01
 - a. Calculate the mean time for the race.
.....
.....
 - b. State the median time for the race.
.....
 - c. State the modal time for the race.
.....
3. A survey of the heights of trees in a forest is conducted by a group of students. What should they do?
 - a. Record the data they collect?
.....
 - b. Represent those data on a graph (what type of graph should be plotted)?
.....
4. The survey in question 3 is expanded to consider leaf shapes found in the forest. What type of graph should be plotted of the data collected?
.....
5. A rectangle is 10 cm wide by 120 cm long. What is the order of magnitude of its area?
.....
6. An experiment to find the relationship between the potential difference ('volts') across a wire and the current through it was conducted. The results are shown in the table below.

Potential difference (V)	0	1	2	3	4	5
Current (A)	0	0.02	0.04	0.05	0.078	0.1

Draw a graph of these data.
State the relationship between the potential difference across the wire and the current through it.

.....



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D1 Graphs

1. What is the general equation of a straight line graph? How would this be different if the graph represented a directly proportional relationship?

.....

.....

.....

2. a. Plot a graph of $y = 3x + 2$ for values of x from 0 to 9.
b. What is the gradient of this graph?

.....

.....

- c. What is the y-intercept of this graph?

.....

.....

3. The power of a resistor is related to the potential difference across it and its resistance by the equation

$$P = \frac{V^2}{R}$$

- a. Plot a graph of P against V for values of V from 0 V to 5 V. Use a value of $R = 10 \Omega$.
b. Measure the gradient of the graph at $V = 4$ volts.

.....

.....

- c. What does this value of the gradient tell you?

.....

.....

.....

.....

.....

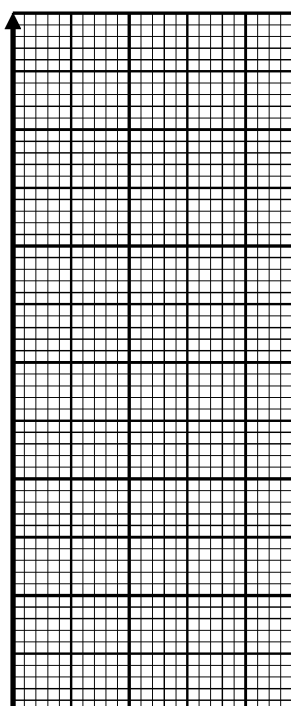
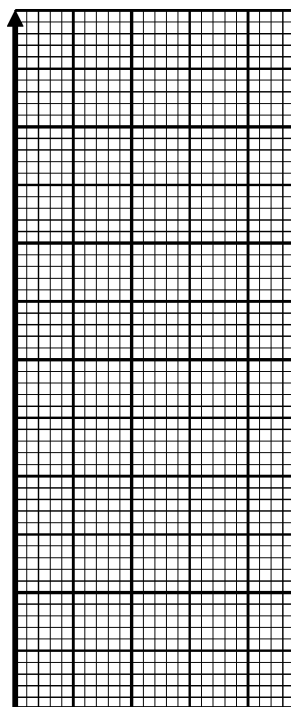
4. Give an example of when you would calculate the area between a graph and the x-axis, and explain the importance of that area in the example you give.

.....

.....

.....

.....

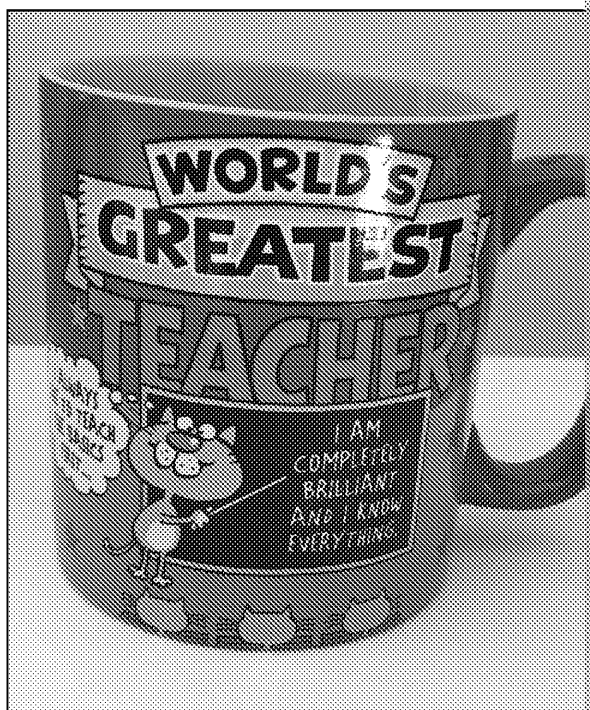


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E1 Geometry and trigonometry

1. What instrument would you use to measure an angle?
.....
2. What units do we use to measure angles?
.....
3. What is the formula for each of the following?
 - a. The area of a rectangle
.....
 - b. The volume of a cube
.....
 - c. The area of a triangle
.....
 - d. The surface area of a cube
.....
4. Here is a picture of a mug. What would it look like in 2D? Draw a diagram

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ARITHMETIC

SPECIFICATION OVERVIEW

- 1a – Recognise and use expressions in decimal form
- 1b – Recognise and use expressions in standard form
- 1c – Use ratios, fractions and percentages
- 1d – Make estimates of the results of simple calculations

THEORETICAL OVERVIEW

This section is about getting the basic toolkit in place for you: recognising numbers (fractions, percentages) and being able to convert between these forms. It also introduces concepts we'll revisit in Chapter 3.

Let's start by being clear about what we mean by each of the terms we're looking at.

Decimal

Expressing a number as a decimal means having a decimal point in it. For example, 1.25.

There are other ways of expressing the same number. You could also say 'ten and

$$4 \times 0.25 = 1$$

$$\text{so } 0.25 \text{ is } 1 \div 4, \text{ or } \frac{1}{4}$$

WORKED EXAMPLE

Express $\frac{5}{2}$ as a decimal.

Solution

$$\frac{5}{2} = \frac{4}{2} + \frac{1}{2} = 2 + \frac{1}{2} = 2.5 \text{ (as } \frac{1}{2} = 0.5 \text{ since } 0.5 \times 2 = 1)$$

Fraction

One number divided by another is a fraction. For example:

$$\frac{30}{4}$$

Of course, we could also express this as a decimal by calculating the value of this

$$\frac{30}{4} = 7.5$$

It can be helpful to remember some fraction to decimal conversions so that you recognise them without using a calculator.

Fraction	decimal	Fraction
$\frac{1}{2}$	0.5	$\frac{1}{10}$
$\frac{1}{3}$	0.33	$\frac{1}{100}$
$\frac{1}{4}$	0.25	$\frac{1}{1000}$
$\frac{1}{5}$	0.2	

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WORKED EXAMPLE 1

Write $\frac{3}{5}$ as a decimal.

Solution

Use your calculator to do $3 \div 5 = 0.6$

When we write fractions, we should always look to simplify them. For example, $\frac{9}{27}$ is a

WORKED EXAMPLE 2

Simplify $\frac{9}{27}$

Solution

Since both 9 and 27 can be divided by 9 (this is the biggest number that 'goes in' as a factor, as you would say in maths), we can write it as $\frac{1}{3}$ because $\frac{9}{9} = 1$ and $\frac{27}{9} = 3$

You could also see this as $\frac{27}{9} = \frac{9 \times 3}{9 \times 1} = \frac{3}{1} = 3$

Percentage

This is a fraction which is evaluated (worked out) and then multiplied by 100. It is a way of expressing a fraction as a percentage. For example, what percentage of a day is three hours?

$$\frac{3}{24} = 0.125$$

$$\text{and } 0.125 \times 100 = 12.5 \%$$

It is possible for the percentage to be greater than 100, of course. The average caffeine content of an energy drink is 64 mg (milligrams). There can be 200 mg of caffeine in an energy drink, so the percentage is

$$\frac{200}{64} \times 100 \% = 313 \% \text{ of the caffeine in one espresso}$$

WORKED EXAMPLE

If we had an electric car with a 77 kWh battery, this means that the battery can store 77 kWh of electrical energy. (1 kWh = 3 600 000 joules.)

We might want to know what percentage of this charge we have when the battery is at 55 kWh.

Solution

So, we express our numbers as a fraction: $\frac{55}{77}$

Then we evaluate this (put the numbers into a calculator):

$$= 0.714$$

So, this is the decimal form of the fraction.

Now we multiply by 100 to get the percentage:

$$0.714 \times 100 = 71.4 \%$$

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Estimation

This is about making an educated guess at the answer. So we might want a rough idea of how many textbooks we could stack floor to ceiling in a room.

WORKED EXAMPLE

Estimate how many Physics textbooks you could stack floor to ceiling in a room.

Solution

We could estimate the thickness of one textbook to be 5 cm, and the height of the room to be 250 cm. So we could stack $\frac{250}{5} = 50$ books between the floor and the ceiling.

This is an estimate – I didn't measure the thickness of the textbook or the height of the room. These are about the sort of size you might expect. Books vary and rooms vary, but this is a reasonable estimate.

Standard form

Standard form is where a number is written so that it is in two parts:

1. A number greater than 1 and less than 10
2. This number is then multiplied by 10 to a power

WORKED EXAMPLE 1

Express 900 in standard form.

Solution

$$900 = 9 \times 10^2$$

9 is a number greater than 1 and less than 10; to make our standard form value the same as 900, we multiply by 100. 100 is 10^2 .

Another example, to illustrate the point further, is 0.9.

WORKED EXAMPLE 2

Express 0.9 in standard form.

Solution

0.9 is less than 1, so we have to write the number as '9'. Now we have to have our standard form value the same as 0.9.

So we are actually dividing the 9 by 10 to get to 0.9: $\frac{9}{10}$

but $\frac{1}{10}$ can be written 10^{-1}

So, 0.9 in standard form is 9×10^{-1}

Look at the pattern in the values of the powers of 10 and the equivalent fraction or decimal.

Power of 10	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10^1
Decimal	0.0001	0.001	0.01	0.1	1	10
Fraction	$\frac{1}{10\,000}$	$\frac{1}{1\,000}$	$\frac{1}{100}$	$\frac{1}{10}$	$\frac{1}{1}$	Not really

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Ratio

A ratio expresses how many of one thing you have, compared to another. For example, with car shampoo, the label on the bottle might instruct you to 'Put 30 ml of shampoo to 5000 ml of water'.

WORKED EXAMPLE 1

What is the ratio of shampoo to water required in the example above?

Solution

We have a ratio of 30 to 5000 (since there is 1000 ml in 1 L).

This is usually written 30 : 5000, or, better still, 3 : 500.

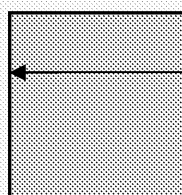
WORKED EXAMPLE 2

What is the ratio of the height of this rectangle to its length?

Solution

The ratio will be expressed height : length because that's how the question is asked.

The height is 7 cm and the length is 21 cm, so the ratio is 7 : 21. However, we can simplify this by dividing both numbers by 7 (since 7 is the highest common factor of both numbers – the highest number that divides exactly into each), giving us 1 : 3 as our final answer.



PRACTICE QUESTIONS

- Copy and complete the table below. Use a calculator to calculate the decimal. Some of the answers will come out as an exact number (e.g. $\frac{1}{10} = 0.1$), while others will not.

If the number is NOT exact, give your answer to three significant figures.

	Fraction	Decimal		Fraction	Decimal	
A	$\frac{1}{16}$		D	$\frac{3}{7}$		G
B	$\frac{1}{20}$		E	$\frac{82}{3}$		
C	$\frac{1}{5}$		F	$\frac{213}{5}$		

- Copy and complete the table below. Calculate the percentages.

	Number	Percentage		Number	Percentage	
A	$\frac{1}{16}$		D	$\frac{3}{7}$		G
B	$\frac{1}{20}$		E	0.22		H
C	$\frac{1}{5}$		F	$\frac{75}{136}$		I

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3. Copy and complete the table below. Express the percentages as fractions. Write the simplest fraction possible.

	Percentage (%)	Fraction
A	5	
B	22	
C	12	
D	40	
E	67.67	
F	75	
G	90	

4. Convert the numbers in the left-hand column into standard form.

	Number	
A	365.25 days in a year	
B	1 500 000 km is the average Earth–Sun distance	
C	0.000 000 000 144 m is the radius of an atom of gold	
D	101 000 N/m ² is atmospheric pressure	
E	24.8 N/kg is the value of gravity on Jupiter	

5. Convert the numbers in the left-hand column from standard form into a decimal.

	Standard form	
A	$1 \times 10^3 \text{ cm}^3$ in one litre	
B	$3.1\ 557\ 6 \times 10^7$ seconds in a year	
C	$1 \times 10^{-5} \text{ m}$ is the diameter of a human hair	
D	$1 \times 10^{-9} \text{ m}$ is one nanometre	
E	$9.1 \times 10^9 \text{ km}$ is the diameter of the solar system	

6. Estimate the values of the following:
- How long it would take to walk 10 km.
 - How long it would take to run 300 m.
 - How far you've travelled if you were on a passenger aircraft for two hours.
 - (Harder!)** How long it would take a ball to hit the ground when dropped from a height of 10 m. The acceleration due to gravity is 9.8 m/s^2 . You might find it helpful to sketch the path the ball would look like.
7. If 75 % of my domestic electricity is used for heating and cooking, and the rest is used for running other electrical devices, what is the ratio of usage for heating and cooking to the rest?
8. The constant π is the ratio of the circumference of a circle to the diameter of the circle. If a circular road is built around the equator, calculate the circumference at the equator, if the radius of the Earth is 6370 km. Use π as 3.14.

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ALGEBRA

SPECIFICATION OVERVIEW

3a – Understand and use the symbols $=$, $<$, $<<$, $>>$, $>$, \propto , \sim

3b – Change the subject of an equation

3c – Substitute numerical values into algebraic equations using appropriate units for

3d – Solve simple algebraic equations

THEORETICAL OVERVIEW

This section is about equations and the relationships they represent. It is also a further put together in Chapter 1.

Let's start by being clear about the meanings of the terms we will be using:

Proportional

When two things are proportional, whichever operation (e.g. multiply, divide, square) happen to the other. For example:

$$\text{distance} = \text{speed} \times \text{time}$$

We could say 'the distance travelled is proportional to the speed the object travels' or 'distance travelled is proportional to the time of travel'.

What this means is that if we travel for $2 \times$ as long, **keeping the speed the same**, the distance travelled is multiplied by 2, or

if we travel at twice the speed, we will cover twice the distance **in the same period of time** by 2.

To look at this in terms of division rather than multiplication, if we were to travel at half the speed, we would cover half the distance **in the same period of time** – both divided by 2.

Notice that in each of the examples, one of the variables on the right-hand side of the equation (time) was kept the same (see the parts of the examples **in bold**), while we looked at changing the other – so one was a **constant**.

Generally, when one thing (let's call it y) is proportional to another (let's call it x), then

$$y \propto x$$

The symbol \propto is the Greek letter alpha and it means 'proportional to' in this case. For example, if we are studying radioactivity and it'll represent a type of radiation then.

So, this means 'y is proportional to x'; that is, whatever we do to x (multiply, divide, square, etc.), the same will happen to y.

Moving from 'proportional to', in order to get an equation, we need that constant

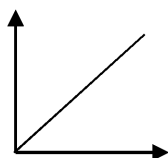
$$y \propto x$$

$$\text{So } y = \text{constant} \times x$$

We normally write the constant as 'k', so our equation looks like this:

$$y = k \times x$$

This is a proportional (sometimes called **directly proportional**) relationship between two quantities. It is represented by a straight line graph which passes through the origin (see 'Graphs' section later).



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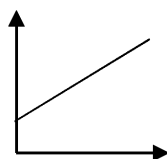
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If we had

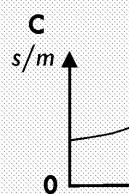
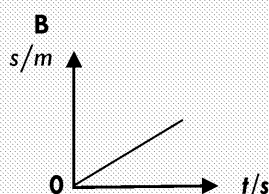
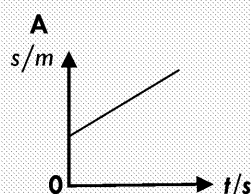
$$y = kx + \text{constant}$$

we would have what is sometimes called **indirect proportion**. This will still give us a straight line, but it will not go through the origin if the constant is not equal to zero. Again, we'll talk about graphs later.



WORKED EXAMPLE

Which of these graphs shows displacement, s , directly proportional to time, t ?



Solution

The answer is B. It is the only straight line which also goes through the origin (0,0).

Symbols

There are some symbols that you just need to recognise and understand. These are read from left to right:

- < means 'less than'. So $y < x$ means 'y is less than x'.
- << means 'much less than'. So $y << x$ means 'y is much less than x'.
- > means 'greater than'. So $a > b$ means 'a is greater than b'.
- >> means 'much greater than'. So $a >> b$ means 'a is much greater than b'.

Note: for each of these arrow symbols, the bigger quantity goes nearest to the smaller quantity goes at the 'point' of the arrow.

- ~ means 'approximately equal to'. So you might say that a T-shirt priced at £9.99 is approximately £10. You would write that the cost of the T-shirt is $\sim £10$.

A more scientific example would be the number pi (π). It has an endless number of decimal places, so we use 3.14, so strictly we should write $\pi \sim 3.14$.

WORKED EXAMPLE

Rewrite the following statement replacing the underlined word with the most appropriate symbol from either <, >, >>, << or ~.

The mains electricity supply is approximately 230 V.

Solution

The mains electricity supply is ~ 230 V.

Rearranging and using equations

Equations tell us how two or more quantities (things) relate to each other. We all know that if you buy (e.g. 1 kg bags of sugar), the heavier your bag of shopping will be. There's a simple equation for this:

$$\text{total weight} = \text{weight of a bag of sugar} \times \text{number of bags}$$

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You might reasonably say ‘Why would I want an equation? That’s just common sense – we need equations. They allow us to:

- Predict the outcome of an event (‘If I increase the number of electrical items I have, will I be tripping the circuit breaker?’)
- Plan how to do something successfully (‘If I build this motorway bridge with the weight of traffic it needs to be able to carry?’)
- Answer exam questions!

You will have lots of equations you need to memorise in order to succeed in your GCSE. You will have a smaller number that you will be given, if you need them.

However, you need to be able to rearrange all of them so that the quantity you’re interested in is the subject. For example, let’s take Ohm’s law, which tells us how the electrical current is related to the voltage across the conductor. It states:

potential difference = current \times resistance

In symbols, $V = I \times R$

So, if we are given the current through a lamp (I) and the resistance of the lamp (R), we can calculate the potential difference (V) by $I \times R$ to get V .

But what if we are given the values of V and I in order to find R ?

Now we need to rearrange the equation so that R is the subject (that is to say the quantity we’re interested in).

Remember, it’s an equation, so, to keep both sides equal, whatever we do to one side we must do to the other.

WORKED EXAMPLE 1

Rearrange $V = I \times R$ to make R the subject.

Solution

On the right we have $I \times R$, but we just want R . So, if we divide this by I , that gives us R .

$$\frac{I \times R}{I}$$

Now, the rules of maths mean that we can divide either of the things on the top line – it doesn’t matter which.

$$\text{In } \frac{I}{I} \times R, \text{ since } \frac{I}{I} = 1 \text{ then } \frac{I}{I} \times R = R$$

Now, since we divided the right-hand side by I , we must do the same to the left-hand side to keep the equation equal. So $\frac{V}{I} \times \frac{I \times R}{I} = R$

We can rewrite this, with the subject (R) on the right:

$$R = \frac{V}{I}$$

What we’ve done, then, is to divide both sides of the equation by the thing we want to get rid of, in order to get just the thing we want on that side.

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Here's another example:

WORKED EXAMPLE 2

$$y = mx + c$$

Let's make x the subject.

Solution

Step 1: deal with the c on the right-hand side – subtract c from both sides:

$$y - c = mx + c - c$$

$$\text{So } y - c = mx$$

Step 2: get x on its own on the right – divide both sides by m :

$$\frac{y - c}{m} = \frac{mx}{m}$$

$$\text{So } \frac{y - c}{m} = x$$

Rate of change and Δ symbol

Rate of change is actually just what it sounds like – how quickly one thing is changing. Velocity is the rate of change of displacement with time (the increase or decrease in displacement divided by the time taken). Acceleration is the rate of change of velocity with time (the increase or decrease in velocity divided by the time taken).

The symbol Δ is the Greek capital letter 'delta' and we use it to say 'change in'. So the change in displacement would be written Δs . This gives us velocity (the change in displacement every second).

$$\frac{\Delta s}{t}$$

where t is the time over which the change Δs took place. Displacement is the vector quantity (see the 'Geometry and trigonometry' section for some help with vectors).

WORKED EXAMPLE

If I drive 200 m (my displacement, s) in a period of 10 seconds, what is the rate of change of my displacement?

Solution

$$\text{rate of change} = \frac{\Delta s}{t}$$

$$= \frac{200}{10} = 20 \text{ m/s}$$

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PRACTICE QUESTIONS

1. Ohm's law states that the voltage across a conductor and the current through it are related by the equation $\text{voltage} = \text{current} \times \text{resistance}$. And that, if the temperature remains constant, the resistance is constant.
- Describe the relationship between the voltage and the current.
 - Sketch the shape of graph you would expect for this relationship.
 - The power developed by an electrical device can be calculated using the equation $\text{power} = \text{current}^2 \times \text{resistance}$
 - Rearrange this equation to make resistance the subject.
 - A certain wire produces 50 W of thermal energy (the power) when a current of 2 A flows through it. Calculate the resistance of the wire.
 - Use the equation $\text{voltage} = \text{current} \times \text{resistance}$ to calculate the potential difference (voltage) across the wire in part (ii) when it is producing 50 W of thermal energy.
 - What is the ratio of the voltage to the current?
 - An electric lamp is marked '40 W'. This represents the electrical power consumed by the lamp. If the efficiency of the lamp is 0.25:
 - Calculate the useful power output (light) of the lamp using the equation $\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}$
 - Express the efficiency as a fraction.
 - Express the efficiency as a percentage.
 - Express the power in standard form.
 - Estimate how long the lamp can be used for with one unit of electricity (1 unit = 1 kWh = 3 600 000 J. 1 W = 1 joule per second).

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HANDLING DATA

SPECIFICATION OVERVIEW

- 2a – Use an appropriate number of significant figures
- 2b – Find arithmetic means
- 2c – Construct and interpret frequency tables and diagrams, bar charts and histograms
- 2f – Understand the terms mean, median and mode
- 2g – Use a scatter diagram to identify a correlation between two variables
- 2h – Make order of magnitude calculations

THEORETICAL OVERVIEW

In this section, we want to develop the use of the skills we've looked at in our first topic: finding the mean, median and mode for a set of data, seeing the pattern in the data (correlation) and how the examiner wants them (significant figures and decimal places).

Arithmetic mean

The arithmetic mean is the average of a set of numbers:

$$\text{mean} = \frac{\text{total of the numbers}}{\text{the number of data points}}$$

You'll use this particularly with data from experiments – you'll be given a table of data and asked to find the mean. The extra challenge here is to spot the anomaly (the value that doesn't fit the pattern) and not include that value in the mean.

WORKED EXAMPLE 1

If we measured the heights of all the boys in an A Level Physics class, we measured the following heights:

Name	Height (m)
Jake	1.72
Peter	1.70
Saul	0.40
Mobashir	3.20
Ahmed	1.71
Kevin	1.70
Sunil	1.67

Find the mean height of the boys in the class.

Solution

We would look at Mobashir and see that he is unusually tall! It would seem that he is an outlier, either incorrectly or that his height has been recorded incorrectly. If we want a mean height for a student in sixth form, we need to leave Mobashir out of the calculation. Equally, we would probably do the same for Saul, so we would not include him in the calculation.

So, our mean would be:

$$\text{mean} = (1.72 + 1.70 + 1.71 + 1.70 + 1.67) / 5 = 1.70 \text{ m}$$

If we were to include both Mobashir and Saul, we would have:

$$\text{mean} = (1.72 + 1.70 + 0.40 + 3.20 + 1.71 + 1.70 + 1.67) / 7 = 1.73 \text{ m}$$

If you look at the table, you can see that only Mobashir is this height or more, so including him would skew the data, whereas 1.70 m does, as we have a spread of heights around this value.

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Significant figures (s.f.)

The point here is about not claiming a level of accuracy you don't have a right to claim. What we mean by 'significant'.

Zeros before a number and zeros after a number aren't significant:

0024 only 2 s.f. as the leading zeros don't count

2400 only 2 s.f. as the trailing zeros don't count

Zeros after a decimal point ARE significant:

24.00 is 4 s.f.

This can all get a bit much, so I ALWAYS write (or at least think about) my numbers in standard form, significant when written in standard form:

1000 is 1 s.f.

1×10^3 is 1 s.f.

1.0×10^3 is 2 s.f.

1.00×10^3 is 3 s.f.

1.000×10^3 is 4 s.f.

Imagine you dropped an earring in an area of long grass and I know where it is, so I tell you to walk 27.425 m north, then turn east and walk 12.552 m, if you follow these directions you'll end up right where the earring is. If I said walk for 27 m north and then 13 m east, you'll probably have to look around a bit to find the earring.

The first set of directions was given to five significant figures, the second to two significant figures.

Now, if I said walk 27.425 m north and 13 m east, how easily you find the earring is a function of the direction.

So, significant figures are important when we are writing down our answers. We can't claim more accurate piece of data we're using to get the answer. For example, if we are working with

length = 1.10 m, width = 1.50 m

Each of these numbers is given to three significant figures, so the answer should be given to three significant figures.

area = $1.10 \times 1.50 = 1.65 \text{ m}^2$ to 3 s.f.

But if we had

length = 1.10 m, width = 1.5 m

then, because the width is given to only two significant figures, our answer can only be given to two significant figures.

area = $1.10 \times 1.5 = 1.65$
= 1.7 m^2 to 2 s.f.

If this isn't clear, imagine a ridiculous scenario – I give you the length (in metres) of a rectangle to the nearest millimetre (1.100 m) but the width to the nearest metre (1 m). Now it just isn't possible to give the area to the nearest mm^2 , only to the nearest m^2 , because you just don't have the level of accuracy in the width limits the accuracy of our answer.

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WORKED EXAMPLE

Let's take a number and consider it to different numbers of significant figures
12356.124561 is to 11 significant figures

Solution

Number of significant figures	Number	Note
1	10 000 or 1×10^4	We look at the second It's a 2, so we round down
2	12 000 or 1.2×10^4	We look at the third It's a 3, so we round down
3	12 400 or 1.24×10^4	We look at the fourth It's a 5, so we round up
4	12 360 or 1.236×10^4	We look at the fifth It's a 6, so we round up
5	12 356 or 1.2356×10^4	We look at the sixth It's a 1, so we round down
6	12 356.1 or 1.23561×10^4	We look at the seventh It's a 2, so we round down
7	12 356.12 or 1.235612×10^4	We look at the eighth It's a 4, so we round down

Frequency tables and diagrams

Frequency tables are just what they sound like – a table showing how often a particular value for a variable occur(s). Frequency diagrams are the graphs (e.g. bar chart, histogram) of the data in the frequency table

We might look at the shoe sizes across a year group at school. The frequency table is as follows:

Shoe size	Frequency
3	3
3½	8
4	21
4½	25
5	58
5½	33
6	18
6½	16
7	49
7½	32
8	16
8½	3

WORKED EXAMPLE

How many students have size 5 feet in the year group above?

Solution

The frequency table shows us the numbers of students with each shoe size in the year group. In the row with shoe size 5, there are 58 students with size 5 feet.

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Bar charts and histograms

These can be confused, but they are different. Bar charts are used to display discrete data. Histograms are used to display continuous data. You need to know the difference! Continuous data can take any value within a range – it can be measured to many decimal places, limited only by the precision of the measuring device. Discrete data can only take certain values – no values in between. For example, the number of children in a class – it can be zero, or one, or two... but never any value in between. Shoe size data, and temperature would be another example of continuous data.

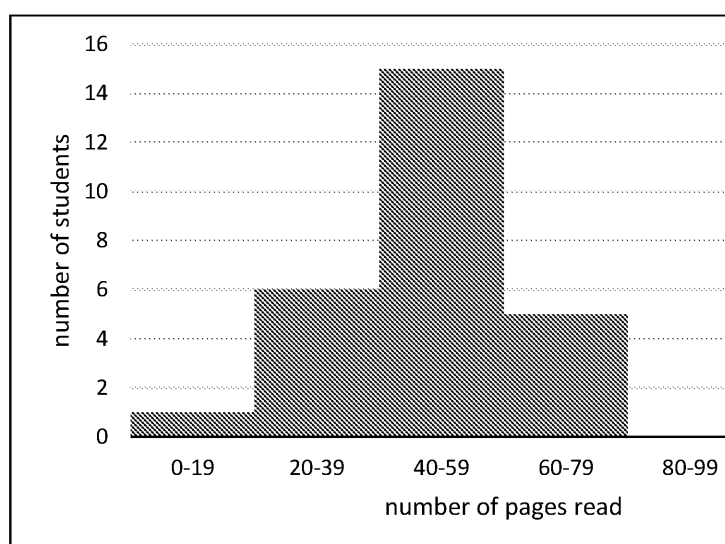
An obvious difference, which you can see at a glance, is that a bar chart has gaps between the bars, while in a histogram the bars are touching (because the data is continuous, so there is no 'break' in the data being plotted horizontally).

So, if you were representing our shoe size data, it would be a bar chart because shoe sizes are discrete.

If you were representing the ages in the school population, it would be a histogram because ages are continuous. If the age being 13, they become 14 – no gap, no break, just a continuous flow from one age to the next.

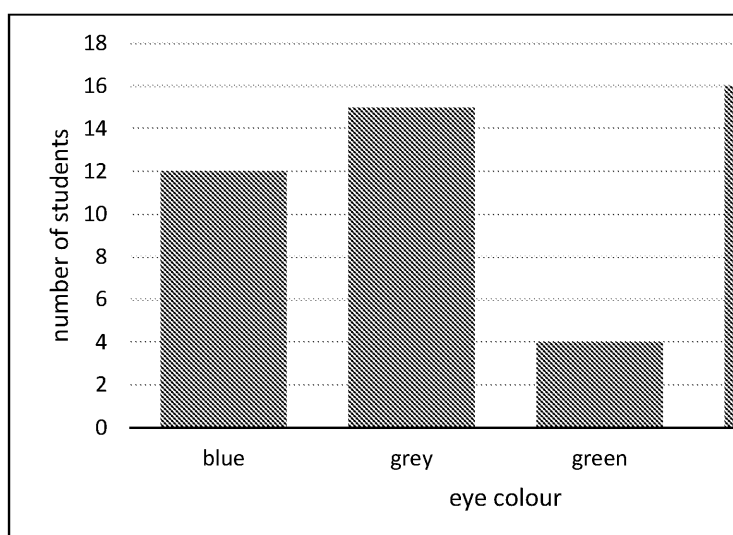
Here's an example of a histogram. The data is a survey of progress through a course of reading by a group of students after one week:

number of pages read	0–19	20–39	40–59	60–79
number of students	1	6	15	5



And here's an example of a bar chart. This is data for the eye colour in a group of students:

eye colour	blue	grey	green
number of students	12	15	4



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Mean, median, mode

These are terms that relate to a set of data.

Mean – the average of a set of numbers

WORKED EXAMPLE 1

Let's take 15 scores in a Physics test as an example:

32 %, 33 %, 45 %, 56 %, 56 %, 56 %, 64 %, 64 %, 72 %, 75 %, 81 %

What is the mean score?

Solution

For our set of 15 results, the mean is:

mean science score = $(32 + 33 + 45 + 56 + 56 + 56 + 64 + 64 + 72 + 75 + 81) / 15$
 $= 68 \% (2 \text{ s.f.})$

Median – the middle number, when they are written in order of size

WORKED EXAMPLE 2

What is the median score?

Solution

For our data, which is already in order of size, the middle number is the eighth.

Now, what if we have an even number of data points – there isn't a 'middle' one then. We take the average of the two in the middle.

WORKED EXAMPLE 3

What is the median of the following set of numbers?

3 5 7 9 11 13

Solution

The median is the average of 7 and 9 because 7 and 9 lie in the middle (two data points).

So the median = $(7+9)/2 = 8$

Mode – the most frequent value in a data set

WORKED EXAMPLE 4

What is the mode of the Physics test scores?

Solution

In our example using test data (above), 56 occurs three times, and 64 occurs twice.

So the mode is 56 because it occurs most frequently.

Probability – the probability of something happening is a number from 0 (it is impossible)

WORKED EXAMPLE 4

What is the probability of throwing any number on a regular six-sided die?

Solution

Dice have numbers 1 to 6 on their sides. When throwing a die, you have a one in six chance of any number coming up. So the probability of each number coming up is $1/6$, or 0.17.

There is a lot more to probability, but this is all we really need to know here.

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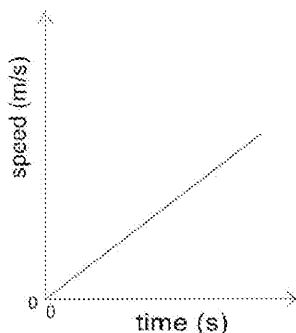


Correlation (between variables)

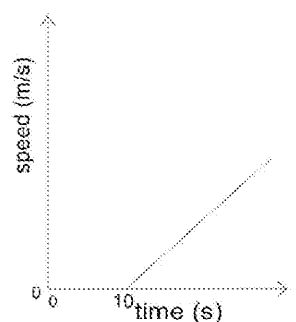
This means a relationship between the variables – ‘as x increases, y increases/decreases’
 ‘there is a linear relationship between x and y ’, and so on.

So, this skill is about looking at the points on a graph and seeing the relationship between them.

WORKED EXAMPLES



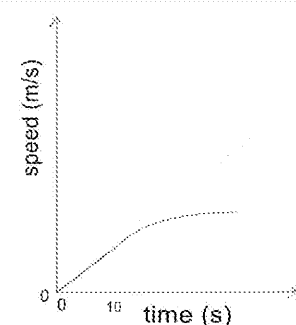
Here we have a simple ‘speed is directly proportional to time’ relationship – as time increases by a factor of 2, the speed also doubles (i.e. if the time doubles, so does the speed).



Here's another:

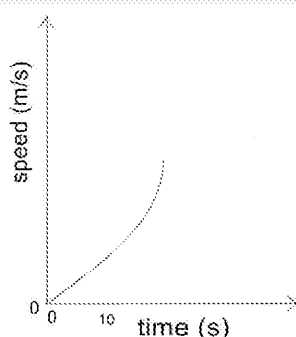
Here the speed remains at 0 m/s until 10 seconds, then increases linearly.

You might see that the relationship changes at 10 seconds. You just need to be aware that there are two different relationships to talk about.



For example

In this example, there are two different sections to the graph. Up to 10 seconds, speed increases linearly. After 10 seconds, the rate of change of speed decreases (i.e. speed increases more slowly), tending to a constant value of speed.



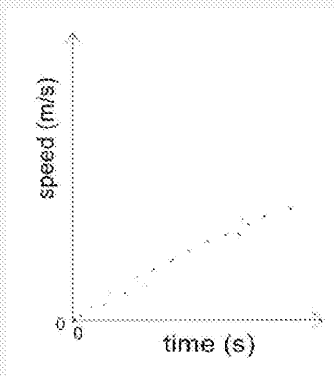
Here's another:

Here, we have the same proportional relationship as before. Then, at 10 seconds, it changes. After 10 seconds, the rate of change of speed increases (i.e. it is increasing more rapidly, increasing acceleration).

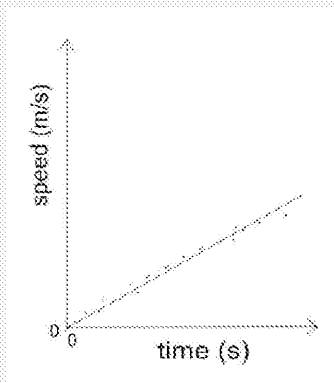
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WORKED EXAMPLES



It might be that you are given a graph, or data, and then you're asked to state the variables. You might have a graph like this:



Now, you can see that these points basically follow a straight line, and also that this line passes through the origin, indicating a proportional relationship.

Drawing lines of best fit is a skill and only really comes with practice and, therefore, you should aim to draw a smooth line that best represents the trend of the data. It may be straight, it may be curved.

So, a good line of best fit will:

- Have as many points on one side of it as it does on the other (see the example)
- Ignore anomalies
- Be a smooth curve or a straight line – don't join the dots!

Orders of magnitude calculations

Here you're giving your answer to the nearest power of 10 – or order of magnitude.

WORKED EXAMPLE

The Sun is 150 400 000 km away from the Moon at one point in its cycle. If the Moon travels at 300 000 km/s, which answer shows the correct order of magnitude for the time taken for the Moon from the Sun, measured in seconds?

- A 10^{13}
- B 10^{40}
- C 10^8
- D 10^3

Solution

The distance, in standard form, is 1.504×10^8 km, so the order of magnitude of the distance is 8.

The speed, in standard form, is 3.00×10^5 km/s, so the order of magnitude of the speed is 5.

The time taken, using $\text{speed} = \text{distance}/\text{time}$, is

$$\text{time} = \text{distance}/\text{speed}$$

so, in terms of orders of magnitude, this is $10^8/10^5$.

This is the same as $10^{(8-5)}$, so 10^3 .

If we do the full calculations, we get an answer of 501 seconds. This rounds up to 500 seconds, which is 10^3 .

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PRACTICE QUESTIONS

1. The data below represent the prices found for a 128 GB USB flash drive:

Retailer	Price (£)
1	9.69
2	16.99
3	14.79
4	26.00
5	28.99
6	25.03
7	1.50
8	11.00
9	13.19
10	16.99
11	18.99
12	13.59
13	26.99
14	1.38
15	24.99
16	14.93
17	23.99

- a. Copy and complete the frequency table below, for these data.

Price (£)	Frequency
0–5.99	
6–10.99	
11–15.99	
16–20.99	
21–25.99	
26–30.99	

- b. Plot a graph of these data.
 c. Calculate the mean price of the USB flash drive.
 d. What is the median price of the USB flash drive?
 e. Taking the prices to the nearest pound, what is the mode in these data?
2. By plotting a scatter graph, determine the correlation between the variables

Current (A)	Force (N)
0.0	0.00
0.5	0.13
1.0	0.25
1.5	0.38
2.0	0.50
2.5	0.63
3.0	0.75
3.5	0.88
4.0	1.00
4.5	1.13
5.0	1.25

3. The force on a wire carrying a current at a right angle to a magnetic field is
force = magnetic field strength × current in the wire × length of wire in the field
 For a certain wire, the current is 1000 A, the length is 22 km and the magnetic field strength is 0.001 T.
 What is the order of magnitude of the force on the wire?

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GRAPHS

SPECIFICATION OVERVIEW

- 4a – Translate information between graphical and numeric form
- 4b – Understand that $y = mx + c$ represents a linear relationship
- 4c – Plot two variables from experimental or other data
- 4d – Determine the slope and intercept of a linear graph
- 4e – Draw and use the slope of a tangent to a curve as a measure of rate of change
- 4f – Understand the physical significance of area between a curve and the x-axis in appropriate units or squares as appropriate

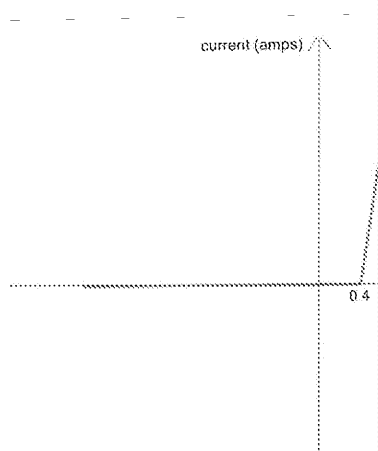
THEORETICAL OVERVIEW

In this section, we are building on the graphical skills we looked at earlier (in the 'Area' sections), and extending them. Previously, we have looked at data and plotted a graph from graphs. We have also looked at the general equation of a straight line ($y = mx + c$). If c does not equal zero, we have indirect proportion. If $c = 0$, the graph is directly proportional, $c = 0$. If c does not equal zero, we have indirect proportion. We call these **linear relationships**. Now, we are looking at how graphs can tell us about the data used to plot them. Once again, let's get our terminology straight.

Intercept

This is the point at which the line of the graph crosses either the y-axis (when the x value = 0). Note carefully that it's where the other variable (x or y) = 0. Look at the graph below. The x-axis is drawn crossing the y-axis at a value other than zero, and vice versa. These points represent the intercept.

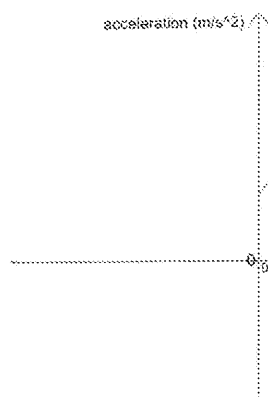
The intercept can tell you that there are other factors at work that you haven't considered or haven't fully allowed for. For example, a diode is a device that only allows current to flow freely in one direction – the resistance in the reverse direction is very high, so current almost doesn't flow at all. BUT, current only starts to flow when there is a potential difference (p.d.) or voltage of around 0.4 V across the diode, so we have an intercept on the potential difference axis (the x-axis because the potential difference is the independent variable – the current depends on it, not the other way around).



So this graph tells us that no current flows until there is a p.d. of 0.4 V across the diode in the forward direction. In the 'reverse' or negative direction, a p.d. does not cause a current to flow.

We can also have intercepts on the y-axis. If we were to conduct an experiment to find the relationship between the force on an object and the acceleration of it, we might get a graph like this:

This graph is expected to be directly proportional – a straight line through (0,0). However, it shows that even though there is no applied force, the object is still accelerating! This must be because there is some other force that we have not considered, which is acting on the object. This might be because we are rolling our object down a ramp, but the ramp is too steep.



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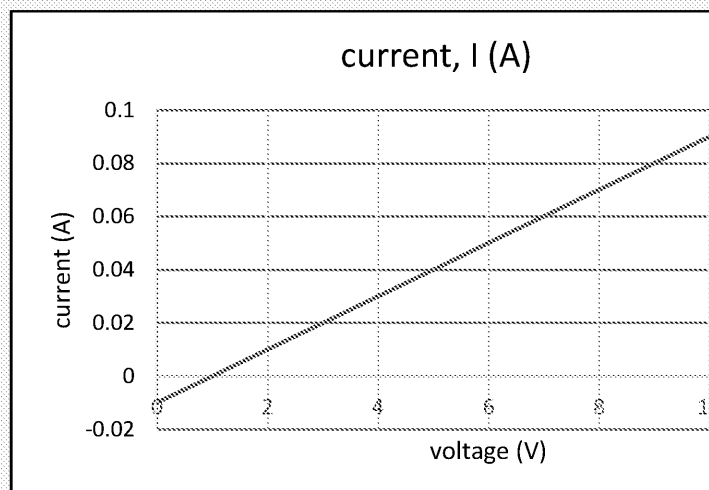
WORKED EXAMPLE

Here is some experimental data, investigating the relationship between the voltage across a wire and the current flowing through it:

voltage, V (volts)	0	1	2	3	4	5	6	7
current, I (A)	-0.01	0	0.01	0.02	0.03	0.04	0.05	0.06

Plot a graph of these data to find the value of the y-intercept.

Solution



The y-intercept is -0.01 A. This tells us that the ammeter had a zero error – though there was no voltage across the wire (and so there couldn't be any current).

Slope/gradient

The slope/gradient is literally the steepness of the line of the graph. Of course, he scale of the graph! So we actually calculate the slope or gradient using

$$\text{gradient} = \frac{\text{change in the y value}}{\text{change in the x value}}$$

Looking back at the earlier graph of acceleration against force, the gradient or slope of acceleration changes for every one newton increase in the force applied.

WORKED EXAMPLE

(continuing with the previous worked example)

Here is some experimental data, investigating the relationship between the voltage across a wire and the current flowing through it:

voltage, V (volts)	0	1	2	3	4	5	6	7
current, I (A)	-0.01	0	0.01	0.02	0.03	0.04	0.05	0.06

Plot a graph of these data to find the gradient of the graph.

Solution

The graph will be the same, of course! The gradient is calculated by
 $\text{gradient} = (\text{change in the current value}) / (\text{change in the potential difference value})$
 So we take the longest section of straight line we can – choosing points as far apart as possible.
 We can choose (0, -0.01) and (10, 0.09):
 $\text{gradient} = (0.09 - (-0.01)) / (10 - 0) = 0.1 / 10 = 0.01$

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Tangent

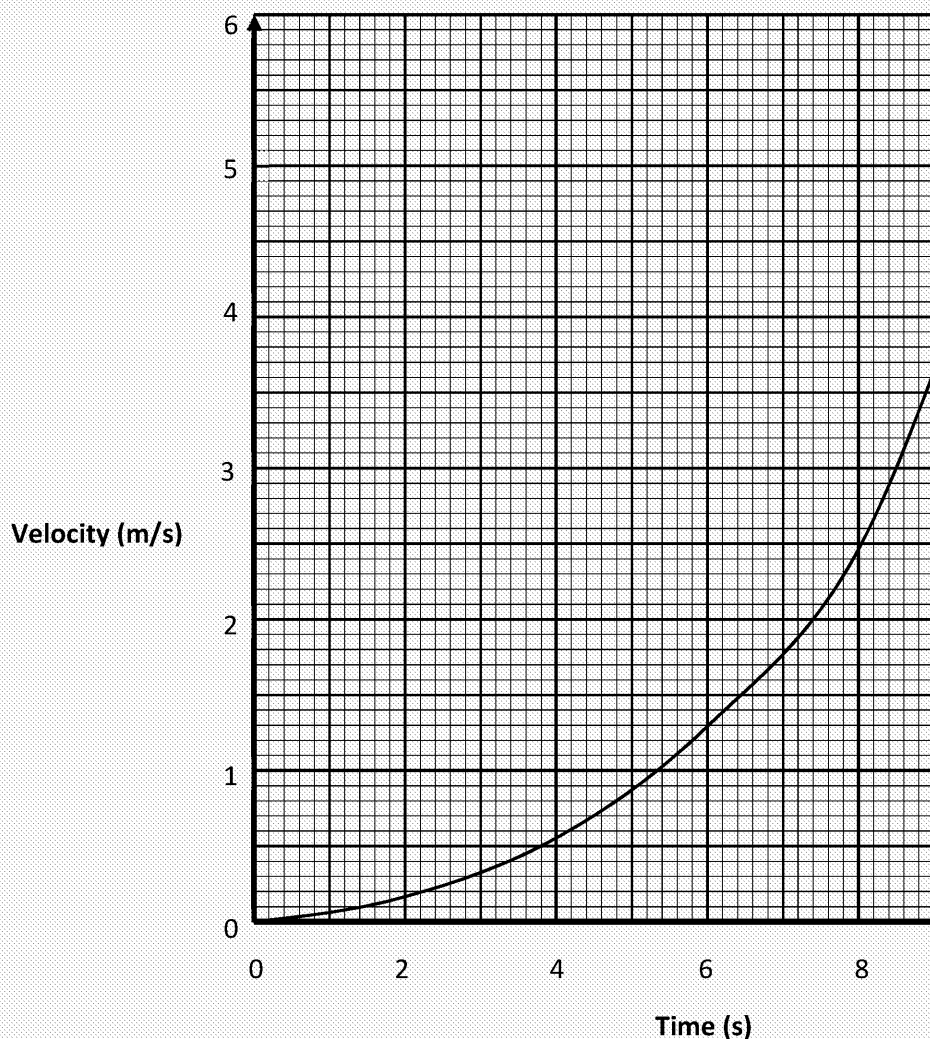
Sometimes, the graph we have isn't straight, but we still need to know how the values of x change. To do this, we take the gradient of the tangent to the graph. A tangent just touches the graph at one point only.

Rate of change – when we are measuring the slope or the gradient of a line, what is the rate of change of the y variable with the x variable – how much the y changes

WORKED EXAMPLE

Here is a graph of velocity/time for a car moving off from a set of traffic lights.

Calculate the acceleration of the car at 8 seconds.



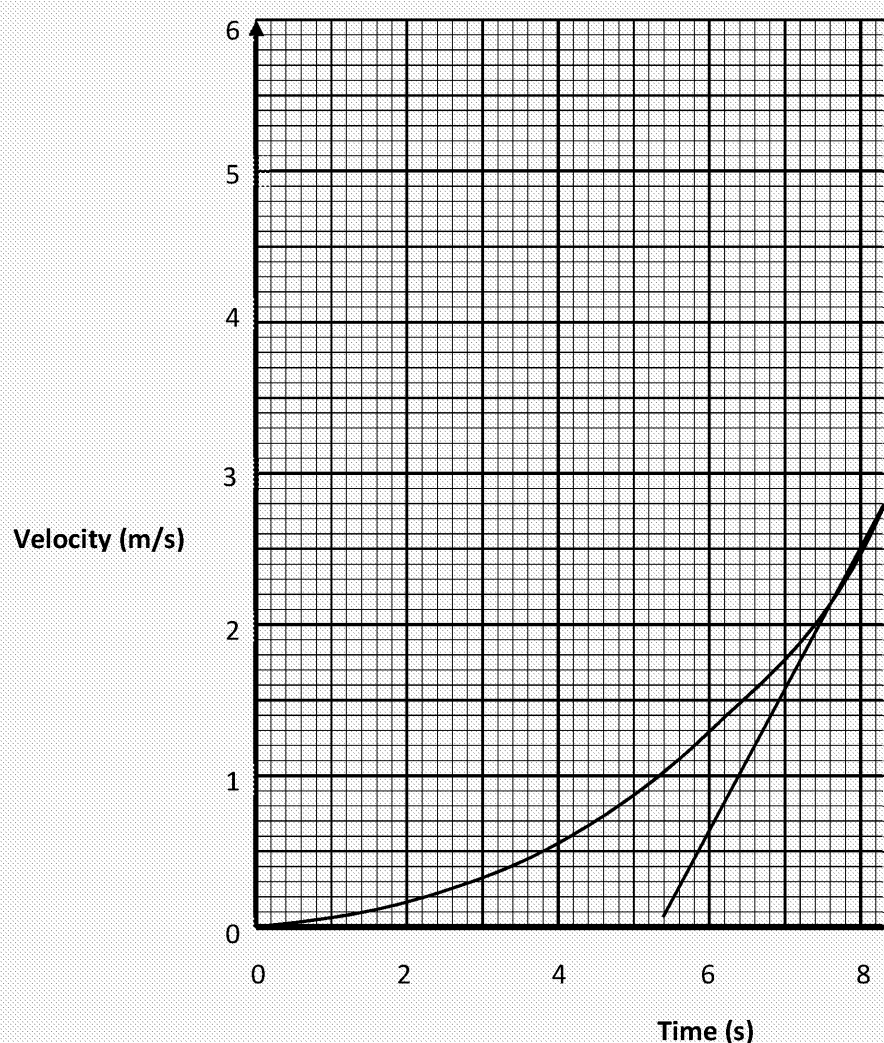
Solution

What we need to do is draw a tangent to the curve at 8 seconds. Note that we are not picking a data point which isn't on the line.

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WORKED EXAMPLE CONTINUED



Now we work out the gradient of the tangent (the rate of change of the velocity what we're asked for). We would choose two points as far apart on the tangent (5.4, 0.2) are ideal. The reason is so that we have the biggest difference between. This means that any error we make in plotting the points / reading the points impact on the answer.

So, the gradient is

$$\begin{aligned}\text{gradient} &= (\text{change in the velocity value}) / (\text{change in the time value}) \\ &= (5.9 - 0.2) / (11.6 - 5.4) \\ &= 0.92 \text{ m/s}^2\end{aligned}$$

The units come from the fact that the units of the y-axis are m/s and the unit. When you divide m/s by seconds, we get $\text{m/s} \div \text{s}$

Which gives us $\text{m}/(\text{s} \times \text{s})$

Which we write as m/s^2

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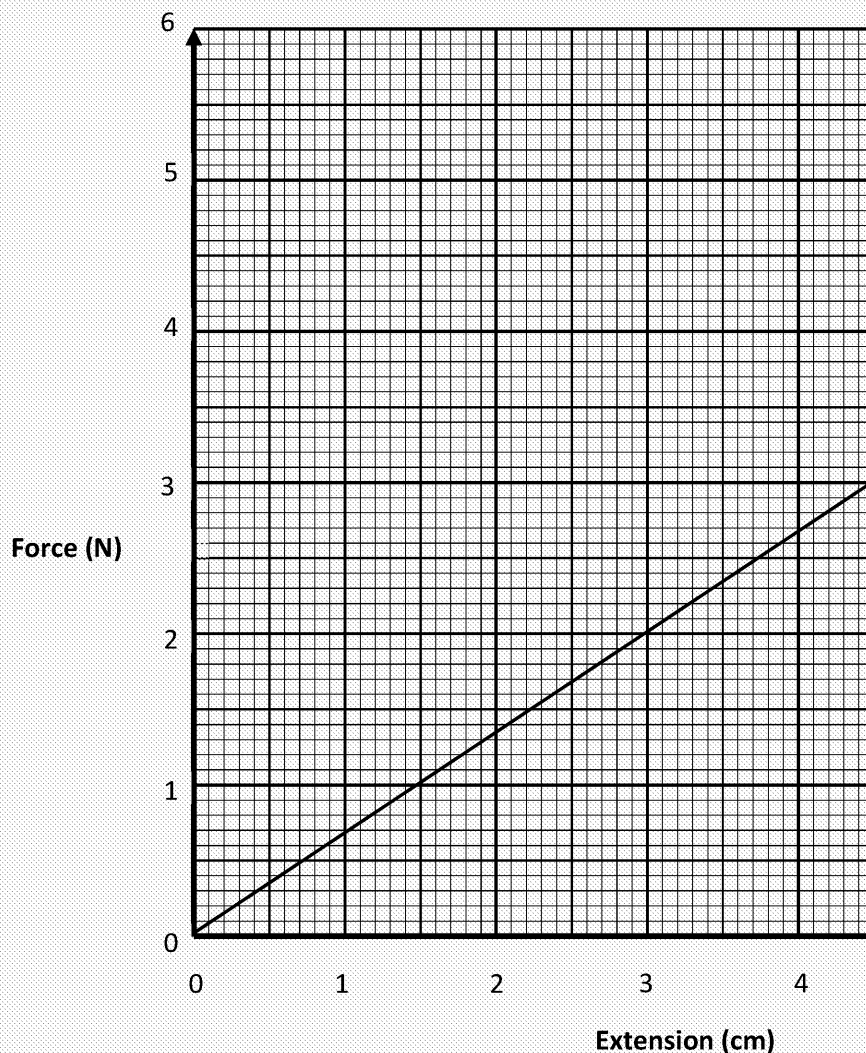


Area under a graph

The area under a graph is basically two numbers multiplied together. If we have a graph of force against extension, the area under it will be a force \times a distance. You might recall that force \times distance is work.

WORKED EXAMPLE 1

What is the area under the graph between 0 cm and 3 cm?



Solution

If we work out the area under the graph between 0 cm and 3 cm, we have the area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$ (see 'Geometry and trigonometry' section) = $0.5 \times 3 \times 2 = 3$

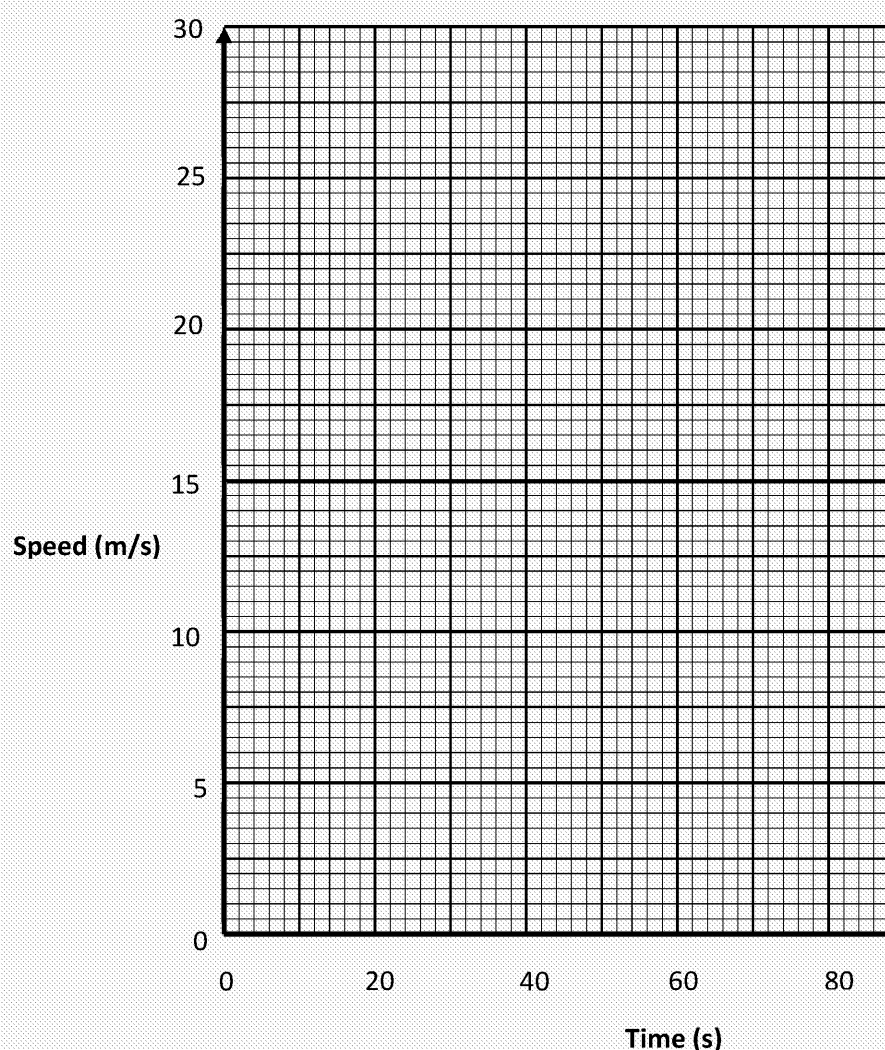
Of course, we know that the unit of work is the joule and if this was going to be in SI (standard units), so extension needs to be in metres. $0.5 \times 0.03 \times 2 = 0.03 \text{ J}$

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WORKED EXAMPLE 2

If we had a car travelling at a constant speed, we'd have a graph like this:



What is the area under this graph?

Solution

From our equation $\text{speed} = \text{distance} / \text{time}$

we know that $\text{distance} = \text{speed} \times \text{time}$, which is the area under the graph!

So, the distance travelled between 40 seconds and 60 seconds is the area of the height 15 m/s and length 20 seconds ($60 - 40$).

$$\text{Distance} = 15 \times 20 = 300 \text{ m}$$

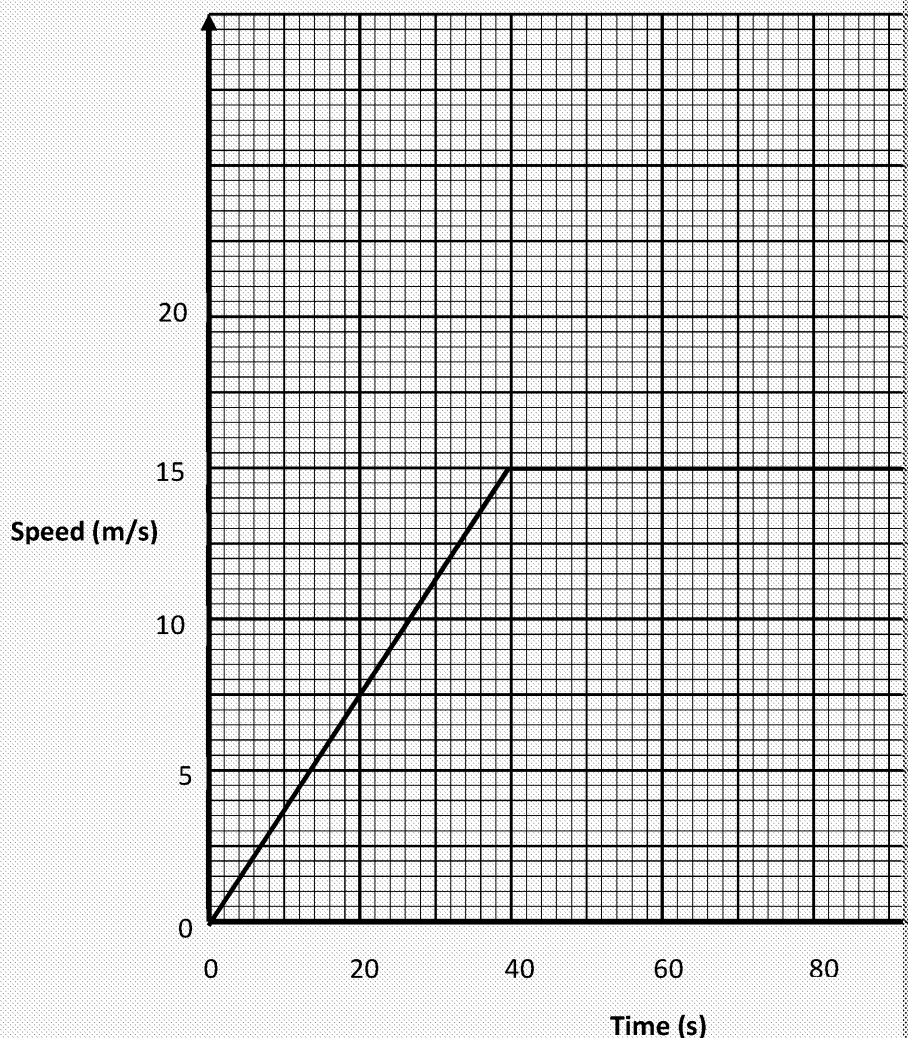
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Sometimes, we get a change in the shape of the graph and so we have two areas o

WORKED EXAMPLE 3

What is the area under this graph?



Solution

In this case, the distance travelled between 0 seconds and 120 seconds is the area under the graph up to 40 seconds and the area from 40 seconds to 120 seconds, added together.

$$\text{Area of triangle: } 0.5 \times 40 \times 15 = 300 \text{ m}$$

$$\text{Area of rectangle: } 80 \times 15 = 1200 \text{ m}$$

$$\text{Total distance} = 1500 \text{ m}$$

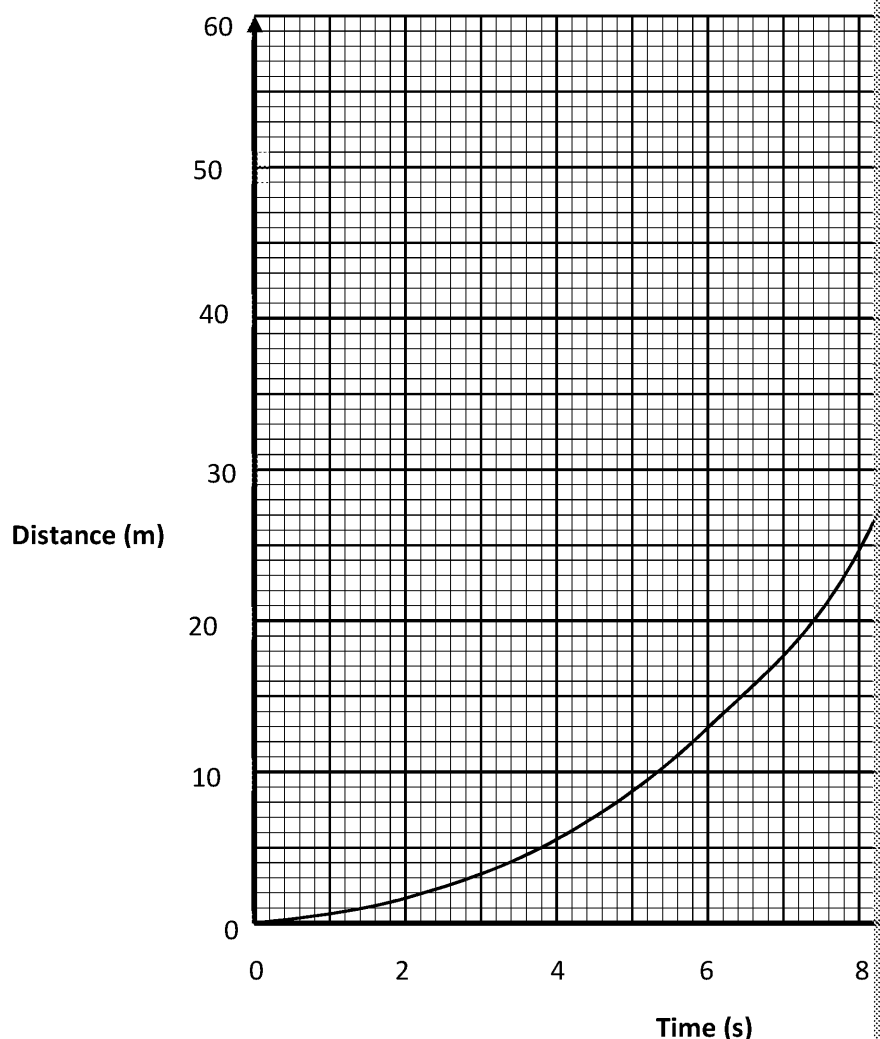
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PRACTICE QUESTIONS

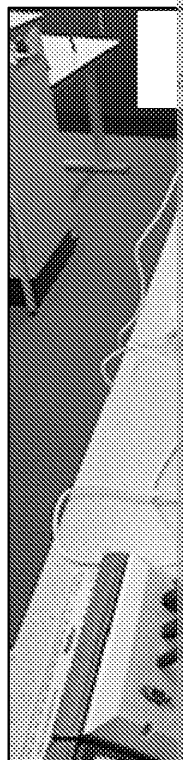
1. a. Find the rate of change of distance with time at time = 7.8 seconds in the



- b. How does this rate of change vary as time gets larger?
2. The experiment shown in the picture to the right was conducted to find the relationship between the force on an object and the acceleration of the object. The table of data collected is shown below:

Force (N)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
Acceleration (m/s^2)	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

- Plot a graph of these data and draw the line of best fit to your data. (4 marks)
- State the relationship between the acceleration and the force. (1 mark)
- What is the general equation which describes a relationship like this? (1 mark)
- Calculate the gradient of the graph. (2 marks)
- Calculate the area under the line between $F = 0 \text{ N}$ and $F = 9 \text{ N}$. (2 marks)
- What is the value of the y-intercept? (1 mark)
- What is the significance of this intercept? (1 mark)



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GEOMETRY AND TRIGONOMETRY

SPECIFICATION OVERVIEW

5a – Use angular measures in degrees

5b – Visualise and represent 2D and 3D forms, including two-dimensional representations

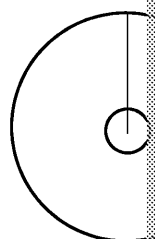
5c – Calculate areas of triangles and rectangles, surface areas and volumes of common solids

THEORETICAL OVERVIEW

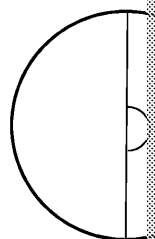
This section wraps up our toolkit. We look at measuring angles and calculating some areas. It's not nothing too scary!

Degrees (°)

There are 360° in a circle.

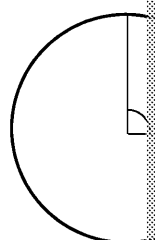


So in a semicircle (half a circle) there are 180° .

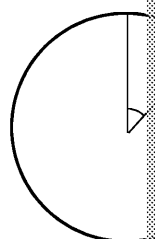


If you think about it, if you go from any point on the circumference of a circle to the point directly opposite, you've just gone along a diameter of the circle. So moving in a straight line!

Continuing this idea, in a quarter of a circle there are $\frac{360}{4} = 90^\circ$ and a quarter of a circle describes a 'right angle'.



Half of 90 is 45 and that is the angle we'd go through going $\frac{1}{8}$ of the way around a circle.



Measurement of angles in a question relies on the skilful use of a protractor. Take care when you're doing it properly and check that your answer makes sense and it will be correct.

In order to state the displacement, velocity or acceleration of an object, we need to state the magnitude of the quantity and the direction. The direction is often found by measuring an angle.

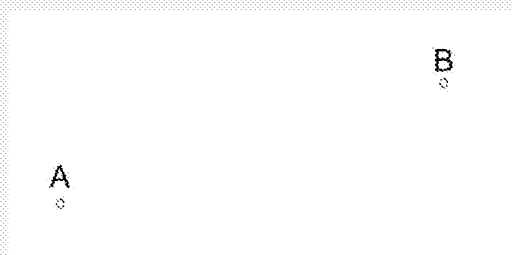
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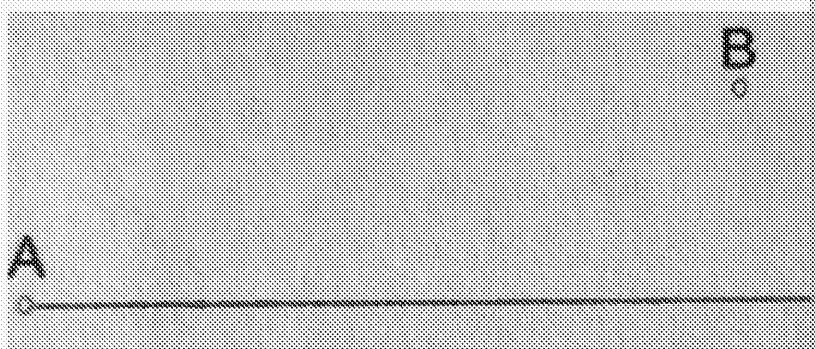
WORKED EXAMPLE

Measure the angle between the horizontal, on which A lies, and point B.

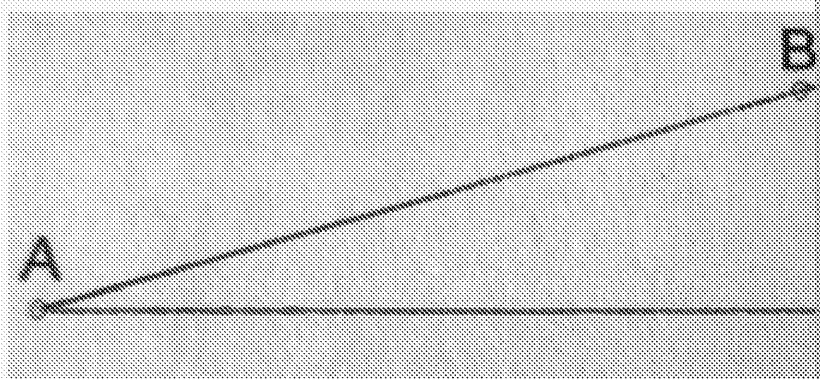


Solution

What we do is to draw a horizontal line through A – this is a reference line for



Then we draw a line from A through B.



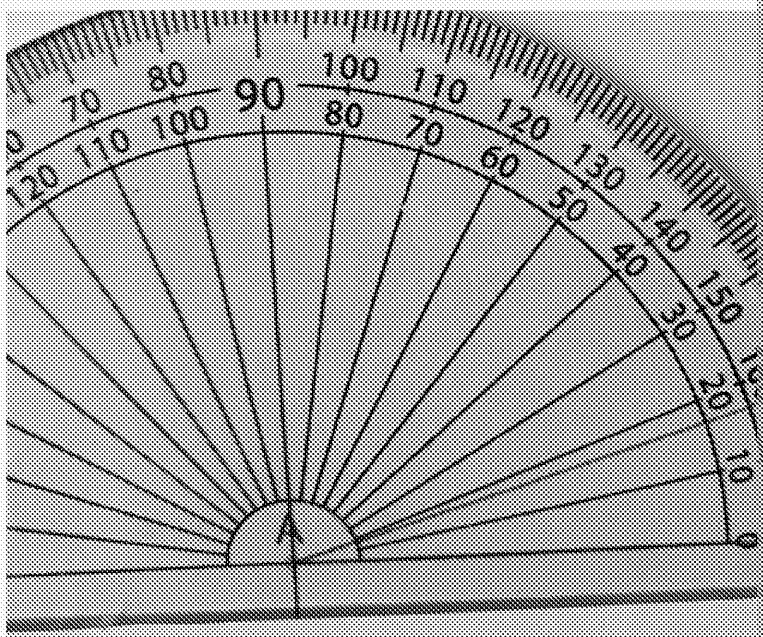
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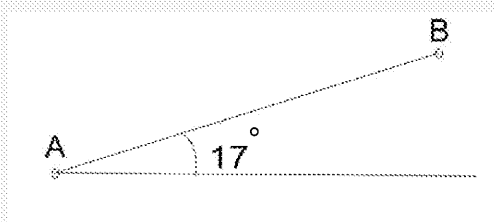
WORKED EXAMPLE CONTINUED

Then we place the protractor so that:

1. The cross in the middle of the 0° line is on A.
2. The horizontal line through A runs right through the middle of the 0° mark.



Then we measure the angle:

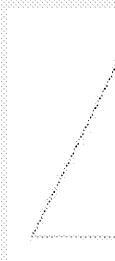


Representing 3D objects as 2D

One of the things that you need to be able to do is to draw a two-dimensional view

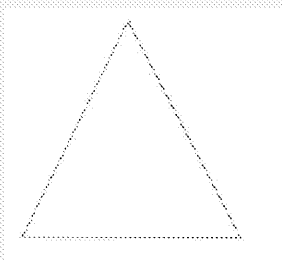
WORKED EXAMPLE

Here is a 3D diagram of a glass prism.
Draw a 2D version of this diagram.



Solution

This will just look like a triangle in 2D.



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Areas and volumes

Formulae you'll need to know:

area of a triangle: $A = \frac{1}{2} \text{base} \times \text{height}$

(the height is the vertical distance from the base to the apex or top point of the triangle)

area of a rectangle: $A = \text{length} \times \text{width}$

surface area of a cube: $A = 6 \times \text{length} \times \text{width} = 6 \times \text{length}^2$

(a cube is a 3D shape where all the sides are the same length – so each face is a square)

volume of a cube: $V = \text{length}^3$

WORKED EXAMPLE 1

Calculate the area of the triangle shown.

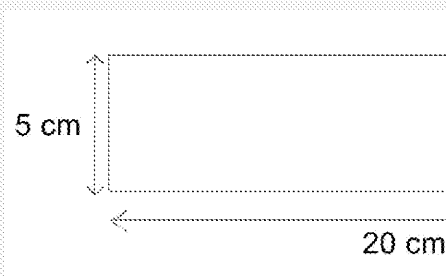


Solution

$$\begin{aligned} \text{area} &= \frac{1}{2} \text{base} \times \text{height} \\ &= \frac{1}{2} \times 7 \times 10 = 35 \text{ cm}^2 \end{aligned}$$

WORKED EXAMPLE 2

Calculate the area of the rectangle shown.



Solution

$$\begin{aligned} \text{area} &= \text{length} \times \text{width} \\ &= 20 \times 5 = 100 \text{ cm}^2 \end{aligned}$$

WORKED EXAMPLE 3

Calculate the surface area of a cube of side 3 cm.

Solution

$$\begin{aligned} \text{surface area} &= 6 \times \text{length}^2 \\ &= 6 \times 3^2 = 54 \text{ cm}^2 \end{aligned}$$

WORKED EXAMPLE 4

Calculate the volume of the cube in worked example 3.

Solution

$$\begin{aligned} \text{volume} &= \text{length}^3 \\ &= 3^3 = 27 \text{ cm}^3 \end{aligned}$$

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Vectors

Although this isn't actually mentioned in the maths skills, it is part of the specification

What a vector is

A vector is a quantity that has both size (magnitude) AND direction. Velocity is a vector, for example. If you just said '3 m/s', you haven't fully said what the velocity is, '3 m/s to the right', for example. If you just said '3 m/s', you haven't fully said what the velocity is. On its own, '3 m/s' is just a speed – which is scalar NOT a vector. To illustrate, if something accelerates, its velocity changes (acceleration = change in velocity/time). A force is required to cause the acceleration ($F = ma$, remember?). Now, since velocity has changed, so has the speed (3 m/s to the left and 3 m/s to the right are NOT the same). The direction, the fact that the object had completely turned around would be missed if you only said speed. A resultant force has been acting to make this happen.

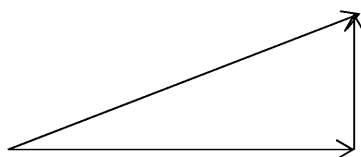
How to describe vectors

The description must include BOTH the size AND the direction. So a distance (scalar) would be '22 m'. A displacement (vector) version would be '22 m north'.

How to take any vector and break it up (resolve it) into two vectors perpendicular to each other (You only need to know how to do this at Higher tier)

For example, here's a vector, representing a force, which makes an angle of 20° to the horizontal:

You can see that it has an effect horizontally to the right and also has an effect vertically upwards. So, we could achieve the same effect with two forces – one horizontally to the right and the other vertically upwards. These three forces form a triangle:



This process of breaking down a vector into two parts (**components**) which are at right angles to each other is called **resolution**. We have resolved this force at 20° into its horizontal and vertical components.

How to add vectors together, using scale drawing (this is finding the resultant) (You only need to know how to do this at Higher tier.)

Let's look at adding two forces together to get the resultant. One is 5 N horizontally to the right and the other is 5 N acting at an angle of 30° to the horizontal and acting upwards.

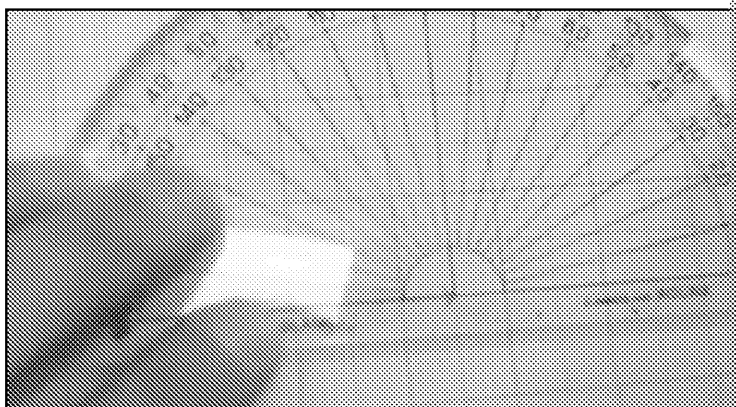
So, the first step is to choose a scale for our diagram – 1 cm to each newton works well here. If we had a force of 500 N, we would probably say 100 N to each cm.

Then draw our 5 N vector – a horizontal line 5 cm long:

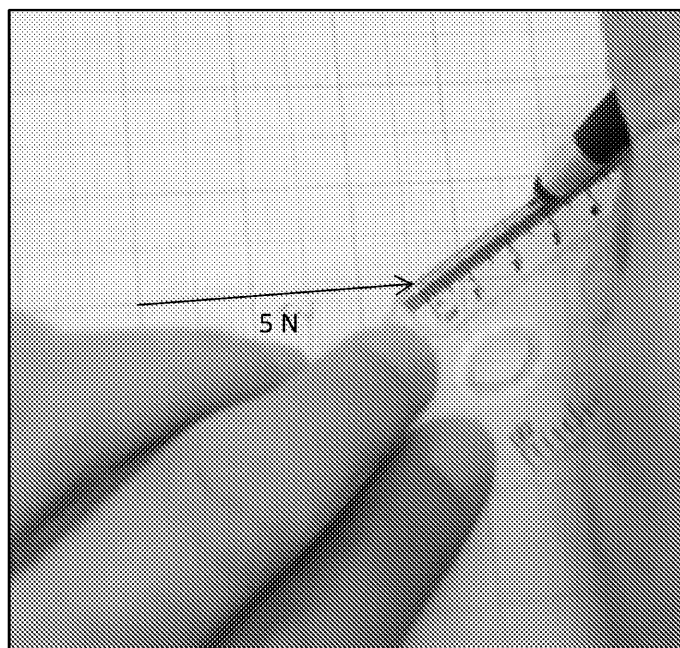
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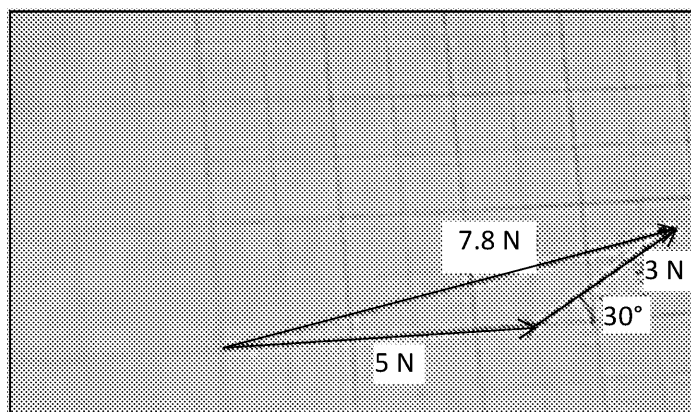
Now we add our 3 N force vector, but first we have to measure the 30° angle we



Now we can draw our 3 N force vector – 3 cm long at 30° to the horizontal:



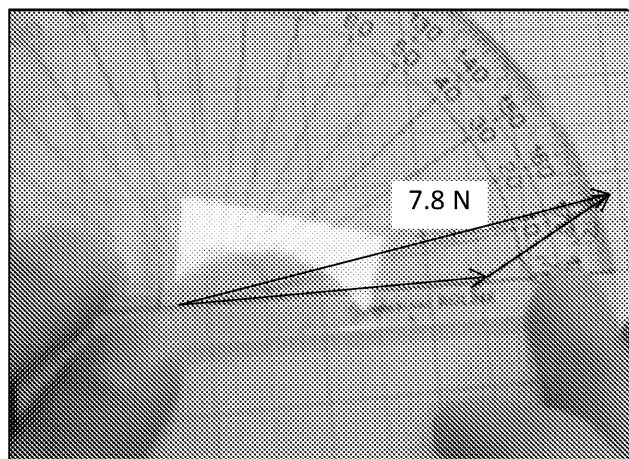
Then we join the point where we started the 5 N force to the point where we finished the resultant of the two. The length is measured and, using the scale we decided on, the



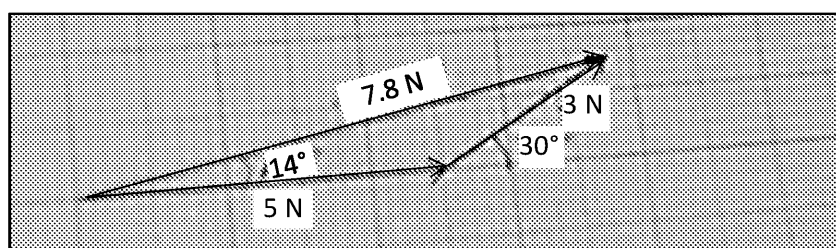
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The final stage of the work is to measure the angle – remember a vector **MUST** be the direction:



So the resultant of our two vectors has been found by scale drawing!



Resultant
 $\theta =$

PRACTICE QUESTIONS

- The displacement is the vector distance between two points. That means we need a straight line between the two points, but also the angle that line makes with the horizontal. By making measurements from the diagram, write down the displacement. The diagram is to scale.

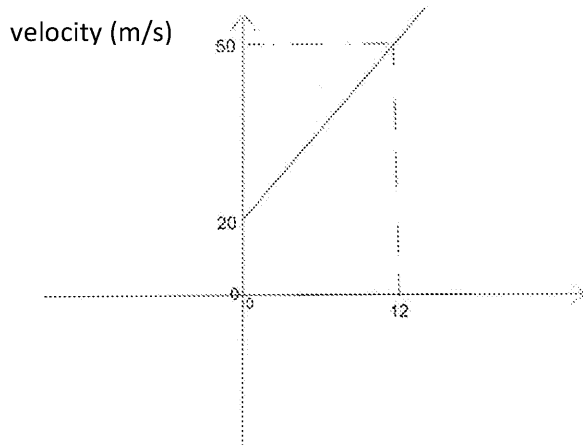
B

A

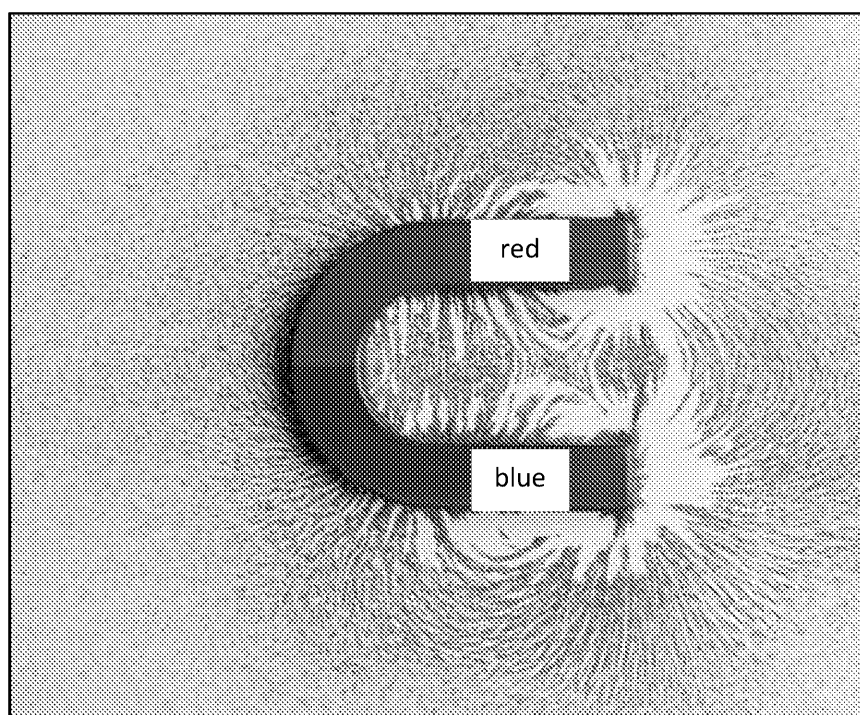
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2. The magnitude (size) of the displacement can be found by working out the area of a moving object. Calculate the displacement of the car for which the velocity–time graph is shown below.



3. In a children's soft play area there are lots of inflated cubes for the children to play on. Each of these cubes has a side of length 50 cm. Taking the density of air as $\rho = 1.2 \text{ kg m}^{-3}$, calculate the mass of air in one cube, in kg.
4. Here is a picture of a magnet's field lines being highlighted using iron filings:



Draw a 2D representation of this picture. Just draw four field lines from the poles. Assume the red side is north and put arrows on your lines accordingly.

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Diagnostic Test 2

A2 Arithmetic

1. Write down $\frac{3}{12}$ as a decimal.
.....
2. Write down 0.2 as a fraction, in its simplest form.
.....
3. Write 512 in standard form.
.....
4. 1 TW is 1×10^{12} watts. Write this number as a decimal.
.....
5. An electric car has a range of 350 miles. The owner drives 37 % of this range.
.....
6. If the car in question 5 is driven 150 miles, what percentage of the range is this?
.....
7. A water tap in my house is dripping at the rate of one drop per second. Estimate how long it will take to fill a 5 L washing up bowl placed underneath.
.....
.....

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B2 Algebra

1. Write the following expression in words: $x \sim y$

.....

2. Make variable u the subject of this equation.

$$v^2 = u^2 + 2as$$

.....

3. A certain constant can be calculated using the equation

$$\text{constant} = \frac{\text{force}}{\text{current} \times \text{length}}$$

Given that the units of force = N, current = A and length = m, what are the units of the constant?

.....

4. A car is travelling at 10 m/s and accelerates at 3 m/s² to 20 m/s. Calculate the time taken during the acceleration. Use the equation in question 2, where $u = 10$ m/s, $v = 20$ m/s and $a = 3$ m/s².

.....

.....

5. If I eat one square of chocolate from a bar of chocolate every minute until I've finished the bar, describe the relationship between the amount of chocolate **remaining** and the time I have taken to eat the bar.

.....

6. Water pours into a bucket at a rate of 50 cm³ per second. If the bucket contains 1000 cm³ of water, describe the relationship between the volume of water in the bucket and the time taken for the bucket to overflow.

.....

7. If the temperature, θ , of a cup of tea falls from 95 °C to 85 °C in 300 seconds, describe the change of the temperature of the tea.

.....

.....

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C2 Handling data

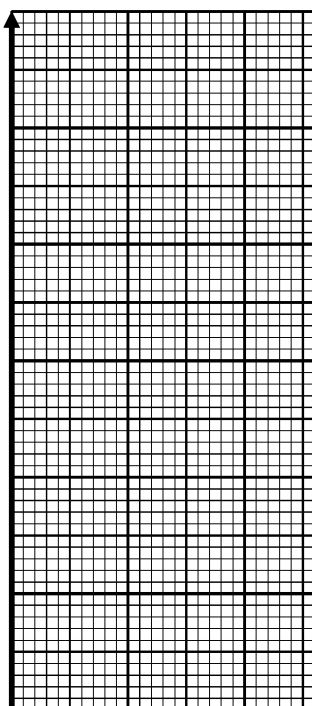
- Write 86 400 to two significant figures.
.....
- A world-class female shot-putter achieves the following distances, in metres, on six successive attempts:
21.91 20.85 22.05 22.50 21.55 22.50 22.50
 - Calculate the mean distance.
.....
.....
 - State the median distance.
.....
 - State the modal distance.
.....
- A survey of the number of books in a library, separated by type (e.g. fiction, non-fiction, reference, etc.). What is the most appropriate way to:
 - Record the data collected?
.....
 - Represent those data on a graph (what type of graph should be plotted)?
.....
- A car is driven 50 km a day for a year. What is the order of magnitude of the distance travelled?
.....

- An experiment to find the relationship between the potential energy of an object and the distance from the centre of a planet is done. The results are shown in the table below:

Distance (km)	6400	12 800	19 200	25 600	32 000	38 400
Energy ($\text{J} \times 10^5$)	6.00	3.00	2.00	1.50	1.20	1.00

Draw a graph of these data.
State the relationship between the force between the objects and the distance between them.

.....
.....
.....
.....



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D2 Graphs

1. What would the graph of a/F (a vs F) look like for the equation $F = ma$? Assume

2. a. Plot a graph of $3y = 4x + 12$ for values of x from 0 to 9.

b. What is the gradient of this graph?

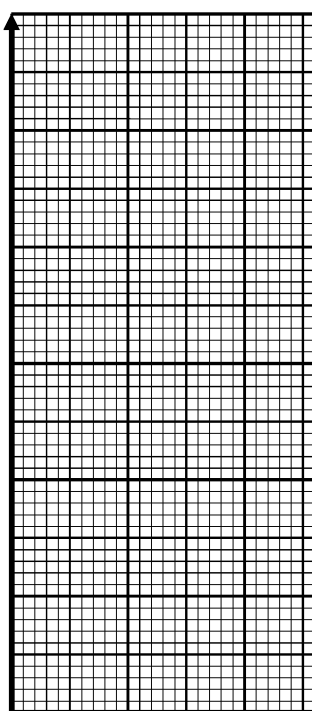
.....

.....

c. What is the y -intercept of this graph?

.....

.....



3. a. Plot a graph of time against speed for the values in the table below:

Speed (m/s)	5.5	11	16.5	22	27.5	33	38.5
Time (s)	1.82	0.91	0.61	0.45	0.36	0.30	0.26

b. Measure the gradient of the graph at speed = 5.5 m/s.

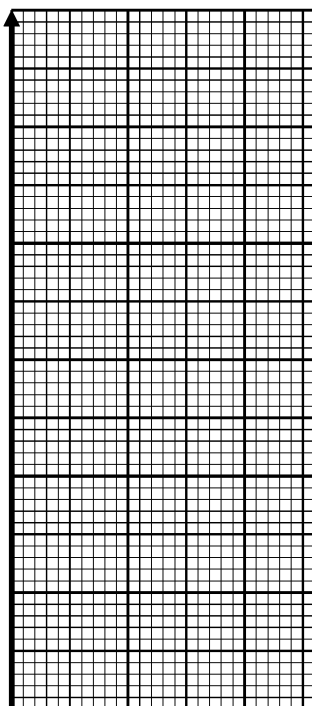
.....

.....

c. What does this value of the gradient tell you?

.....

.....



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E2 Geometry and trigonometry

1. Measure the angle shown in the diagram.



2. Calculate:

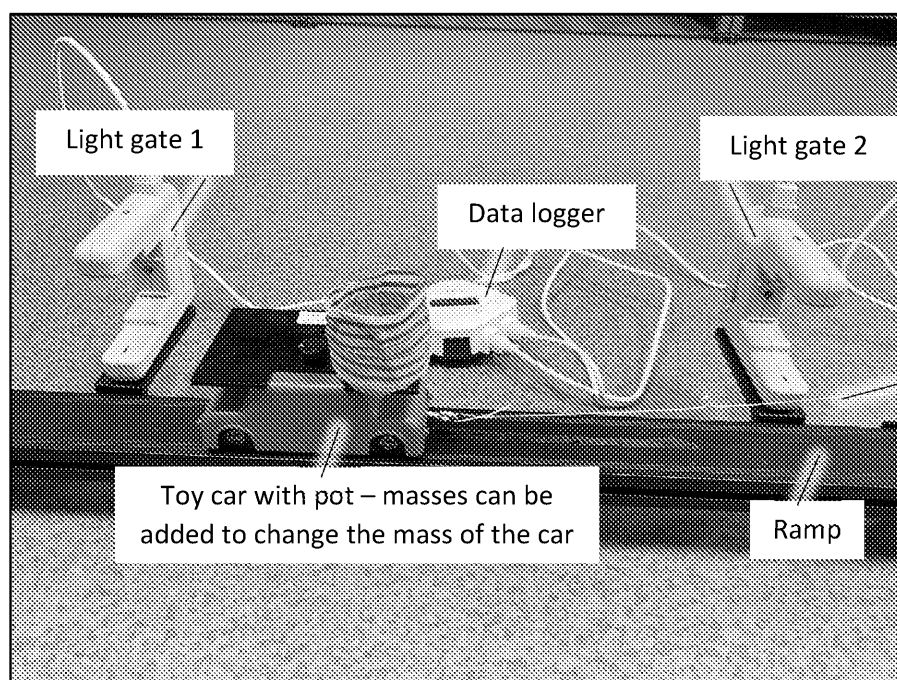
- a. The area of a rectangle of length 5 cm and width 2 cm

- b. The volume of a cube of side 5 cm

- c. The area of a triangle of height 20 cm and a base of 10 cm

- d. The surface area of a cube of side 5 cm

3. Here is a picture of the equipment used to measure the acceleration of a toy car. What would it look like in 2D from this perspective? Draw a diagram to show data logger.



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SOLUTIONS TO QUES

DIAGNOSTIC TEST 1

A1 Arithmetic

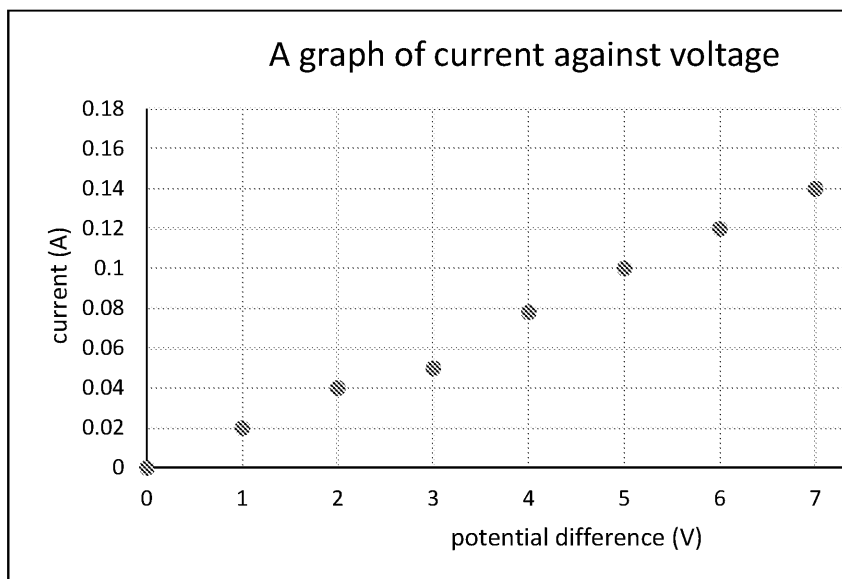
- 0.25
- $\frac{1}{3}$
- 1.024×10^3
- 1 000 000 000 bytes
- 0.125
- 20 %
- Estimate the mass of an adult male as 80 kg. Estimate the mass of a small car as 1200 kg. Estimate the mass of 1200 males to have the same mass as the car. There are no absolutely correct answers here but values that can reasonably be taken as a value between 70 kg and 100 kg, and that of the car as 1200 kg.

B1 Algebra

- x is much greater than y
- $t = \frac{v-u}{a}$
- Units are Nm^2/kg^2
- 6.7 m/s^2 (6.67)
- Linear
- Proportional
- $\Delta I = 5 \text{ A}$
 - 2500 A/s

C1 Handling data

- 12 300
- 10.25 s
 - 10.22 s
 - 10.01 s
- Frequency table
 - Histogram
- Bar chart
- In metres, the dimensions are 0.1 m and 1.2 m
The area is then
 $0.1 \times 1.2 = 0.12 \text{ m}^2$
which is 1.2×10^{-1} , so the order of magnitude is -1
-



- The current is directly proportional to the potential difference

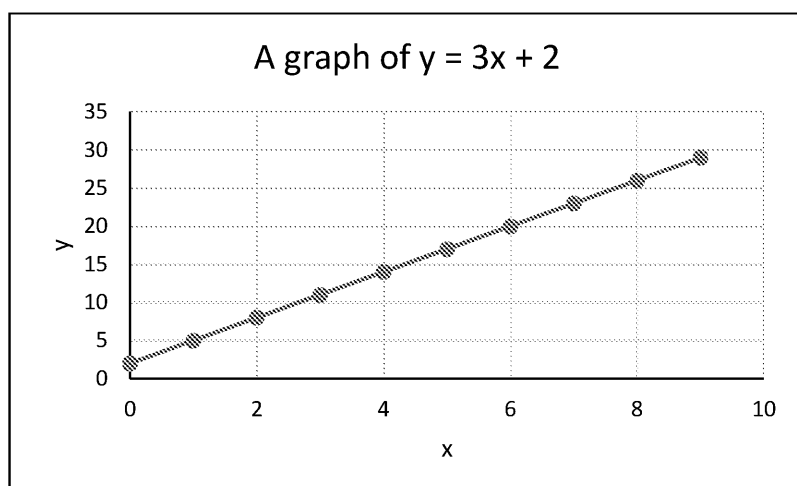
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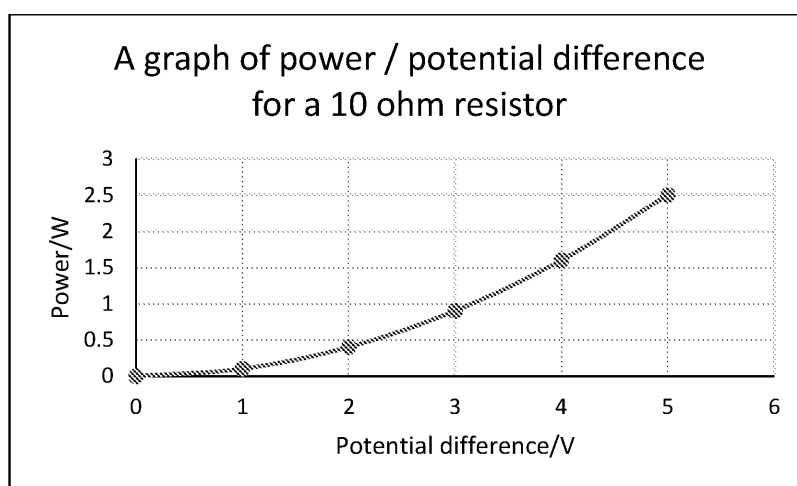


D1 Graphs

1. $y = mx + c$. It would become $y = mx$ for a directly proportional graph.
2. a.



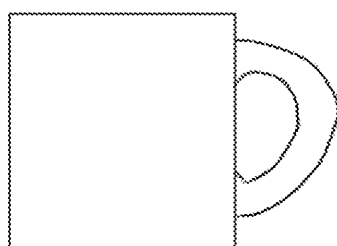
- b. 3
- c. 2
3. a.



- b. 0.8 (in range 0.7–0.9)
- c. The rate of change of power with potential difference when $V = 4$ volts
4. There are several examples you could give here. Examples include:
 A graph of current/time – the area under the graph is the charge that has flowed
 A graph of velocity/time – the area under the graph is the displacement
 A graph of force/time – the area under the graph is the impulse or the change in momentum
 A graph of force/distance moved – the area under the graph is the work done
 A graph of power/time – the area under the graph is the work done

E1 Geometry and trigonometry

1. Protractor
2. Degrees
3. a. area = length \times width
 b. volume = length³
 c. $\frac{1}{2}$ base \times height
 d. $6 \times$ length²
- 4.



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PRACTICE QUESTIONS

Arithmetic

1.

	Fraction	Decimal		Fraction	Decimal		Fraction	Decimal
A	$\frac{1}{16}$	0.0625	D	$\frac{3}{7}$	0.429	G	$\frac{1002}{17}$	58.9
B	$\frac{1}{20}$	0.05	E	$\frac{82}{3}$	27.3			
C	$\frac{1}{5}$	0.2	F	$\frac{213}{5}$	42.6			

2.

	Number	Percentage		Number	Percentage		Number	
A	$\frac{1}{16}$	6.25	D	$\frac{3}{7}$	43 (42.9)	G	$\frac{55}{127}$	
B	$\frac{1}{20}$	5	E	0.22	22	H	$\frac{133}{100}$	
C	$\frac{1}{5}$	20	F	$\frac{75}{136}$	55	I	$\frac{49}{8}$	

3.

	Percentage (%)	Fraction		Percentage (%)	Fraction
A	5	$\frac{1}{20}$	E	67.67	$\frac{2}{3}$
B	22	$\frac{11}{50}$	F	75	$\frac{3}{4}$
C	12	$\frac{3}{25}$	G	90	$\frac{9}{10}$
D	40	$\frac{2}{5}$			

4.

	Number	Standard form
A	365.25 days in a year	3.65
B	1 500 000 km is the average Earth–Sun distance	1.5
C	0.000 000 000 144 m is the radius of an atom of gold	1.44
D	101 000 N/m ² is atmospheric pressure	1.01
E	24.8 N/kg is the value of gravity on Jupiter	2.48

5.

	Standard form	
A	1×10^3 cm ³ in one litre	
B	3.15576×10^7 seconds in a year	3.15576
C	1×10^{-5} m is the diameter of a human hair	0.00001
D	1×10^{-9} m is one nanometre	0.000000001
E	9.1×10^9 km is the diameter of the solar system	9 100 000 000

6. a. 10 km = 10 000 m. 10 000 m at 2 m/s: 5000 s (about 83 minutes)
 b. 300 m at 5 m/s: 60 s
 c. 800 km/h average aircraft speed, 800 km/h for 2 hours = 1600 km
 d. Gravity is 9.8 m/s², so after one second the ball would be travelling at 9.8 m/s
 graph is the distance travelled = $\frac{1}{2} \times 1 \times 9.8 = 4.9$ m, so it takes about 1 s to

7. 3 : 1

8. circumference = $3.14 \times 12\,756 = 40\,054$ km

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Algebra

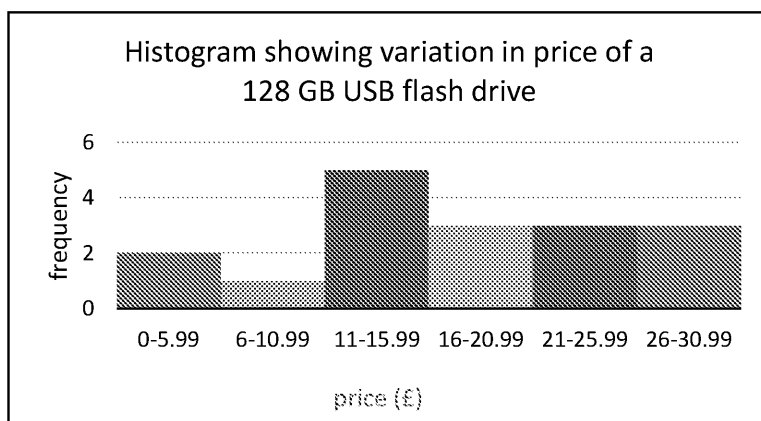
1. a. (Directly) Proportional
- b. The graph is a straight line (1 mark), passing through the origin (0,0)
- c. (i) $\text{resistance} = \frac{\text{power}}{\text{current}^2}$
- (ii) $\text{resistance} = \frac{50}{5^2}$
= 2 Ω (1 mark)
- (iii) $\text{voltage} = \text{current} \times \text{resistance}$
= 5×2
= 10 V (1 mark)
- (iv) 2 : 1
- d. (i) 10 W
- (ii) $\frac{1}{4}$
- (iii) 25 %
- (iv) 4.0×10^1 W
- (v) 9×10^4 seconds = 25 hours

Handling data

1. a.

Price (£)	Frequency
0–5.99	2
6–10.99	1
11–15.99	5
16–20.99	3
21–25.99	3
26–30.99	3

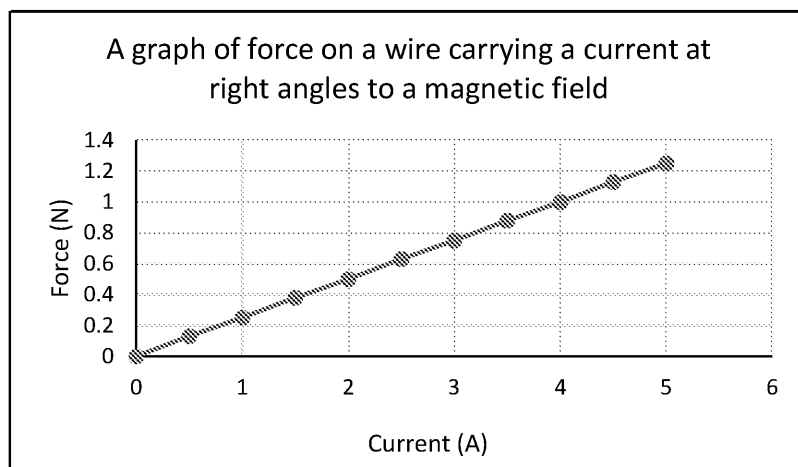
- b.



(axes labelled 1 mark; bars in

- c. £17.00
- d. £16.99
- e. £15, £17 and £25 as each occurs twice

- 2.



(scales 1 mark; points plotted 1 mark, losing 1 mark for any

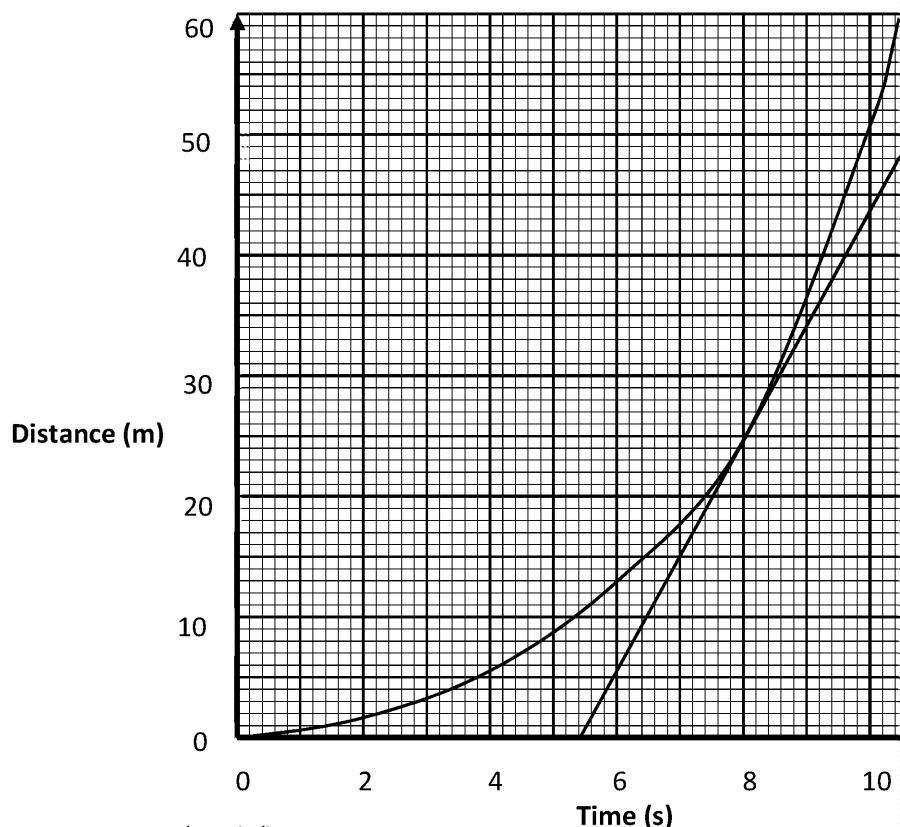
3. The graph shows a directly proportional relationship
- Conversion of km to metres
 10^3

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Graphs

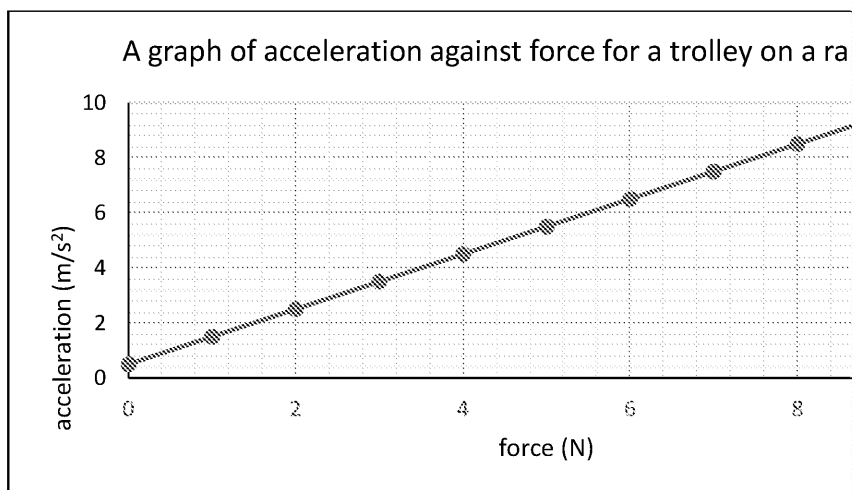
1. Gradient of the tangent to the graph at time = 7.8 seconds = rate of change of distance



a. $\text{gradient} = \frac{\text{change in distance}}{\text{change in time}}$
 $= \text{gradient} = \frac{60 - 0}{11.2 - 5.4}$ (allow ± 0.5 small square in values read from graph)
 $= 10.3 \text{ (m/s)}$

- b. Increases

2. a.



Mark allocation: scales (1), labels (1), points plotted within half a small square (1)

- b. Linear

c. $y = mx + c$

d. $\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$
 $\text{gradient} = \frac{9.5 - 1.0}{9.0 - 1.5}$
 $\text{gradient} = 1.0 \text{ (m/Ns}^2\text{)}$

e. $\text{area} = \text{area of parallelogram} = \frac{1}{2} \times \text{sum of parallel sides} \times \text{right angle distance}$
 $= \frac{0.5 + 9.5}{2} \times 9 = 45$

Or $\text{area} = \text{area of rectangle of side } 0.5 \text{ m/s}^2 \text{ and length } 9 \text{ N} + \text{area of triangle of base } 8.5 \text{ N and height } 9 \text{ m/s}^2$
 $= 4.5 + 4.5 \times 9 = 45$

- f. 0.5 m/s^2

- g. It shows that the trolley is accelerating even though there is no force applied to it

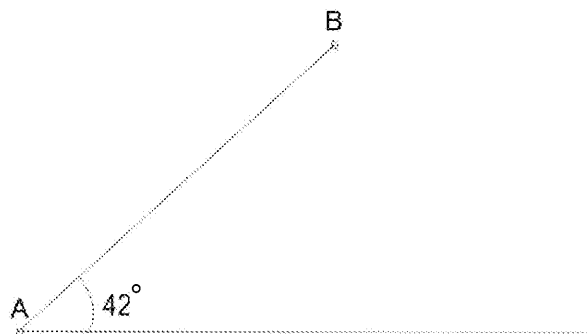
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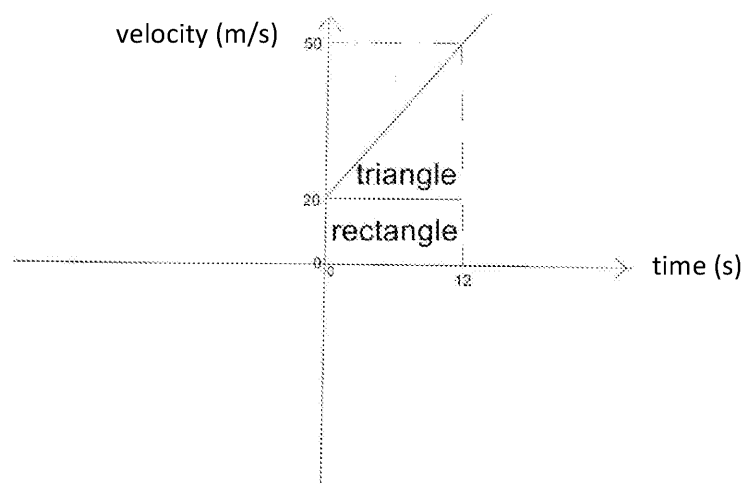


Geometry and trigonometry

1. $AB = 6.4 \text{ cm}$



2. area = area of rectangle of length 12 s and width 20 m/s + area of triangle of base 12 s and



$$= (12 \times 20) + (\frac{1}{2} \times 12 \times 30)$$

$$= 240 + 180 = 420 \text{ m}$$

Or

$$\text{area} = \frac{1}{2} \text{ sum of the parallel sides} \times \text{right angled distance between them}$$

$$= \frac{1}{2} \times (20 + 50) \times 12 = 420 \text{ m}$$

3. $\text{density} = \frac{\text{mass}}{\text{volume}}$

So, rearranging to make mass the subject: $\text{mass} = \text{density} \times \text{volume}$

convert 50 cm into metres = 0.5 m

$\text{volume} = \text{length}^3$

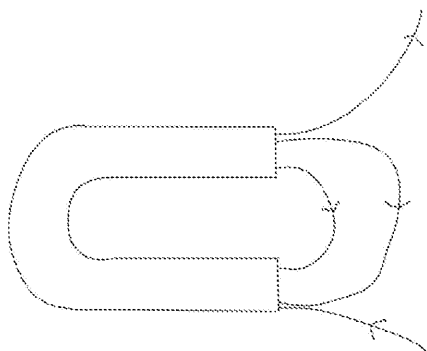
$$\text{volume} = 0.5^3 = 0.5 \times 0.5 \times 0.5$$

$$\text{volume} = 0.125 \text{ m}^3$$

$$\text{mass} = 1.2 \times 0.125$$

$$\text{mass of air in one cube} = 0.15 \text{ kg}$$

- 4.



(1 mark)

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DIAGNOSTIC TEST 2

A2 Arithmetic

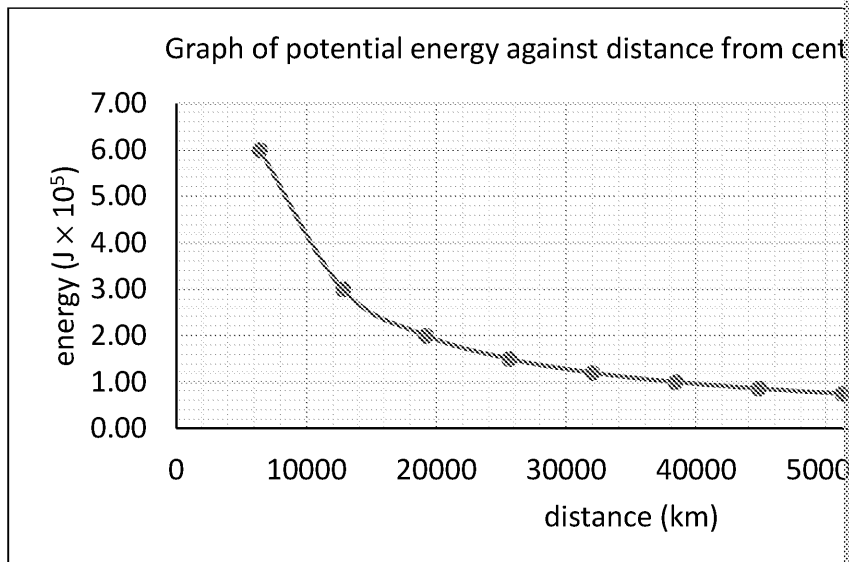
- 0.25
- $\frac{1}{5}$
- 5.12×10^2
- 1 000 000 000 000 W
- 0.37
- $\frac{150}{350} \times 100 = 43\%$
- Estimate the volume of each drip between 0.25 cm^3 and 2 cm^3 , giving the time to deliver seconds (5 h 33 min 20 s) and 2500 seconds (41 min 40 s) respectively.

B2 Algebra

- x is approximately equal to y
- $v = \frac{2}{\sqrt{2\alpha S}} \sqrt{v^2}$
- N/Am
- 50 m
- Inverse proportion
- Indirect proportion
- 10°C
 - 0.033°C/s

C2 Handling data

- 86 000
- 22.03 m
 - 22.05 m
 - 22.50 m
- Frequency table
 - Bar chart
- $50 \times 365 = 18\,250$, which is 10^4 , so the order of magnitude is 4
-



- Inverse proportion

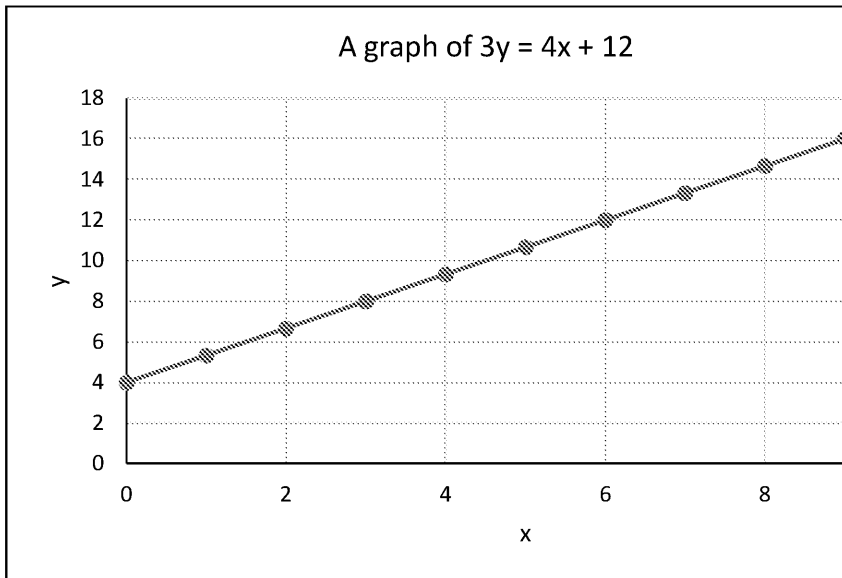
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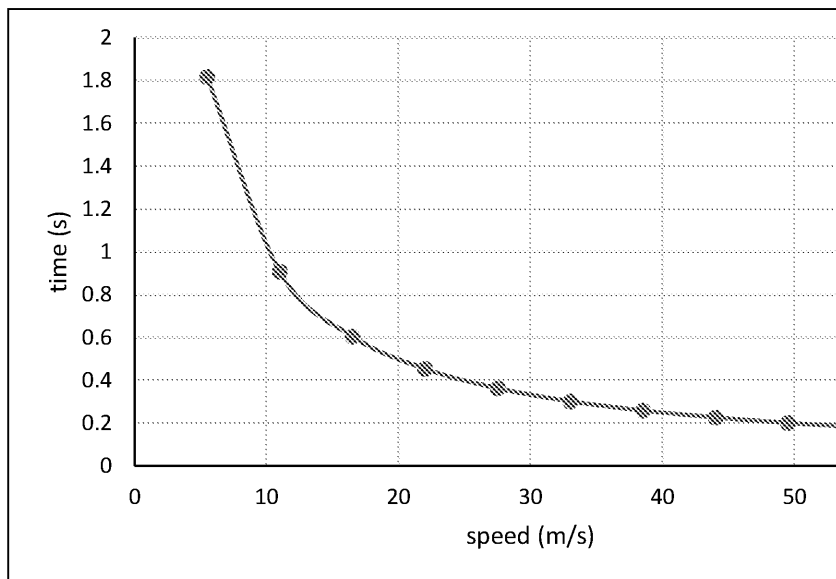


D2 Graphs

1. Straight line through the origin (directly proportional)
2. a.



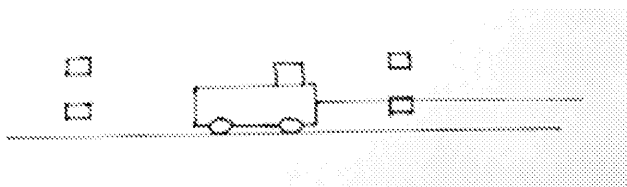
- b. 1.3 or $4/3$
- c. 4
3. a.



- b. Gradient = -0.11
- c. How quickly the time for the journey is decreasing at a speed of 5.5 m/s

E2 Geometry and trigonometry

1. 20° in the range of 19° to 21°
2. a. 10 cm^2
b. 125 cm^3
c. 100 cm^2
d. 150 cm^2
- 3.



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