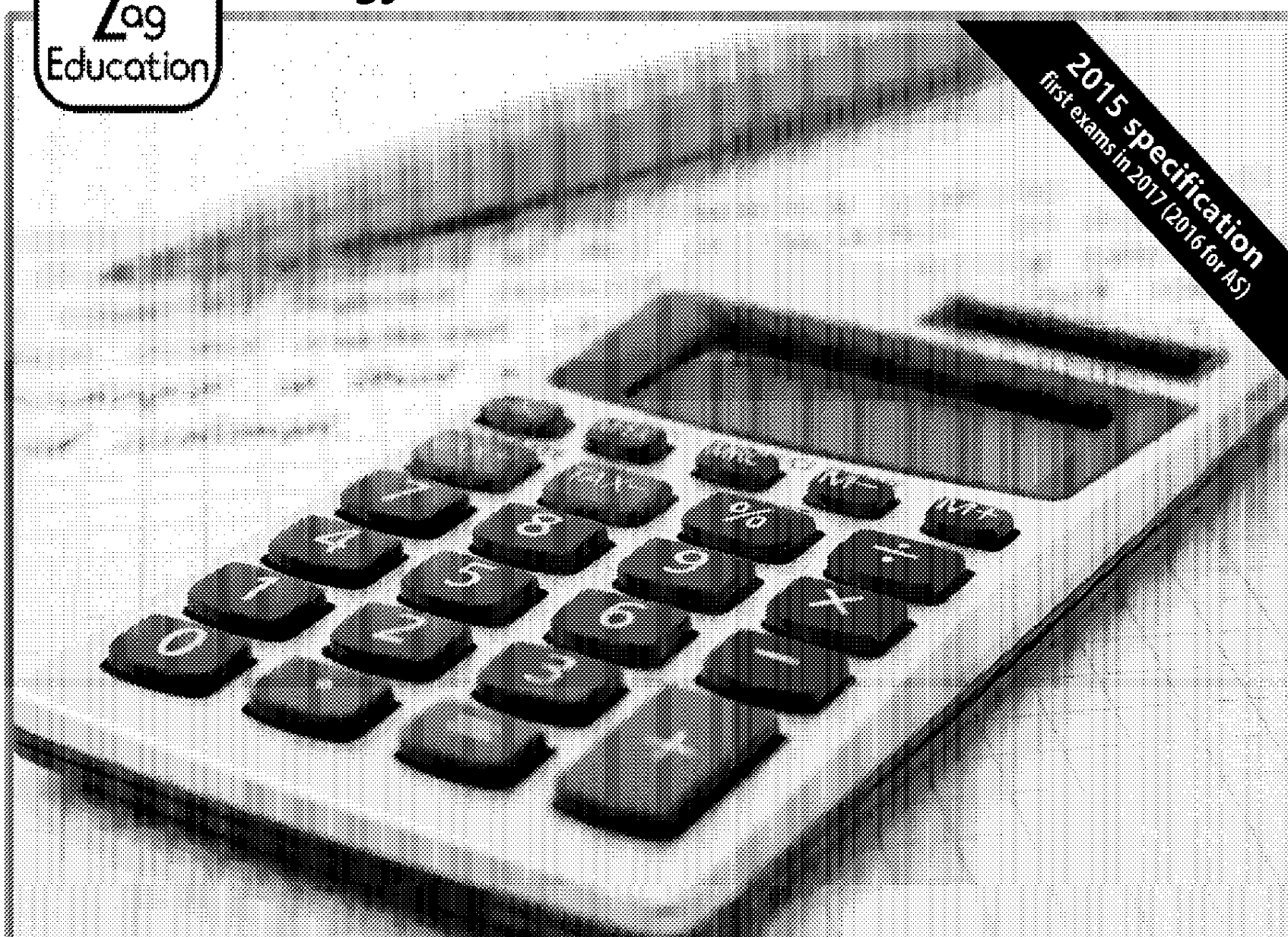




**Biology**

AS and A Level



# Mastering Maths

for AS and A Level Biology

Suitable for AQA, OCR, Edexcel, Eduqas and WJEC

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# Teacher's Introduction

Each of the following exam boards has published a list of the mandatory mathematical skills required for its Biology courses. These skills at AS & A Level are identical across all of the following specifications:

- AS and A Level AQA Biology (7401 and 7402)
- AS and A Level OCR Biology A (H020 and H420)
- AS and A Level OCR Biology B (H022 and H422)
- AS and A Level Edexcel Biology B (8BI0 and 9BI0)
- AS and A Level WJEC Eduqas Biology (B400QS and A400QS)
- AS and A Level WJEC Biology (2400 series and 1400 series)

Students sometimes find the mathematical skills required for success at AS & A Level a challenge, especially when expected to apply them to the context of Biology. This Mastering Maths resource has been designed with the intention of providing students with the opportunity to review the mathematical skills familiar to them from GCSE higher-tier courses, and to develop their understanding of new skills, such as statistical tests. The key aim of this resource is to allow students to master the core mathematical skills **so you can focus on the Biology!**

Some sections are relatively basic, and serve to boost confidence and eradicate any bad habits. Others will provide even the brightest students with the opportunity to practise the more challenging mathematical skills. As all biological contexts are explained, these sheets may be used at any time during the course. Some will be beneficial right at the start of Year 12, while others will provide support for Year 13 students who are dropping maths marks in the run-up to the final exams.

The resource includes a table mapping each basic maths skill outlined in the exam boards' published requirements lists to each specification point where the skill is found. The required mathematical skills are driven by the Department for Education. The assessment marks of quantitative skills in both AS & A Level papers will comprise a minimum of 10% of the required mathematical skills for Biology (Level 2 or above).

## Skills Sections

Each section covers all the core mathematical skills prescribed at AS/A Level. Some skills are treated relatively briefly (e.g. mathematical symbols), while others are given several chapters (e.g. rearranging equations).

Each chapter contains:

- mathematical guidance on the skill
- worked examples, including examples in a biological context
- a mix of simple questions and in-context questions to practise the relevant skill

## Diagnostic Test

This section includes a diagnostic test that is designed to give an assessment of students' comfort with different mathematical skills. This could be used at the start of Year 12 to gain an idea of different students' background knowledge and ability.

The test indicates the mathematical skills tested in each question; therefore, the specific skills with which the students are still struggling can be identified.

*April 2020*

MAPPING MATHS SKILLS TO SPECIFIC

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AQA	OCR (A/B)	Edexcel (B)	WJEC/ Eduqas	
Arithmetic and numerical computation				
MS 0.1	M 0.1	A.0.1	No reference numbers	Recognise and make use of appropriate units in calculations
MS 0.2	M 0.2	A.0.2		Recognise and use expressions in decimal and standard form
MS 0.3	M 0.3	A.0.3		Use ratios, fractions and percentages
MS 0.4	M 0.4	A.0.4		Estimate results
MS 0.5	M 0.5	A.0.5		Use calculators to find and use power functions, exponential and logarithmic functions (A Level only)
Handling data				
MS 1.1	M 1.1	A.1.1	No reference numbers	Use an appropriate number of significant figures
MS 1.2	M 1.2	A.1.2		Find arithmetic means
MS 1.3	M 1.3	A.1.3		Construct and interpret frequency tables and diagrams, bar charts and histograms
MS 1.4	M 1.4	A.1.4		Understand simple probability
MS 1.5	M 1.5	A.1.5		Understand the principles of sampling as applied to scientific data
MS 1.6	M 1.6	A.1.6		Understand the terms mean, median and mode
MS 1.7	M 1.7	A.1.7		Use a scatter diagram to identify a correlation between two variables
MS 1.8	M 1.8	A.1.8		Make order of magnitude calculations
MS 1.9	M 1.9	A.1.9		Select and use a statistical test
MS 1.10	M 1.10	A.1.10		Understand measures of dispersion, including standard deviation and range
MS 1.11	M 1.11	A.1.11		Identify uncertainties in measurements and use simple techniques to determine uncertainty when data are combined
Algebra				
MS 2.1	M 2.1	A.2.1	No reference numbers	Understand and use the symbols: $=, <, \ll, \gg, >, \infty, \sim$
MS 2.2	M 2.2	A.2.2		Change the subject of an equation, including non-linear equations
MS 2.3	M 2.3	A.2.3		Substitute numerical values into algebraic equations using appropriate units for physical quantities
MS 2.4	M 2.4	A.2.4		Solve algebraic equations
MS 2.5	M 2.5	A.2.5		Use logarithms in relation to quantities that range over several orders of magnitude (A Level only)
Graphs				
MS 3.1	M 3.1	A.3.1	No reference numbers	Transfer information between graphical, numerical and algebraic forms
MS 3.2	M 3.2	A.3.2		Plot two variables from experimental or other data
MS 3.3	M 3.3	A.3.3		Understand that $y = mx + c$ represents a linear relationship
MS 3.4	M 3.4	A.3.4		Determine the intercept of a graph (A Level only)
MS 3.5	M 3.5	A.3.5		Calculate rate of change from a graph showing a linear relationship
MS 3.6	M 3.6	A.3.6		Draw and use the slope of a tangent to a curve as a measure of rate of change
Geometry and trigonometry				
MS 4.1	M 4.1	A.4.1	No reference numbers	Calculate the circumferences, surface areas and volumes of regular shapes

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# MAPPING CHAPTERS TO SPECIFICATION

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Chapter in this resource	
1. Decimals and Standard Form	
2. Units I – Common Units and Prefixes	
3. Units II – Units with Powers	
4. Significant Figures	
5. Fractions, Percentages and Ratios	
6. Scaling Quantities	
7. Calculating Means, Medians and Modes	
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20. Constructing Graphs	
21. Analysing Graphs	
22. Surface Area and Volume of Shapes	
Appendix – Using a Calculator	

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# 1. DECIMALS AND STANDARDS

## LEARNING OUTCOME

Be comfortable with using both decimals and standard form, and converting between them

## THEORETICAL OVERVIEW

Biologists need to be able to manage both large and small numbers. Sometimes they find them difficult to use in calculations.

e.g. a prokaryotic ribosome has a diameter of... 0.000000075 m  
there are approximately... 200 000 000 000 000 synapses

### Standard form

When doing calculations, it is a lot easier to write these numbers in standard form. The decimal point gives the size of the number as a power of 10.

### Converting numbers into standard form

Numbers in standard form are written as:

$$a \times 10^x$$

where  $a$  is a number from 1 to 9, and  $x$  is the number of decimal places the decimal point has moved from the first non-zero digit of the number.

If the decimal point moves to the **left** then  $x$  is a **positive number**. If the decimal point moves to the **right** then  $x$  is a **negative number**.

For example:

$$\begin{array}{c} 234000000.0 \\ \underbrace{\phantom{234000000.0}}_8 \\ 234000000.0 = 2.34 \times 10^8 \end{array}$$

$$\begin{array}{c} 0.000137 \\ \underbrace{\phantom{0.000137}}_4 \\ 0.000137 = 1.37 \times 10^{-4} \end{array}$$

### Converting numbers back into decimals

To convert from standard form to a decimal, you move the decimal  $x$  times in the opposite direction.

For example:

$$2.89 \times 10^{-5} = 0.0000289 = 0.0000289$$

### Rounding

Rounding a number is a way of shortening numbers so they are easier to use in calculations. A number is rounded to a certain number of decimal places (d.p.).

4.560	
3 d.p.	4.560 ←
2 d.p.	4.56 ←
1 d.p.	4.6
0 d.p.	5

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## Standard form on your calculator

To be able to make calculations involving standard form, you will need to know how to use standard form on your calculator.

Standard form button  $\boxed{\times 10^x}$

Here are some examples of how to use this button:

$\boxed{3} \boxed{\cdot} \boxed{5} \boxed{\times 10^x} \boxed{4}$

which inputs  $3.5 \times 10^4$

$\boxed{4} \boxed{\times 10^x} \boxed{3} \boxed{\times} \boxed{6} \boxed{\times 10^x} \boxed{5}$

which inputs the calculation  $4 \times 10^3 \times 6 \times 10^5$

### WORKED EXAMPLE

A DNA molecule has a radius of  $1.2 \times 10^{-9}$  m and is  $4.93 \times 10^{-8}$  m long.

- Write the length of the DNA molecule as a decimal to 8 decimal places.
- The formula for the volume of the DNA molecule is given as  $\pi \times r^2 \times l$ , where  $r$  is the radius and  $l$  is the length. Perform the following calculation on your calculator to find the volume to 3 significant figures:

$$\pi \times (1.2 \times 10^{-9})^2 \times 4.93 \times 10^{-8}$$

- $4.93 \times 10^{-8} = 0.0000000493$  (move the decimal place eight places to the left)  
 $0.00000005$  (round up to one significant figure)
- $2.230279457 \times 10^{-25}$  (in standard form)  
 $= 2.23 \times 10^{-25}$  (rounded down to three significant figures)

### PRACTICE QUESTIONS

- Write the following in standard form:
  - 2 150 000 J
  - 0.084 m
  - 573 000 000 000 cells
  - 0.0000006345 kg
- Write the following numbers out in full:
  - $6.42 \times 10^{-6}$  s
  - $3.51 \times 10^4$  bases
  - $6.94 \times 10^{-4}$  g dm<sup>-3</sup>
  - $8.159 \times 10^2$  cm<sup>3</sup>
- Round the following to the given number of decimal places (d.p.):
  - 1.465 kJ to 2 d.p.
  - 7.9624 g to 3 d.p.
  - 3.1234 g to 2 d.p.
  - 0.1234 to 1 d.p.
- An ecologist samples an area of 11.628 ha, and counts nine foxes in the area.
  - Write down the area in hectares to two decimal places.
  - Calculate the density of the foxes in foxes ha<sup>-1</sup>, by dividing the number of foxes by the area. Give your answer in standard form to two decimal places.
- In a food chain, the productivity of the producers is 9400 kJ m<sup>-2</sup> year<sup>-1</sup>, and the productivity of the primary consumers is 12 %. Find the productivity of the primary consumers in kJ m<sup>-2</sup> year<sup>-1</sup>.

$$\text{primary consumers productivity} = \frac{\text{producers productivity} \times \text{biomass transfer efficiency}}{100}$$

Give your answer in standard form.

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# 2. UNITS I – COMMON UNITS AND CONVERSIONS

## LEARNING OUTCOME

Understand different units, convert between them, and understand why it is important

## THEORETICAL OVERVIEW

### Types of unit

In Biology, common measurements include length, volume and mass. You will also eventually come across other measurements, such as energy and water potential. The table on the right shows some common units for different types of measurement.

Type of measurement
length
mass
volume
temperature
time

### Converting between units

To do a calculation you might need to convert the values you are given into other units, like converting volumes given in  $\text{cm}^3$  into  $\text{dm}^3$ , or masses from kg to g. In Biology, converting between units often involves multiplying or dividing by powers of 10.

The different prefixes represent different powers of 10.

To convert to prefixes which are **larger** / higher powers of 10, you need to **divide** by 10 for every difference in the power.

To convert to numbers which are **smaller** / negative powers of 10, you need to **multiply** by 10 for every difference in the power.

For example, a **centimetre** ( $10^{-2}$ ) is 100 times smaller than a metre.

$$\begin{aligned} 1 \text{ cm} &= 0.01 \text{ m} \\ 100 \text{ cm} &= 1 \text{ m} \end{aligned}$$

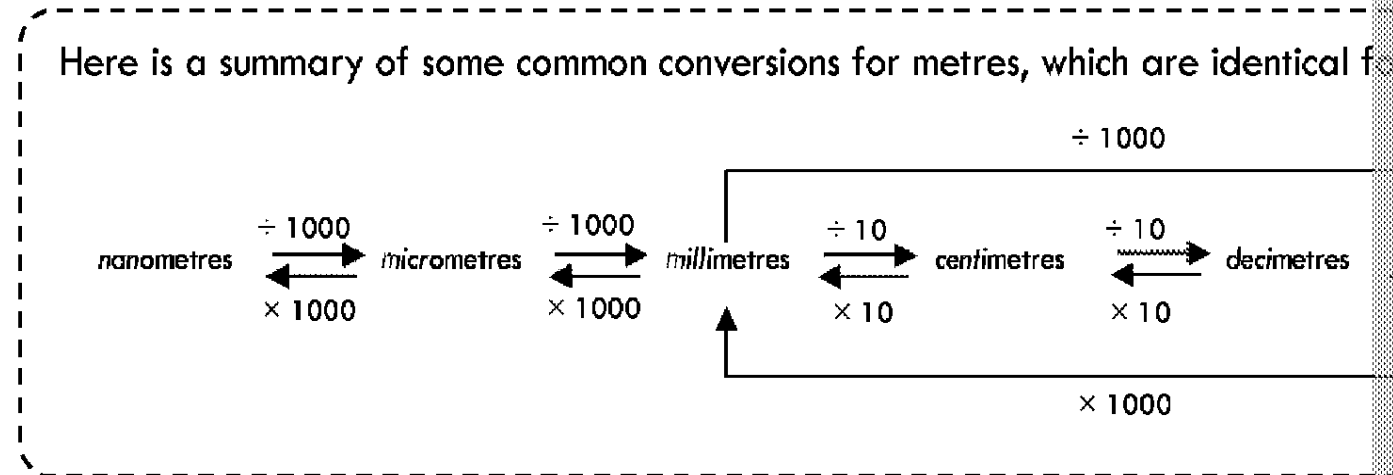
To convert centimetres to metres, divide the value in centimetres by 100 ( $10^2$ ).  
To convert metres to centimetres, multiply by 100.

$$\begin{array}{c} \div 100 \\ \curvearrowright \\ 1.3 \text{ cm} = 0.013 \text{ m} \\ \curvearrowleft \\ \times 100 \end{array}$$

Whether it's metres, grams or litres, converting between the different prefixes works the same way. For example, a **kilogram** is 1000 times (or  $10^3$  times) bigger than a gram.

$$\begin{aligned} 1 \text{ kg} &= 1000 \text{ g} \\ 1 \text{ g} &= 0.001 \text{ kg} \end{aligned}$$

$$\begin{array}{c} \div 1000 \\ \curvearrowright \\ 4.8 \text{ g} = 0.0048 \text{ kg} \\ \curvearrowleft \\ \times 1000 \end{array}$$



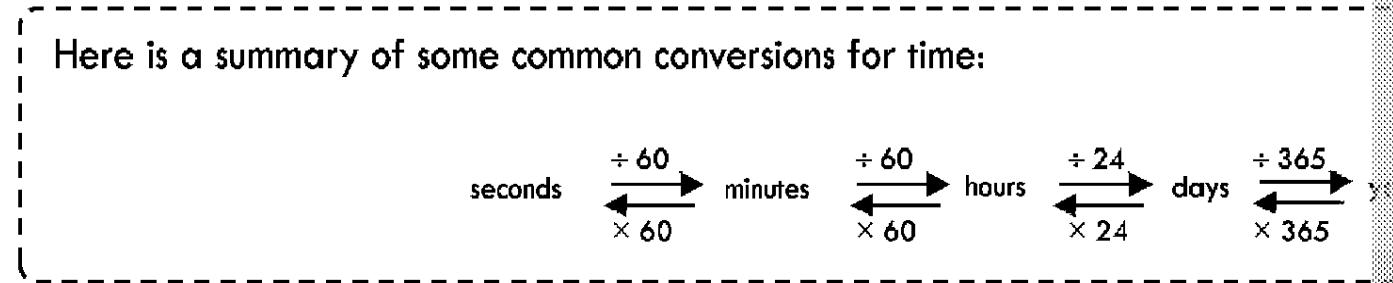
**WORKED EXAMPLES**

Convert 149 kiloseconds to milliseconds.

Convert 289 nanometres to millimetres.

**Time**

Time is often measured in hours in Biology. You can multiply or divide to convert between units.



**Units in equations**

To use equations, it may be necessary to convert values into suitable units for the equation. Most equations should use values in SI units, but there are some exceptions, e.g. minutes and seconds (s). These should usually be stated within a question in an examination.

**WORKED EXAMPLE**

A student is recording how distance changes during an experiment, and records the following data. Draw a second table which has values in centimetres and seconds.

Distance (m)	Time (min)
0.02	1:10
0.24	1:40
0.65	2:20
1.10	2:50

Distance (cm)	Time (s)
$0.02 \times 100 = 2$	$1 \times 60 + 10 = 70$
$0.24 \times 100 = 24$	$1 \times 60 + 40 = 100$
$0.65 \times 100 = 65$	$2 \times 60 + 20 = 140$
$1.10 \times 100 = 110$	$2 \times 60 + 50 = 170$

## PRACTICE QUESTIONS

1. Convert the following quantities:

- a) 400 nm into  $\mu\text{m}$
- b) 0.03 kg into mg
- c) 1.75 h into min
- d)  $6.48 \times 10^{-7}$  m into nm
- e)  $0.22 \times 10^8$  J into kJ
- f)  $3.05 \times 10^{12}$  nm into cm
- g) 712 000 pm into nm

2. Convert the following masses into milligrams:

- a)  $m = 25$  g
- b)  $m = 0.004$  kg
- c)  $m = 360$   $\mu\text{g}$

3. The speed of conduction,  $c$ , down a given nervous pathway is calculated from

$$c = \frac{d}{t}$$

*$d$  = distance in m;  $t$  = time in s*

Convert the values to the correct units for the following data:

- a)  $d = 25$  cm
- b)  $d = 3800$   $\mu\text{m}$
- c)  $t = 520$  ms
- d)  $d = 6 \times 10^8$  nm
- e)  $t = 0.0042$  min

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# 3. UNITS II – UNITS WITH

## LEARNING OUTCOME

Understand units with powers, and convert between them.

## THEORETICAL OVERVIEW

### Converting units with powers

Units for area (e.g.  $m^2$ ) and volume (e.g.  $m^3$ ) have powers (i.e.  $^2$  and  $^3$ ). It is more complex to convert between units with multiple dimensions. It may surprise you that  $1\text{ m}^3$  is 1 000 000 times larger than  $1\text{ cm}^3$ .

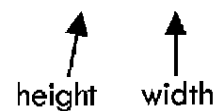
#### Areas

Square 2 has 10 times the width and height of square 1.

However, square 2 does **not** have 10 times the area of square 1.

Square 2 has **100 times the area** of square 1.

This is 10 squared ( $10^2$ ) which is  $10 \times 10$ .



$1\text{ dm}^2$   
square 1

#### Volumes

Cube 2 has 10 times the width, height and depth of cube 1.

However, cube 2 does **not** have 10 times the volume of cube 1.

Cube 2 has **1000 times the volume** of cube 1.

This is 10 cubed ( $10^3$ ) which is  $10 \times 10 \times 10$ .



$1\text{ cm}^3$   
cube 1

To convert a value in  $\text{dm}^3$  to a value in  $\text{cm}^3$ , you have to multiply by 1000, and divide for the reverse calculation.

e.g.  $3\text{ dm}^3 = 3000\text{ cm}^3$

$\div 1000$   
 $\text{mm}^3$   $\longleftrightarrow$   
 $\times 1000$

Although most volumes are usually given in terms of centimetres cubed or decimetres cubed, some may be given in terms of litres (l).

However, this is easy to deal with because  $1\text{ l} = 1\text{ dm}^3$ , and consequently  $1\text{ l} = 1000\text{ cm}^3$ .

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## WORKED EXAMPLES

1. Convert  $800\text{ cm}^3$  to  $\text{dm}^3$ .

This is going from a smaller unit to a larger unit, so we need to divide.

$$800 \div 10^3 = 0.8\text{ dm}^3$$

2. Convert  $6.3 \times 10^{-17}\text{ m}^2$  to  $\text{nm}^2$ .

This is going from a larger unit to a smaller unit, so we need to multiply.

$$6.3 \times 10^{-17} \times (10^9)^2 = 6.3 \times 10^{-17} \times 10^{18} = 63\text{ nm}^2$$

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## Inverse units

Inverse units include units such as 'per gram' and 'per second'. These are represented as ' $/\text{g}$ ' or ' $/\text{s}$ ', or more commonly at A Level, ' $\text{g}^{-1}$ ' and ' $\text{s}^{-1}$ '.

These are important for working with compound units, which are made up of two or more different units, like metres per second ( $\text{m s}^{-1}$ ), or grams per centimetre cubed ( $\text{g cm}^{-3}$ ). You will notice that the second of these uses an inverse unit to the power in the original unit ( $\text{cm}^3$ ) was to the power 3. **An inverse unit just has the negative power.** The power will be minus 1 if the original did not have a power (as  $\text{unit} = \text{unit}^1$ ).

When converting between inverse units, the conversion works the other way round to normal units.

To go from a **smaller** unit to a **larger** unit you need to **multiply**.  
To go from a **larger** unit to a **smaller** unit you need to **divide**.

For example, to go from grams to kilograms, you multiply by 1000. But to convert per gram to per kilogram, you divide by 1000.

### WORKED EXAMPLE

Convert  $950 \text{ s}^{-1}$  to  $\text{ms}^{-1}$ .

*This is going from a smaller unit to a larger unit so we need to divide.*

$$950 \div 10^3 = 0.950 \text{ ms}^{-1}$$

## PRACTICE QUESTIONS

- How many times bigger is:
  - $1 \text{ m}^3$  than  $1 \text{ dm}^3$ ?
  - $10 \text{ m}^3$  than  $1 \text{ dm}^3$ ?
  - a cube with sides 2 cm long than a cube with sides 1 cm long?
  - $2 \text{ m}^2$  than  $1 \text{ m}^2$ ?
  - $5 \text{ m}^2$  than  $10 \text{ cm}^2$ ?
- Convert the following quantities:
  - $4 \text{ m}^3$  into  $\text{mm}^3$
  - $7 \text{ cm}^3$  into  $\text{m}^3$
  - $20 \text{ m}^2$  into  $\text{cm}^2$
  - $500 \text{ m}^2$  into  $\text{mm}^2$
  - $8.8 \times 10^7 \text{ m}^3$  into  $\text{km}^3$
  - $14.65 \text{ cm}^3$  into  $\text{dm}^3$
  - $0.320 \text{ dm}^3$  into  $\text{cm}^3$
  - $4.9 \text{ J g}^{-1}$  into  $\text{J kg}^{-1}$
  - $18 \text{ g cm}^{-3}$  into  $\text{g dm}^{-3}$
- Convert the following values into centimetres cubed:
  - $v = 5 \text{ dm}^3$
  - $v = 0.02 \text{ l}$
  - $v = 24 \text{ m}^3$
- A subject's heart rate is recorded as  $2.45 \text{ beats s}^{-1}$ . Convert this value into beats per minute.

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# INSTRUCTION COPY

Use and understand significant figures, and give an appropriate number of significant

$$1 \div 7 = 0.142857142857142857142...$$

## Significant figures

A significant figure is a digit in a number that gives you information about its value. The first significant figure is the first **non-zero** digit. If a zero appears after this, it counts as a significant figure because it is a placeholder.

You can **round** a value to a number of significant figures. To round to two significant figures: if it is larger than or equal to 5, round up; if it is smaller than 5, round down.

For example, the two numbers on the right (above) rounded to two significant figures

34 000 (2 s.f.)

0.0072 (2 s.f.)


Rounding values to a number of significant figures makes calculations simpler, but is potentially less accurate, answer if the rounding is done too early.

Two key points to remember when using significant figures in calculations are:

1. Don't round any numbers until **the very end** of the calculation.
2. Give your final answer to the **smallest number** of significant figures used in

You cannot give your final answer to more significant figures than you have used in the calculation. This would mean giving a **more accurate** answer than the values you have used to calculate it.

**Example:**

  $1 \text{ m}^3 + 1.0 \text{ cm}^3 = 2.0 \text{ cm}^3$        $1 \text{ cm}^3 + 1.0 \text{ cm}^3 =$

$\uparrow$        $\uparrow$        $\uparrow$        $\uparrow$        $\uparrow$

2 s.f.      2 s.f.      2 s.f.      1 s.f.      2 s.f.

The answers have the **same number of significant figures** as the values with the **small**

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**WORKED EXAMPLE**

A potometer was used in an experiment to study the rate of transpiration in a plant. 30 seconds after starting the experiment, Stan took a reading. The air bubble was at 1.65 cm. 90 seconds after starting the experiment, Graeme took another reading. The air bubble was at 5.2 cm.

- Give Stan's reading to two significant figures.
- Calculate the average rate of transpiration in the plant per minute between the two readings, using the **unrounded** numbers and then giving your answer to an appropriate number of significant figures.

a) The first two significant figures are 1.6, but the next digit is a 5, so you have to round up. Stan's reading = 1.7 cm (to two significant figures).

b) The average rate of transpiration =  $(5.2 - 1.65) \div 1 = 3.55 \text{ cm min}^{-1}$ . Remember to round until the end.

The smallest number of significant figures used in the calculation is two, so the answer must be given to two significant figures.

= 3.6 cm min<sup>-1</sup>

**PRACTICE QUESTIONS**

- Round the following numbers to the given number of significant figures:

- 56 499 to 2 s.f.
- 0.0016382 to 3 s.f.
- 18 990 to 3 s.f.
- 0.040052 to 2 s.f.
- 0.0072087 to 4 s.f.
- 3.9999 to 4 s.f.

- The primary productivity of an area can be calculated using:

$$\text{primary productivity} = \frac{\text{biomass}}{\text{area} \times \text{time}}$$

Calculate the primary productivity of the following areas in  $\text{kJ ha}^{-1} \text{ yr}^{-1}$ , giving your answer to the appropriate number of significant figures:

- Biomass produced = 71 500 kJ; area = 2.1 ha; time = 1.0 yr
- Biomass produced = 4230 kJ; area = 0.36 ha; time = 0.75 yr
- Biomass produced = 14 350 kJ; area = 1.01 ha; time = 0.4167 yr



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# 5. FRACTIONS, PERCENTAGES

## LEARNING OUTCOME

Use and convert between fractions, percentages and ratios.

## THEORETICAL OVERVIEW

### Fractions, percentages and ratios

Fractions, percentages and ratios are different ways of representing proportions.

#### Fractions

$$\frac{\text{number}}{\text{total number}}$$

#### Percentages

$$\frac{\text{number}}{\text{total number}} \times 100$$

### WORKED EXAMPLE

A water molecule has the chemical formula  $\text{H}_2\text{O}$ , so it consists of two hydrogen atoms and one oxygen atom. There are three atoms in the molecule overall. You can express this information using a fraction, a percentage or a ratio.

#### Fraction

The fraction of atoms which are hydrogen atoms is:

$$\frac{\text{number of hydrogen atoms}}{\text{total number of atoms}} = \frac{2}{3}$$

#### Percentage

The percentage of atoms which are hydrogen atoms is:

$$\frac{\text{number of hydrogen atoms}}{\text{total number of atoms}} \times 100 = 66.7\%$$

Each of these representations contains all of the relevant information to describe

### Converting between fractions, percentages and ratios

As well as being able to calculate fractions, percentages and ratios, it is useful to be able to convert between them.

#### Fractions and percentages

To convert  $\frac{3}{4}$  into a percentage, multiply the fraction by 100:

$$\frac{3}{4} \times 100 = 75\%$$

It is not always easy to convert from a percentage to a fraction, but some percentages that are useful to recognise are:

5 %	10 %	20 %	25 %	33.33... %	50 %
$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

#### Ratios and fractions

Imagine sodium and potassium ions in the ratio 3 : 2. There are three sodium ions for

The fraction of sodium ions is:

$$\frac{3}{3+2} = \frac{3}{5}$$

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# Ratios and percentages

To convert a ratio into a percentage, do both of the previous conversions, i.e.:

1. Convert the ratio to a fraction
2. Multiply by 100

## WORKED EXAMPLE

For an eye colour trait with a phenotypic ratio of 7 brown : 1 blue:

1. Use the ratio to find the fraction of individuals with brown eyes:  $\frac{\text{brown}}{\text{brown} + \text{blue}}$
2. Multiply the fraction of brown by 100 to get the percentage of individuals with brown eyes:

$$\frac{7}{8} \times 100 = \frac{700}{8} = 87.5 \%$$

So 87.5 % of individuals have brown eyes

## Simplify

There are different ways of writing the same fraction or ratio, and some fractions can be simplified. For example,  $\frac{1}{2}$  and  $\frac{2}{4}$  are equal (they are both 50 %).  $\frac{1}{2}$  is a simplified version of  $\frac{2}{4}$ .

Fractions are normally written in their *simplest form*. To simplify a fraction, identify a number that both the top and bottom are divisible by the same number, e.g. 6 and 4 are both divisible by 2. Divide all the numbers by this number to get the simplest form.

Ratios are often given in the form  $x : 1$  or  $x : y : 1$ . To get a ratio into this form, divide all the numbers by the smallest number in the ratio.

For example:

- a fraction of  $\frac{4}{6}$  (divide top and bottom by 2) is written as  $\frac{2}{3}$
- a ratio of 12 : 6 : 3 (divide all numbers by 3) is written as 4 : 2 : 1

## WORKED EXAMPLE

Maltose is a disaccharide formed from two units of glucose.

It has the chemical formula  $C_{12}H_{22}O_{11}$ .

- a) What fraction of the atoms in a maltose molecule is oxygen atoms?
- b) What is this as a percentage?
- c) What is the ratio of carbon atoms to other atoms?

- a) The fraction of oxygen atoms in maltose is the number of oxygen atoms (11) divided by the total number of atoms (45):

$$\frac{\text{number of oxygen atoms}}{\text{total number of atoms}} = \frac{11}{12 + 22 + 11} = \frac{11}{45}$$

- b) Then, to convert this to a percentage, you need to multiply the fraction of oxygen atoms by 100:

$$\frac{11}{45} \times 100 = 24.4444... = 24 \% \text{ oxygen (to the nearest whole number)}$$

- c) To calculate the number of atoms that aren't carbon, you subtract the number of carbon atoms from the total number of atoms:

$$\text{number of carbon atoms} = 12$$

$$\text{number of atoms that aren't carbon} = 45 - 12 = 33$$

$$\begin{aligned} \text{ratio of carbon atoms to other atoms} &= 12 : 33 \\ &= 0.36 : 1 \end{aligned} \quad \leftarrow \text{divide both sides by 33}$$

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## PRACTICE QUESTIONS

1. Write the following as decimals:
  - a) 65 %
  - b)  $\frac{1}{4}$
  - c) 0.2 %
  - d)  $\frac{0.8}{3.2}$
  - e)  $\frac{11}{12}$
2. Write the following as percentages to one decimal place:
  - a)  $\frac{3}{7}$
  - b)  $\frac{6}{19}$
  - c)  $\frac{9}{10}$
  - d)  $\frac{2}{9}$
  - e)  $\frac{24}{100}$
3. Write the following as their simplest fractions:
  - a) 20 %
  - b) 110 %
  - c) 75 %
  - d) 9 %
  - e) 15 %
4. Two types of snail, white-lipped and dark-lipped, exist in the ratio 3 : 1 in a park.
  - a) What fraction of the snails in the park are white-lipped?
  - b) What percentage of the snails in the park are dark-lipped?
5. A rat has a surface area of  $150 \text{ cm}^2$  and a volume of  $120 \text{ cm}^3$ . Express its surface area to volume ratio in the form  $x : 1$ .
6. A certain disease has a transmission success rate of 35 %. Express the proportion as a fraction in its simplest form.

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# 6. SCALING QUANTITIES

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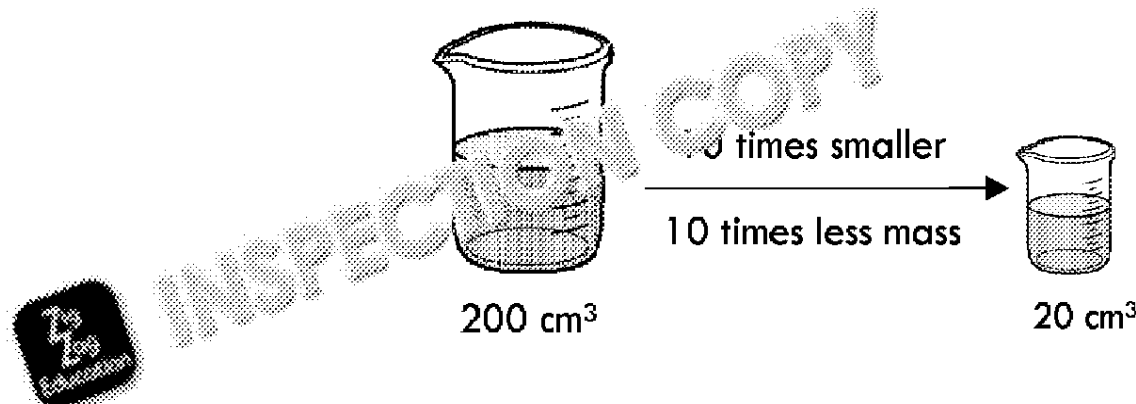
## LEARNING OUTCOME

Scale up and down when doing extended calculations.

## THEORETICAL OVERVIEW

### Scaling up and down

In Biology it can be important to scale a quantity up or down in proportion to another.



The trick is to:

- divide by the original value so you know how much there is in, e.g. 1 cm<sup>3</sup>, and
- multiply by the new value to find out how much there is at the end.

### WORKED EXAMPLE 1

2.0 g of a substance is dissolved in 100 cm<sup>3</sup>. 12.5 cm<sup>3</sup> samples are taken. What mass of substance is in each sample?

100 cm<sup>3</sup> contains 2.0 g

1 cm<sup>3</sup> contains  $\frac{2}{100} = 0.02$  g

12.5 cm<sup>3</sup> contains 0.25 g

The mass of substance in the samples is:  $\frac{2}{100} \times 12.5 = 0.25$  g

divide by  
multiply

### WORKED EXAMPLE 2

A population of 210 individuals is at 70 % of its carrying capacity. How many individuals are at 100 % carrying capacity?

The full carrying capacity is:  $\frac{210}{70} \times 100 = 300$

divide by 70 to  
then multiply

### WORKED EXAMPLE 3

Leaf A is 7.0 cm long. Leaf B is 8.4 cm long. How many times longer than leaf A is leaf B?

The number of times that leaf B is longer than leaf A is:  $\frac{8.4}{7.0} = 1.2$

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# Yield and scaling yields

Percentages may also be used in Biology to represent the yield of a reaction. If there is a competitor, some reactants in an organism or a cell may not react to form the desired product, which decreases the yield.

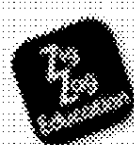
To calculate percentage yield:

$$\text{percentage yield} = \frac{\text{actual amount}}{\text{theoretical amount}} \times 100$$

- The actual yield is the amount of product **actually made** in the reaction.
- The theoretical yield is the amount of product which **could have been made** if all the reactants reacted to form the desired product.

## WORKED EXAMPLE

A bacterial colony forms 2.7 g of a product. The theoretical mass of product that the colony is 7.2 g. Find the percentage yield of the colony.



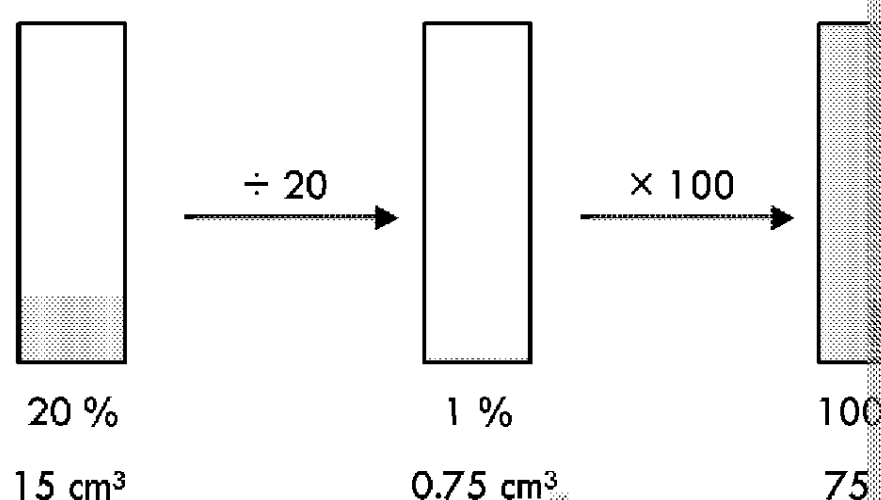
$$\begin{aligned} \text{percentage yield} &= \frac{\text{actual yield}}{\text{theoretical yield}} \times 100 \\ &= \frac{2.7}{7.2} \times 100 \\ &= 37.5\% \end{aligned}$$

If you know the percentage yield, you can use it to scale up to find the total yield.

Imagine a reaction with an actual yield of 15 cm<sup>3</sup>, and a percentage yield of 20 %

$$\text{percentage yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100$$

You can find out the theoretical yield by dividing the volume by 20 (to find the volume by 100 (to find the volume of 100 %). This diagram might help visualise the process.



The theoretical yield is, therefore, 75 cm<sup>3</sup>.

This concept can also be shown by rearranging the equation for percentage yield:

$$\begin{aligned} \text{percentage yield} &= \frac{\text{actual yield}}{\text{theoretical yield}} \times 100 \\ \text{theoretical yield} \times \text{percentage yield} &= \text{actual yield} \times 100 \quad \leftarrow \text{multiply both sides by theoretical yield} \\ \text{theoretical yield} &= \frac{\text{actual yield}}{\text{percentage yield}} \times 100 \quad \leftarrow \text{divide both sides by percentage yield} \end{aligned}$$

Adding numbers to the calculation:

$$\begin{aligned} \text{theoretical yield} &= \frac{\text{actual yield}}{\text{percentage yield}} \times 100 \\ &= \frac{15}{20} \times 100 \quad \leftarrow \text{In by 100} \\ &= 75 \text{ cm}^3 \end{aligned}$$

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## PRACTICE QUESTIONS

1. A solution is diluted to 14 % of its original concentration so that it has a dissolved mass of 0.025 g. Work out the mass of dissolved reactant in the following to two significant figures.
  - a) 50 % of the sample
  - b) 85 % of the sample
  - c) 2 % of the sample
2. 1.20 dm<sup>3</sup> of a solution contains 1.50 g of a substance. Work out how many dm<sup>3</sup> of the following. Give your answers to two significant figures.
  - a) 1 gram of the substance
  - b) 0.075 grams of the substance
  - c) 2.6 grams of the substance
  - d) 0.85 grams of the substance
3. A fungus has a predicted percentage yield of 42 % and produces 5.27 kg of protein. Calculate the theoretical yield of protein.
4. Sarah responds to an audio stimulus on an average of 0.18 seconds. She responds to a visual stimulus on an average of 0.23 seconds. Calculate how many times longer it takes Sarah to respond to a visual stimulus than an audio stimulus on average.
5. A drug increases a patient's red blood cell count from 3.5 million cells  $\mu\text{l}^{-1}$  to 4.5 million cells  $\mu\text{l}^{-1}$ . Find the percentage increase in the patient's red blood cell count after taking the drug.

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# 7. CALCULATING MEANS, MEDIANS

## LEARNING OUTCOME

Calculate mean, median and mode averages by selecting appropriate values from given data.

## THEORETICAL OVERVIEW

### Repeating experiments

It is common in Biology to repeat an experiment and take an average of the results. If the results of multiple experiments, the result will be more **accurate**.

### Mean

To calculate the mean average, you add up the numbers and divide by the number of numbers.

The mean of numbers 1–4 is:

$$\frac{1 + 2 + 3 + 4}{4} = 2.5$$

Note that the result should be to the same number of significant figures as (or to one more than) the raw data values.

### Outliers (anomalies)

You might repeat an experiment to find a more accurate value. However, sometimes the data you have collected might have an outlier. This could be caused by differences in conditions, or an error in reading or recording a measurement.

Repeat number 2 looks like an outlier and should be removed when calculating the mean.

$$\text{mean} = \frac{2.3 + 2.3 + 2.4 + 2.2}{4} = 2.3 \text{ s}$$

## WORKED EXAMPLE

The water potential inside a cell is calculated and recorded under the same conditions.

	Calculation 1	Calculation 2	Calculation 3
Water potential (kPa)	−210	−225	−205

Find the mean water potential of the cell.

Calculation 4 is discarded as it looks like an outlier. The mean of calculations 1, 2 and 3 is:

$$\frac{-210 + -225 + -205}{3} = -213.3 \text{ kPa}$$

### Median and mode

While the mean is often the most useful measure of central tendency for a data set, you can also identify the median or the mode.

To calculate the median, you arrange the data in order and identify the middle value.

When there is an odd number of data values, finding the middle value is often intuitive. The median of the numbers 1–5 is 3:

$$1 \quad 2 \quad (3) \quad 4 \quad 5$$

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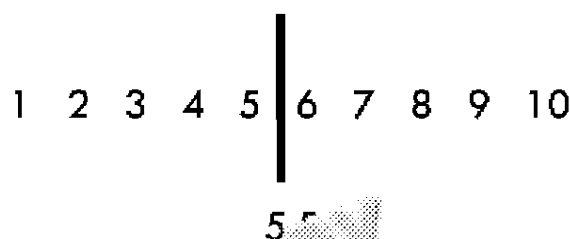


When there is a greater number of values, particularly if the total number of values is even, the formula:

$$\text{median} = \frac{n+1}{2} \text{th value}$$

For example, with 10 data points, the median will be the  $(10 + 1)/2 = 5.5^{\text{th}}$  value. This is the value that is halfway between the fifth value and the sixth value – and this makes 5 values on either side of the median.

The median of the numbers 1–10 is 5.5:



The mode is the most common value in the data set.

This is usually the value that appears most often once the data is ordered – just identify the value with the highest frequency.

### WORKED EXAMPLE

A class of 10 students each determined their resting heart rate in beats per minute.

76 82 98 102 85 82 90 92 83 86

- Find the modal resting heart rate of the class.
- Find the median resting heart rate of the class.
- The teacher had a resting heart rate of 95 bpm. Find the median resting heart rate of the class when the teacher's data value is added.

a) The data in order is: 76 82 82 83 85 86 90 92 98 102. 82 appears twice, so is the mode.

b) With 10 data points, the median value is the  $(10 + 1)/2 = 5.5^{\text{th}}$  value. The fifth and sixth values in the ordered list are 85 and 86, so the median is 85.5.

c) Adding the teacher's resting heart rate makes the ordered list:

76 82 82 83 85 86 90 92 95 98

### PRACTICE QUESTIONS

- Calculate the mean of the following sets of data:
  - The whole numbers 1–8
  - 6, 4, 3, 9
  - 6.5, 6.2, 6.6, 6.8
  - 230, 200, 180, 160, 140, 120, 100, 80, 60, 40, 20, 0, 250
  - 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.05, 0.01
- Calculate the median and the mode (where applicable) of the following sets of data:
  - 6, 6, 8, 3, 5
  - 12.2, 8.7, 9.2, 10.0, 7.4, 10.0, 8.9
  - 92, 83, 64, 71, 99, 72, 74, 88
  - 12, 4, 8, 10, 2, 8, 2, 12, 4, 4, 10, 0

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# 8. USING EQUATION REARRANGING SIMPLE EC

## LEARNING OUTCOME

Use and rearrange simple equations to calculate values for physical quantities.

## THEORETICAL OVERVIEW

### Substituting values

In Biology, equations are used to calculate values from other values. The following 'y is four times bigger than x':

$$y = 4x$$

When  $x = 2$ , the equation can be used to calculate that  $y = 8$ :

$$y = 4 \times 2 = 8$$

### Rearranging equations

If you know the value of  $y$  and want to find  $x$ , the equation can be rearranged:

$$\begin{array}{l} y = 4x \\ \frac{y}{4} = x \end{array} \quad \begin{array}{l} \text{divide both sides} \\ \text{by 4} \end{array}$$

The key to rearranging equations is that if you do something to one side of the equation, you must do the same to the other side.

### Magnification

A key equation you will be using for Biology is the magnification formula, used to calculate the size of an object from the size of its image and the magnification used.

$$\text{magnification} = \frac{\text{size of image}}{\text{size of object}}$$

If you know that the magnification used is  $\times 400$ , and the size of the image is 4.8 mm, you can rearrange the equation, and then substitute in this information to find the size of the object:

$$\begin{array}{l} \text{magnification} = \frac{\text{size of image}}{\text{size of object}} \\ \text{magnification} \times \text{size of object} = \text{size of image} \\ \text{size of object} = \frac{\text{size of image}}{\text{magnification}} \end{array} \quad \begin{array}{l} \text{multiply} \\ \text{of object} \\ \text{divide by} \\ \text{'magnification'} \end{array}$$

$$\text{size of object} = \frac{4.8}{400}$$

$$\text{size of object} = 0.012 \text{ mm}$$

$$= 12 \mu\text{m}$$

substitute  
perform  
convert

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## WORKED EXAMPLE

The equation for cardiac output is:

$$CO = SV \times HR$$

where: CO = cardiac output in  $\text{l min}^{-1}$   
SV = stroke volume in l  
HR = heart rate in  $\text{beats min}^{-1}$

A patient has a resting heart rate of 75 bpm and a cardiac output of  $4.8 \text{ l min}^{-1}$ . Calculate the stroke volume of the patient in ml.

$$\begin{aligned} CO &= SV \times HR && \xrightarrow[\text{dividing by HR}]{\text{rearrange by}} SV = \frac{CO}{HR} && \xrightarrow{\text{substitute in}} SV = \frac{4.8}{75} \\ &= 0.064 \text{ l} \\ &= 64 \text{ ml} \end{aligned}$$

## PRACTICE QUESTIONS

1. The rate of diffusion can be calculated using the equation:

$$\text{rate} = \frac{\text{concentration of substance moved}}{\text{reaction time}}$$

Rearrange the equation to show how the concentration of substance moved depends on the reaction time.

2. Rearrange the following equations to make  $x$  the subject of the equation:

- a)  $R = 0.5x$
- b)  $aB = yx$
- c)  $18 = 6x$

3. The respiratory quotient is:  $RQ = \frac{\text{volume of CO}_2 \text{ produced}}{\text{volume of O}_2 \text{ consumed}}$

- a) Rearrange the equation to make volume of  $\text{O}_2$  consumed the subject.
- b) Rearrange the equation to make volume of  $\text{CO}_2$  produced the subject.

4. The retention factor in thin-layer chromatography is calculated as  $R_f = \frac{\text{distance moved by solute}}{\text{distance moved by solvent}}$

- a) Calculate the retention factor when the solvent moves 4.2 cm and the solute moves 1.5 cm.
- b) Calculate the distance moved by the solvent when a solute which moved 2.5 cm has a retention factor of 0.6.

5. Find the magnification used to view a 0.2 mm object as an 8 mm image.

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# 9. USING EQUATION

## EQUATIONS WITH +, -, >

### LEARNING OUTCOME

Make variables the subject of an equation in equations involving multiplications, divisions

### THEORETICAL OVERVIEW

Equations are a really useful way of looking at the relationships between quantities. For example, this equation tells you how the quantity  $y$  depends on the quantity  $x$ .

$$y = 2x + 4$$

But what if we want to know how  $x$  depends on  $y$ ? To see this, we need to **rearrange** the equation to make  $x$  the subject:

$$y = 2x + 4$$

$$y - 4 = 2x$$

The second step is to get the  $x$  completely on its own. At the moment, you have  $2x$  on the right-hand side of the equation, so divide both sides by 2.

$$\frac{y - 4}{2} = x$$

This equation can be written the other way around:

$$x = \frac{y - 4}{2}$$

Now that you have rearranged the equation, you can substitute a value of  $y$  straight into the equation for  $x$ , e.g. when  $y = 12$ :

$$x = \frac{12 - 4}{2} = \frac{8}{2} = 4$$

### WORKED EXAMPLE

An experiment measuring the volume of gas produced gave the following data:

Repeat	1	2	3
Gas produced (cm <sup>3</sup> )	26.1	24.5	25.2

A student calculated the mean value as 25.5 cm<sup>3</sup>. Find the value of  $x$ .

The mean value is given by:

$$\frac{26.1 + 24.5 + 25.2 + x}{4} = 25.5 \quad \text{we can simplify to} \quad \frac{75.8 + x}{4} = 25.5$$

Rearrange the formula to make  $x$  the subject, and calculate the value.

$$\frac{75.8 + x}{4} = 25.5$$

multiply both sides by 4

$$75.8 + x = 25.5 \times 4$$

subtract 75.8 from both sides

$$x = 25.5 \times 4 - (75.8)$$

perform the calculation

$$= 26.2 \text{ cm}^3$$

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PRACTICE QUESTIONS

1. Make  $x$  the subject of the following equations:

a)  $y = \frac{x-5}{3}$

b)  $y = 10x + 8$

c)  $y = mx + c$

d)  $y = \frac{3}{x}$

e)  $4x + 2 = 6x + 2y$

f)  $5y + 3x + 2 = 3y + 7x - 2$

g)  $8xy = 1$

h)  $2xy + 4 = 2y - 2$

2. For the following data, a student calculated the mean to be  $7.2 \text{ cm}^3$ :

Volume ( $\text{cm}^3$ )	1	2	3	4
	$x$	6.1	8.6	7.2

Find the value of  $x$  (repeat 1).

3. Water potential is calculated as  $\varphi = \varphi_p + \varphi_s$ .

A student calculated the mean water potential of a cell to be  $-209.5 \text{ kPa}$ :

Repeat	1	2	3
Pressure potential ( $\varphi_p$ , kPa)	12.0	$x$	9.5
Solute potential ( $\varphi_s$ , kPa)	-220.0	-216.5	-211.5

Find the value of  $x$  (repeat 2).

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# 10. USING EQUATIONS

## EQUATIONS WITH POWERS AND ROOTS

### LEARNING OUTCOME

Rearrange equations containing powers and roots.

### THEORETICAL OVERVIEW

Some equations in Biology are quite complex, and can be hard to rearrange. For example, the volume of a spherical viral particle is given by:

$$V = \frac{4\pi r^3}{3}$$

where  $r$  is the radius of, for instance, a spherical viral particle.

To make  $r$  the subject of the equation:

$$V = \frac{4\pi r^3}{3}$$

multiply both sides by 3

$$3V = 4\pi r^3$$

divide both sides by  $4\pi$

$$\frac{3V}{4\pi} = r^3$$

take the cube root of both sides

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

Now  $r$  is the subject of the equation, so the equation can be used to easily calculate the radius of a spherical viral particle.

\* The cube root finds the number which, multiplied by itself three times, gives that number.

For example  $\sqrt[3]{8} = 2$  because  $2 \times 2 \times 2 = 8$ .

### WORKED EXAMPLE

The volume of a vein, which can be modelled as a cylinder, is given as  $V = \pi r^2 l$

where  $r$  represents the radius of the vein, and  $l$  represents the length of the vein.

Rearrange the formula to make  $r$  the subject.

$$V = \pi r^2 l$$

divide both sides by  $\pi l$

$$\frac{V}{\pi l} = r^2$$

take the square root of both sides

$$\sqrt{\frac{V}{\pi l}} = r$$

### PRACTICE QUESTIONS

1. Given that  $a = 3$  and  $b = 2$ , find  $c$  for each of the following equations:

a)  $c = a^b + b^3$

b)  $cb = b^a$

c)  $c^2 = a^b$

d)  $abc = \frac{b^4}{a^2}$

e)  $a^2 b^3 = a^2 + a^3 + c^2$

f)  $b^2 c^3 = a^2$

2. Make  $x$  the subject of the following equations:

a)  $y = \frac{1}{3}x^2$

b)  $y = \frac{4\pi}{x^2}$

c)  $y = \sqrt{\frac{x}{9}}$

d)  $y = \frac{1}{x^3}$

# 11. MATHEMATICAL SYMBOLS

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## LEARNING OUTCOME

Be able to use the symbols  $<$ ,  $<<$ ,  $>$ ,  $>>$ ,  $\propto$  and  $\sim$ .

## THEORETICAL OVERVIEW

### Less than and greater than: $<$ and $>$

The symbol points towards the smaller number:  $2 < 5$

These symbols can be rearranged in a similar way to an equals sign ( $=$ ).

For example:

$$x - 3 < 5$$

add 3 to both sides

You can solve this as if it was an equation:

$$x < 8$$

The only difference is when reversing equations. You have to swap the symbol around.

$$x < 2$$

$$2 > x$$

### WORKED EXAMPLE

An ecologist sampled a population of 400 individuals and found that 240 of them had a certain characteristic.

Write an expression for the frequency of the heterozygous phenotype in the population.

$$X < \frac{400 - 240}{400} \text{ because homozygous dominant individuals cannot be heterozygous for the characteristic.}$$

$$X < 0.4$$

### Much less than and much greater than: $<<$ and $>>$

$<<$  means much less than  $>>$  means much greater than

For example:  $5\,000\,000\,000\,000\,000 >> 5$

### Approximately equal to: $\sim$

For example:  $5.001 \sim 5$

### Directly proportional to: $\propto$

This symbol means that as the value of one side of an equation increases, so does the other.

$$y \propto x$$

Therefore, if  $y$  is doubled,  $x$  is also doubled. If  $x$  is divided by 10,  $y$  is also divided by 10.

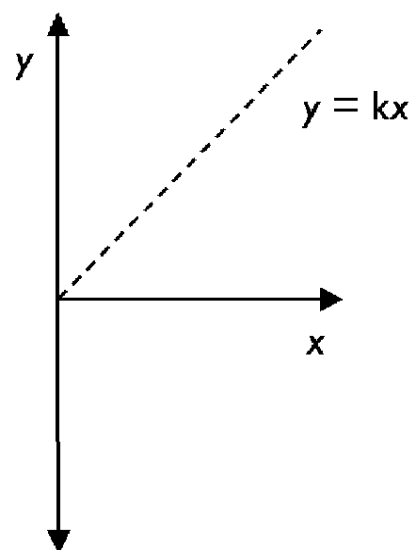
Another way of writing this is using an equals symbol, using 'k' which is a constant (or a value that does not change when  $x$  and  $y$  change):

$$y = kx$$

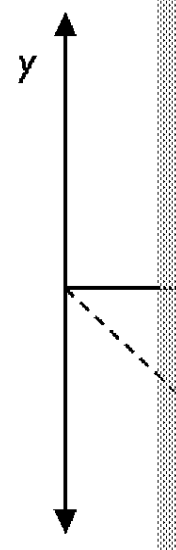
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If k is positive  
x increases proportionally as y increases



If k is negative  
x increases proportionally as y decreases



In the example below, ab is proportional to c:

$$a \times b \propto c$$

$$ab \propto c$$

$$a \propto \frac{c}{b}$$

which can be rearranged to:

From this, you can tell that:

- doubling a doubles c (if b is constant)
- doubling a halves b (if c is constant)

### Directly proportional to $x^2$

y can also be proportional to  $x^2$ :

$$y = kx^2$$

which means that when x is multiplied by a factor (e.g. x doubles), y is multiplied by the square of that factor (e.g. y quadruples).

change in x	change in y
$\times 2$	$\times 2^2 = \times 4$
$\times 3$	$\times 3^2 = \times 9$
$\times 4$	$\times 4^2 = \times 16$
$\times \frac{1}{2}$	$\times \left(\frac{1}{2}\right)^2 = \times \frac{1}{4}$

### PRACTICE QUESTIONS

- Write the following statements using the correct symbols:
  - 3 cm<sup>3</sup> is greater than 1 cm<sup>3</sup>
  - 2800 mg is less than 3000 mg
  - 5000 is much greater than 0.001
  - The rate of photosynthesis is proportional to temperature
- For the relationship  $AB \propto CD$ :
  - What happens to B if C is doubled, assuming that A and D stay the same?
  - What happens to B if D is halved, assuming that A and C stay the same?
  - What happens to D if C is doubled, assuming that A and B stay the same?
  - What happens to C if A is doubled and B and D stay the same?
  - What happens to D if A and B are both halved and C stays the same?
  - What happens to B if A, C and D are all tripled?
- Sketch a graph of the following expressions:
  - a vs b for ' $a \propto b$ '
  - xy vs z for  $xy \propto z$
  - Rate vs y for  $\text{Rate} \propto y^2z$

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# 12. UNCERTAINTY

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## LEARNING OUTCOME

Understand the concept of uncertainty, and be able to calculate uncertainty for different measurements and experiments.

## THEORETICAL OVERVIEW

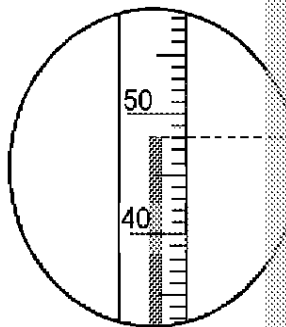
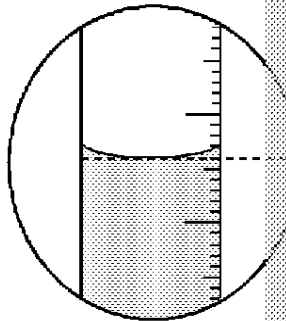
In an experiment, the readings made can never be exact. The true amount can always be higher or lower than the value given. The amount of 'inexactness' is called the uncertainty.

### Readings

A reading is one value recorded from an experiment. The value from a thermometer, a balance or on a measuring cylinder are all readings.

The uncertainty in a reading from many pieces of apparatus is half the distance between the lines. For example, if a thermometer has a reading every 1 °C (the distance between lines is 1 °C), then the uncertainty in a reading is 0.5 °C.

This means that for a temperature of 48 °C, the temperature could be as high as 48.5 °C or as low as 47.5 °C.



### Writing uncertainties

Uncertainties can be written in the form 'reading ± uncertainty'. This is called the absolute uncertainty.

For a temperature of 48 °C with an uncertainty of 0.5 °C, you would write:

$$48 \pm 0.5 \text{ °C}$$

NB  
lines

### Measurements

A measurement is the combination of two readings.

For example, you can measure a temperature change by taking two readings and subtracting the other.

Reading 1	Reading 2
Start temperature	End temperature
22 °C	36 °C

The actual values for readings 1 and 2 could be up to 0.5 °C above or below the value given.

This table shows the maximum and minimum temperature change that could have occurred.

	Start temperature	End temperature
Maximum change	21.5 °C	36.5 °C
Minimum change	22.5 °C	35.5 °C

Another way to show the result is like this:

Start temperature	End temperature
$22 \pm 0.5 \text{ °C}$	$36 \pm 0.5 \text{ °C}$

The absolute uncertainty in the measurement (the temperature change) is  $\pm 1$ . It is the sum of the uncertainties in the two readings added up.

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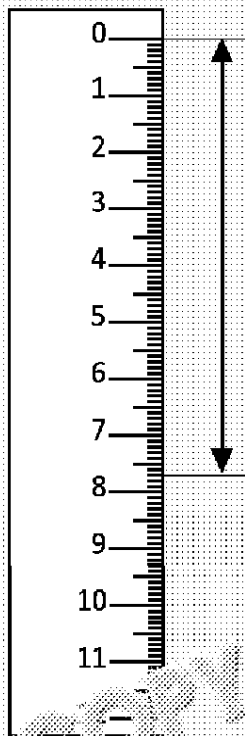
WORKED EXAMPLE

A value from a ruler is actually a measurement, not a reading, because there are two readings: one at the value, and one at zero.

$7.7 \pm 0.1 \text{ cm}$

or

$77 \pm 1 \text{ mm}$



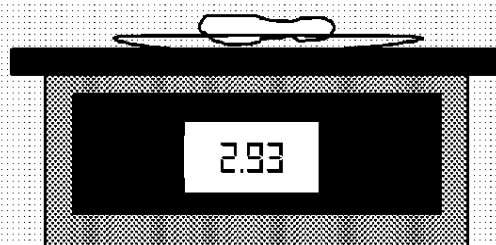
- When taking a reading, you must include:
- the smallest value
  - the uncertainty
  - the uncertainty

WORKED EXAMPLE

Two reactants are needed for a reaction and are weighed out.

What is the uncertainty in their combined mass?

The uncertainty in a digital balance is written on the balance. For the readings shown:



- The value for the combined mass in the reaction is  $2.93 + 1.67 = 4.60$
- The uncertainty for the total mass is  $2 \times 0.01 = 0.02 \text{ g}$
- The value for the mass used in the reaction is written as  $4.60 \pm 0.02 \text{ g}$

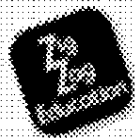
Repeated measurements

Repeating an experiment multiple times and finding the mean result is a method used to reduce uncertainty. Uncertainty is calculated differently for repeated experiments.

When finding the uncertainty from repeat experiments, you find the value of half the range is the difference between the highest value and the lowest value.

WORKED EXAMPLE

Find the mean of the following volumes, giving the uncertainty in your answer.



Reading	1	2	3
Volume (cm <sup>3</sup> )	21.2	21.3	21.3

The result for this data is:

Mean =  $\frac{21.2 + 21.3 + 21.3 + 21.2}{4}$

Half the range =  $\frac{21.3 - 21.2}{2}$

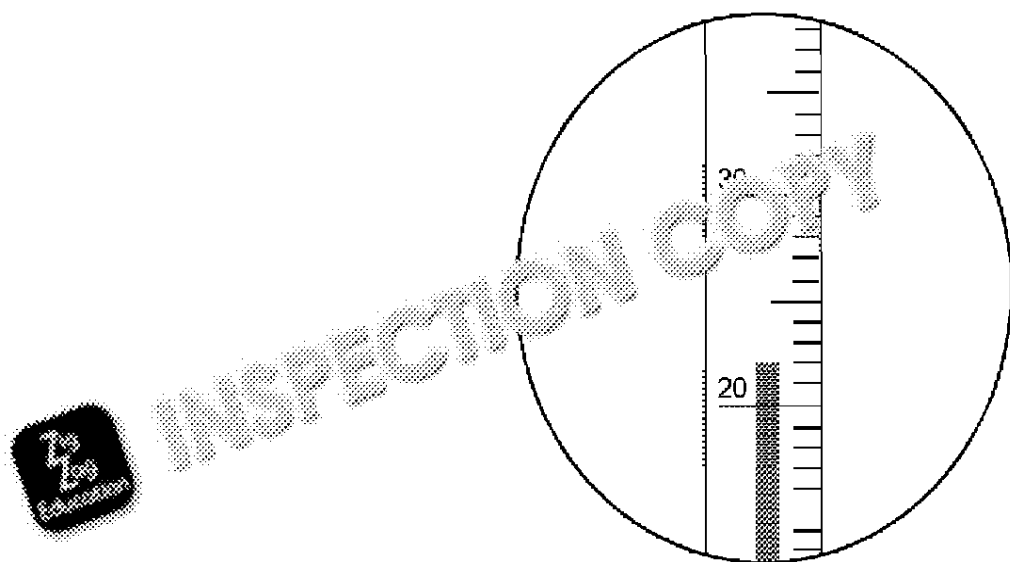
Mean =  $21.25 \pm 0.05 \text{ cm}^3$

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PRACTICE QUESTIONS

1. Write the value with the absolute uncertainty of the following measurements:
- a) A volume of 56.0 cm<sup>3</sup> measured in a measuring cylinder with 1 cm<sup>3</sup> markings
  - b) A 12.0 cm leaf measured with a ruler with 0.1 cm markings
  - c) A temperature change of 18 °C measured with a thermometer with 1 °C markings
  - d) A change in distance of 9.50 cm measured on a potometer with markings every 0.1 cm
  - e) A mass change given by a digital balance with an uncertainty of 0.01 g given a start mass as 5.25 g and end mass as 4.50 g
  - f) The temperature from the following thermometer:



2. a) Calculate the mean result from the following data:

Reading	1	2	3
Temperature (°C)	26.2	25.8	26.0

- b) Calculate the absolute uncertainty of the mean result using the repeated readings method.

3. Calculate the mean value with absolute uncertainty to one decimal place for the following data:

Reading	1	2	3
Volume (cm <sup>3</sup> )	78.2	69.5	74.1

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# 13. UNCERTAINTY

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## LEARNING OUTCOME

Calculate relative uncertainty and percentage change in measurements.

## THEORETICAL OVERVIEW

### Relative uncertainty

An uncertainty of  $\pm 0.1$  m is low for a value of a kilometre, but high for a value of a metre. **Relative uncertainty** in a measurement is useful as it compares the uncertainty to the value.

The relative uncertainty of a measurement is calculated as a percentage using:

$$\text{relative uncertainty} = \frac{\text{absolute uncertainty}}{\text{value}} \times 100\%$$

### WORKED EXAMPLE

For this value:  $21.25 \pm 0.05 \text{ cm}^3$

The relative uncertainty is:  $\frac{0.05}{21.25} \times 100\% = 0.235\%$

The **absolute uncertainty** of a value can be determined by working backwards if the relative uncertainty is known (simply by rearranging).

Remember that the absolute uncertainty should be the **sum** of the individual uncertainties. Involving two readings the absolute uncertainty is twice what it is for a single reading.

### Percentage change

**Percentage change** indicates an increase or a decrease in a quantity, as a percentage of the original value.

Percentage changes are calculated using:

$$\text{percentage change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

If the original value has **increased**, the percentage change will reflect this by being positive.

However, if the original value has **decreased**, the percentage change will be negative. Remember that  $-X\% = \text{decrease of } X\%$  by noticing this beforehand and putting a minus sign in front of the original value on the top line of the equation above.

### WORKED EXAMPLE 1

The rate of a reaction without an enzyme was recorded as  $0.82 \text{ cm}^3 \text{ min}^{-1}$ . After an enzyme was added to the reaction vessel, the rate increased to  $1.96 \text{ cm}^3 \text{ min}^{-1}$ .

Calculate the percentage increase in the rate of reaction (to three significant figures) when the enzyme concentration was added.

$$\text{Percentage change} = \frac{1.96 - 0.82}{0.82} \times 100\% = 139\% \text{ (3 s.f.)}$$

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WORKED EXAMPLE 2

If aerobic respiration can produce up to 38 molecules of ATP per glucose molecule, and anaerobic respiration can produce up to two molecules of ATP per glucose molecule, find the percentage decrease in the number of molecules of ATP produced when an organism is forced to respire anaerobically rather than aerobically.

Give your answer to two significant figures.

Percentage change is  $\frac{2 - 38}{38} \times 100 \% = -95 \%$

The percentage is negative, which indicates a decrease, so the percentage change is 95%.

PRACTICE QUESTIONS

1. Calculate the percentage uncertainty of the following readings to two significant figures.
- a)  $24.50 \pm 0.50 \text{ mm}$
  - b)  $37.90 \pm 0.10 \text{ }^\circ\text{C}$
  - c)  $23.40 \pm 0.15 \text{ cm}^3$
  - d)  $1.2 \pm 1.1 \text{ s}$
  - e)  $5.4 \pm 0.2 \text{ ml}$
  - f)  $0.150 \pm 0.050 \text{ dm}^3$

2. Calculate the mean percentage changes below to three significant figures, stating whether it is an increase or a decrease:

a)

	1	2	3
Initial reading (cm <sup>3</sup> )	50.00	50.00	50.00
Final reading (cm <sup>3</sup> )	66.05	68.15	65.90
Change (cm <sup>3</sup> )			
Change (%)			

b)

	1	2	3
Initial reading (°C)	42.1	44.7	42.9
Final reading (°C)	22.3	26.2	23.5
Change (°C)			
Change (%)			

c)

	1	2	3
Initial reading (mm)	20.5	16.5	18.0
Final reading (mm)	22.5	17.0	20.0
Change (mm)			
Change (%)			

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# 14. LOGARITHMS

## LEARNING OUTCOME

Understand how and why logarithms are used, and use them in calculations.

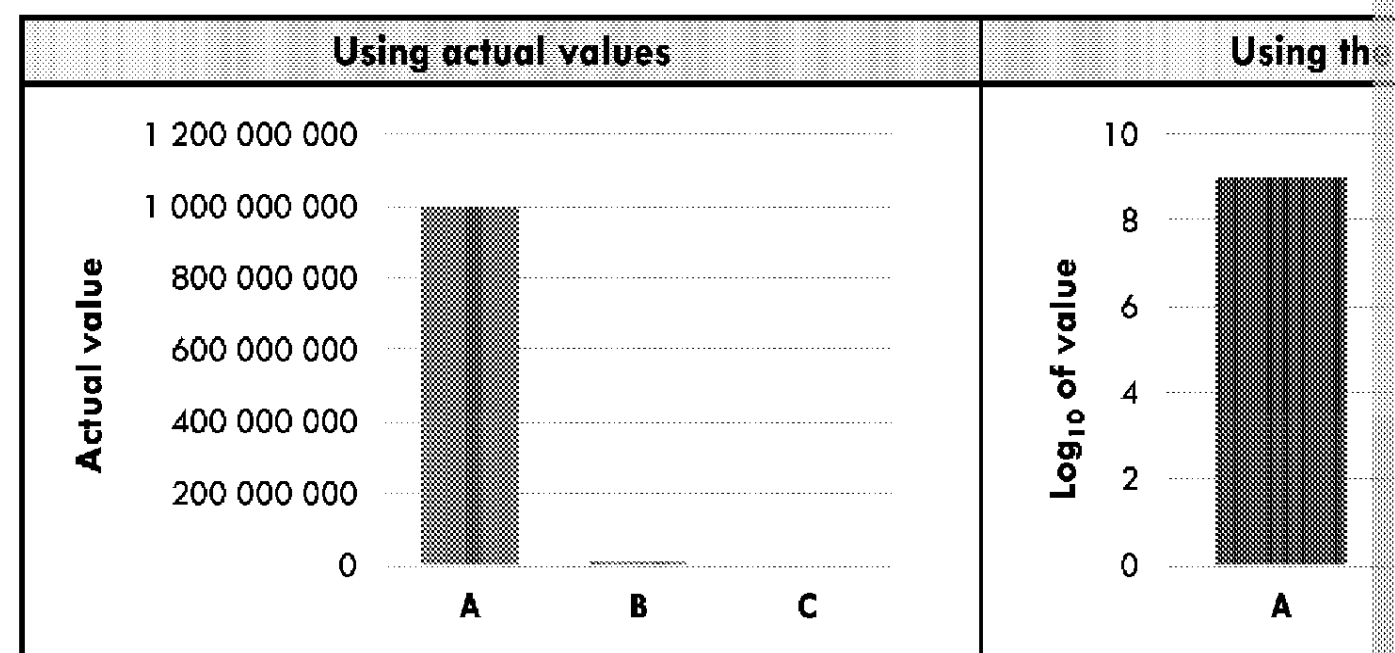
## THEORETICAL OVERVIEW

### Why logarithms are useful

Numbers can get very big or very small in Biology. Sometimes this makes it difficult to compare them.

This data is plotted on the graphs below. These two graphs are plotting the same data using logarithms to make comparing the numbers easier.

Number	Value	Log <sub>10</sub> of the value
A	1 000 000 000	9
B	1 000 000	6
C	1000	3



As you can see, the left-hand graph is not useful for comparing the sizes of B and C to A, making them unreadable. This means it isn't possible to see any patterns. By 'taking the log' and plotting those log values, we can see that A is much bigger than B and C, and B is much bigger than C.

### Mathematics of log<sub>10</sub>

#### Rule 1

The log<sub>10</sub> of a number is the power you need to raise 10 to in order to get that number.

$$\log_{10}(10^x) = x$$

For example:

$$\begin{aligned}\log_{10}(1000) &= \log_{10}(10^3) = 3 \\ \log_{10}(10\,000) &= \log_{10}(10^4) = 4\end{aligned}$$

#### Rule 2

This is also related. Raising 10 to the power of a log gives the number that is 'logged'. In other words, put 10 to the power of the logged number.

$$10^{\log(a)} = a$$

For example:

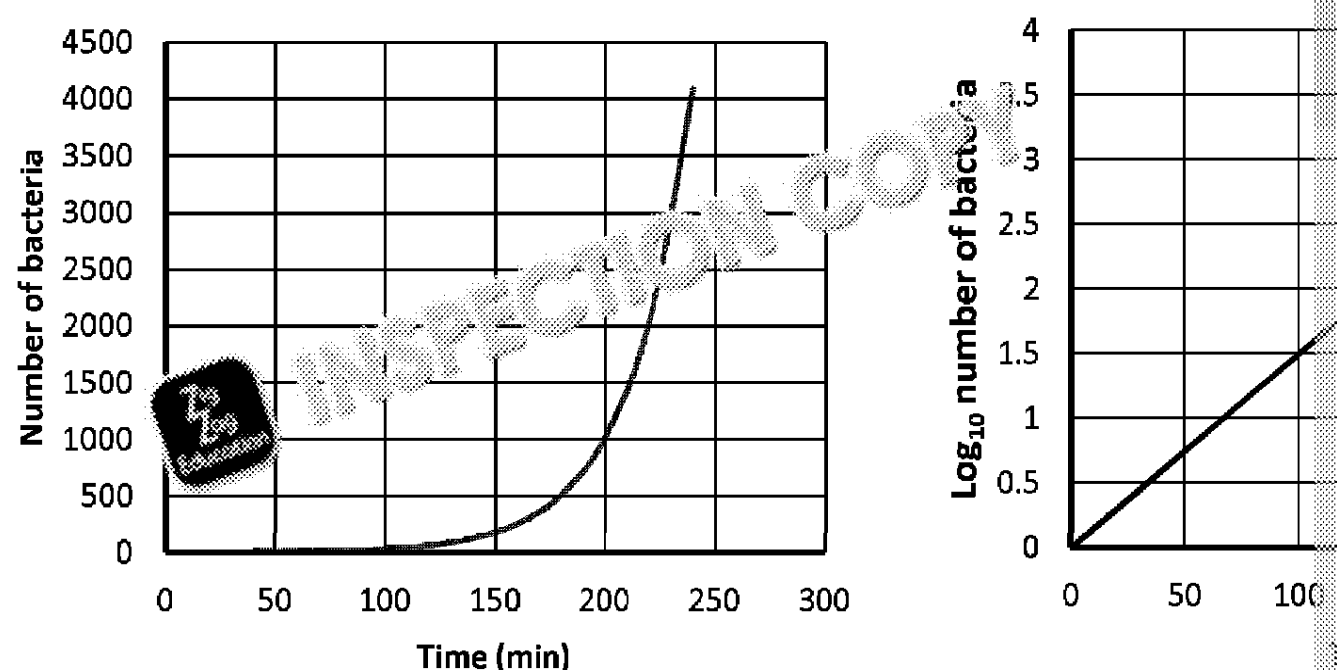
$$10^{\log(9)} = 9$$

Biology example: microbial growth

An example in Biology where numbers are hard to compare is in populations of microorganisms that divide very rapidly (usually about once every 20 minutes under ideal conditions), so that their population can grow very quickly. For example:

	After 1 hour	After 10 hours
Population size:	8	1 073 741 824

This data would be impossible to plot and read usefully using the actual numbers; the logarithm number transforms the data into a straight line graph:



From the second graph, it is possible to work out the number of bacteria at a particular log value (y):

Number of bacteria at x mins = 10<sup>y</sup>

WORKED EXAMPLES

1. Using the graph on the right above, estimate the number of bacteria after 150 minutes, giving your answer to two significant figures.

At  $x = 150$ ,  $y \sim 2.25$   
Number of bacteria =  $10^{2.25}$   
 $= 177.8$   
 $= 180$  (2 s.f.)

2. Using logarithms, calculate at what time (to three significant figures) the number of bacteria will be 5000.

$5000 = 10^y$   
 $\log_{10}(5000) = \log_{10}(10^y)$  ————— see rule 1 on p. 34 if  
 $3.70 = y$  ←  
From the graph, at  $y = 3.70$ ,  $x = 247.5$  so  $y = 0.015x$  ————— work out the increase  
in  $y$ , e.g. using the previous  
 $x = \frac{3.70 - 0}{0.015} = 247$  minutes (3 s.f.) ————— chapters on graphs if

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Mathematics of natural logs, ln

Many relationships in nature have an **exponential** pattern. This is represented mathematically by the equation:

ln, or log<sub>e</sub>, is an important type of logarithm because of the relationship:

$$\ln(e^x) = x$$

It is particularly helpful to remember that this also means that **ln(e) = 1**.

WORKED EXAMPLE

The value of  $\ln A$  in a reaction was found by a graphical method to be 6.2.

Find the value of  $A$  for this reaction.

$A = e^{\ln A}$   
 $= e^{6.2}$   
 $= 492.7$

PRACTICE QUESTIONS

1. Use a calculator to find the following values to four significant figures:
- a)  $10^{3.1}$
  - b)  $\log(140\,000)$
  - c)  $\log(0.0005)$
  - d)  $10^{\log 5}$
  - e)  $\log_{10} 5$
  - f)  $10^{\log(5)}$
2. Calculate the value of  $x$  to four significant figures:
- a)  $\log(x) = 4$
  - b)  $\log(x) = 6.9$
  - c)  $10^x = 0.002$
  - d)  $10^x = 7.45$
  - e)  $10^{(x+2)} = 0.00567$
3. The table shows the number of flies in the first 60 days of a breeding experiment.
- a) Use log rules to calculate the missing values.

Days since start	0	10	20	30	40
Number of flies	4	46	230		900
$\log_{10}(\text{number of flies})$	0.6021			2.790	2.954

- b) Explain why using logs is useful in this situation.

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# 15. UNDERSTANDING SIMPLE P

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## LEARNING OUTCOME

Understand simple probability, and use it in the context of inheritance.

## THEORETICAL OVERVIEW

The probability of an event can range from impossible (0) to certain (1).

When events are **fair** (all outcomes are equally likely), **random** (impossible to predict), and **independent** (not affected by other events), the probability of a **repeated** event occurring is:

probability of an event occurring =  $\frac{\text{number of ways this particular outcome can occur}}{\text{total number of outcomes}}$

where N is the number of times the event is repeated.

For example, when rolling a normal six-sided dice twice, the probability of getting

### Independent events

Coin flips and dice rolls, among other things, are considered to be **independent** events. The outcome of one event has no effect on the other(s).

If you flip a coin and then flip it again, the outcome of the first flip has no impact on the second. You will have the same chance of getting a head on the second flip as you had getting a head on the first flip.

The probability of the outcome of an event X can be written as P(X), and the combined probability of two **independent events** occurring simultaneously can be determined by multiplying the probabilities of each event occurring.

If X and Y are both **independent events** then the probability of event X and event Y occurring simultaneously is:

$$P(X \text{ and } Y) = P(X) \times P(Y)$$

In the context of Biology, probability can be used to determine the chance of seeing a particular phenotype in a genetic cross. For example, if eye colour is determined by a single gene in a species, the allele for brown eyes is the dominant B and the allele for blue eyes is the recessive b, then each individual can have one of the following genotypes: BB, Bb or bb.

If a brown-eyed parent with the genotype Bb and a blue-eyed parent with the genotype bb have children, their offspring will have one of the possible genotypes shown in the Punnett square below.

		Parent Bb	
		B	b
Parent bb	b	Bb (brown eyes)	bb (blue eyes)
	b	Bb (brown eyes)	bb (blue eyes)

As you can see, two of the four possible resulting phenotypes are brown eyes, so the probability of a brown-eyed offspring is 2/4 or 1/2. Their second offspring will have the same chance of having brown eyes as the first because the fertilisation of gametes happens independently for each offspring. This means that, using the equation above, the probability of two parents having two blue-eyed offspring is  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ .

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## WORKED EXAMPLE

What is the probability of the same parents having one blue-eyed and one brown-eyed offspring?

It is important to remember that there could be two cases for this as the question asks for one blue-eyed or brown-eyed offspring, just that there should be one of each.

Therefore:

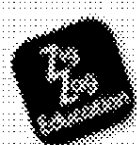
The first offspring is blue-eyed and the second offspring is brown-eyed.

OR

The first offspring is brown-eyed and the second offspring is blue-eyed.

Both of these options would satisfy the condition stated in the question.

$$P(bb \text{ and } Bb) \text{ OR } P(Bb \text{ and } bb) = (P(bb) \times P(Bb)) + (P(Bb) \times P(bb))$$



$$P(bb \text{ and } Bb) \text{ OR } P(Bb \text{ and } bb) = \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{2}$$

## Mutually exclusive events

Two events are **mutually exclusive** if it is **impossible** for them to happen at the same time. For example, a coin can't land on heads and tails simultaneously.

The combined probability of **multiple mutually exclusive events** occurring can be found by adding their individual probabilities together:

If X and Y are **mutually exclusive** then the probability of event X **OR** event Y happening is:

$$P(X \text{ or } Y) = P(X) + P(Y)$$

A tree diagram can be used as a method of determining how to combine the probabilities of **mutually exclusive** outcomes, as discussed above.

Take our eye colour example.

The rules for tree diagrams dictate that you:

- add vertically across the branches (mutually exclusive outcomes)

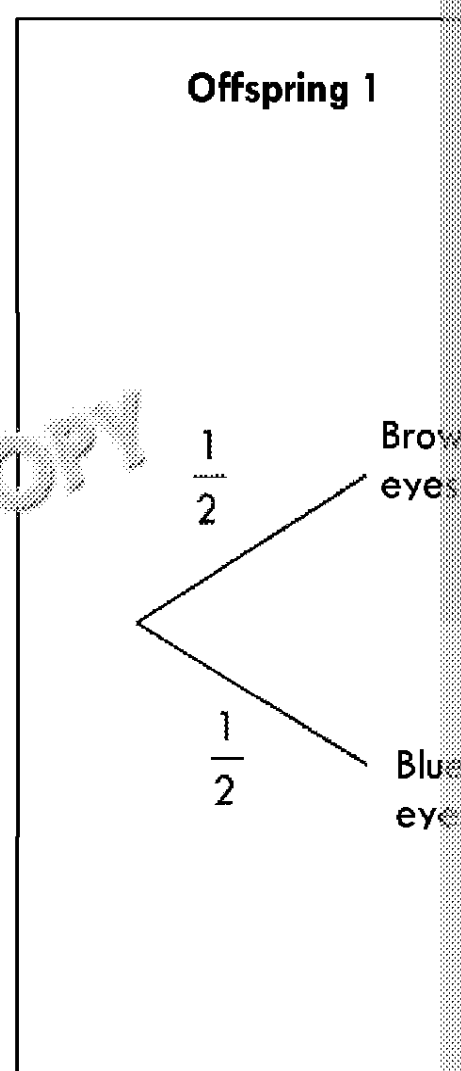
**NB** This is what we had before; the probability that X or Y will occur is  $P(X \text{ or } Y) = P(X) + P(Y)$

- multiply horizontally across the branches (independent outcomes)

**NB** This is what we had before; the probability that X and Y will occur is  $P(X \text{ and } Y) = P(X) \times P(Y)$

Additionally, if X and Y are the **only** possibilities for a certain event, then  $P(X) + P(Y) = 1$ .

You will revisit this shortly.



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# WORKED EXAMPLE

What is the probability of the two parents having one blue-eyed offspring, and the

$$P(\text{blue and brown}) = \left( \frac{1}{2} \times \frac{1}{2} \right) = \frac{1}{4}$$

# WORKED EXAMPLE

In a certain species, the attachment of an individual's earlobes is determined by a single gene. The allele for attached earlobes is A, and the allele for unattached earlobes is a. The A allele is dominant over the a allele.

The three genotypes AA, Aa and aa are present in the population in the ratio 3:2:1.

Find the relative probability of the a allele in the population.

Use a tree diagram to predict the probability of the Aa genotype in the next generation.

Use the same tree diagram to predict the relative probability of the a allele in the next generation.

- a) For six individuals, the alleles in the gene pool are AA, AA, AA, Aa, Aa, Aa. This is a total of eight A alleles and four a alleles, so the relative probability of the a allele is  $\frac{4}{12} = \frac{1}{3}$ .

- b) We need to find the probability of Aa + aA. Allele from parent 1  
We do this by multiplying along the branches and adding between the branches.

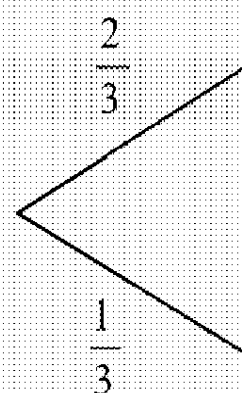
$$P(Aa) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

$$P(aA) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

$$\text{So } P(Aa \text{ or } aA) = P(Aa) + P(aA)$$

$$= \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$$

- c) To find the relative probability of a, we also need:  
 $P(aa) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$   
Then we add this to half of each of  $P(Aa)$  and  $P(aA)$  to get  $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9}$  or  $\frac{1}{3}$  (which is the same as in the original population)



# The Hardy–Weinberg principle

For any gene which comes in two forms, A and a, the Hardy–Weinberg equations relate the relative frequency of particular alleles or genotypes. By letting the relative frequency of the allele A be p, and the relative frequency of the allele a be q, you get the equations:

$$p + q = 1$$

$$p^2 + 2pq + q^2 = 1$$

The first equation comes from the principle above that the gene can only be in the form A or a, so their combined probability must **add to 1**. The second equation comes from squaring the first equation and represents the fact that the probability of both homozygous genotypes is the probability squared, whereas the probability of the heterozygous genotype is **twice** the product of the probabilities of the two alleles.

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**WORKED EXAMPLE**

The colour of a plant's petals is determined by a single gene. The dominant Y allele is for yellow petals and the recessive y allele is for red petals. The Y allele occurs in the population of plants at a relative frequency of 0.7.

Use the Hardy–Weinberg equations to find the probability of each of the genotypes in the population.

Let the relative frequency of Y be  $p$ , and the relative frequency of y be  $q$ .

$p = 0.7$ , and  $p + q = 1$ , so  $0.7 + q = 1$

Therefore,  $q = 0.3$

In the second equation,  $p^2 + 2pq + q^2 = 1$ :

$p^2$  represents the probability of the genotype YY

$q^2$  represents the probability of the genotype yy

$2pq$  represents the probability of the genotypes Yy or yY

$p^2 = 0.7^2 = 0.49$ , so  $P(YY) = 0.49$

$q^2 = 0.3^2 = 0.09$ , so  $P(yy) = 0.09$

$2pq = 2 \times 0.7 \times 0.3 = 0.42$ , so  $P(Yy \text{ or } yY) = 0.42$

We can confirm that this looks correct by the fact that  $0.49 + 0.09 + 0.42 = 1$

**PRACTICE QUESTIONS**

- What is the probability of an event that is certain to happen?
- In a certain large population, an individual's chance of being born with blonde hair is 0.2. What is the probability that two randomly chosen babies both have blonde hair?
- In a different population, an individual's face shape is determined by a single gene. The R allele is dominant and is responsible for a person having a round face. The r allele is recessive and results in a person having a long face.  
If two parents with the genotype Rr mate. Use a Punnett square to find the probability that their child will have a round face.
- Wing colour in a population of dragonflies is determined by a single gene. The B allele is dominant and results in blue wings, and the recessive b allele results in colourless wings. The three genotypes are present in the population in the ratio 3 : 1 : 1.  
Use a tree diagram to predict the probability of the Bb genotype in the next generation.
- The probability of the homozygous recessive genotype xx in a population is 0.04. Use the Hardy–Weinberg equations to find the probability of the heterozygous genotype Xx in the same population.

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# 16. SAMPLING, FREQUENCY AND HISTOGRAMS

## LEARNING OUTCOME

Understand the principles of sampling, and construct and interpret frequency diagrams

## THEORETICAL OVERVIEW

Sampling is a way of collecting information about a population **without** having to check every individual. It is important to remember the purpose of your sample in deciding what to sample.

### Random sampling

**Random sampling** can be used for estimating the total population size of a species in an area. Unlike **opportunistic sampling**, in which you sample the individuals you come across, random sampling is **unbiased** as it uses a random number generator to create a list of sampling coordinates to use.

For example, you could place tape measures along the edges of a 10 m × 10 m area to use as references for coordinates, and then generate eight sampling points by finding eight pairs of random numbers from 0 to 9 to use as the bottom-leftmost corner of a quadrat:

(8, 1) (0, 8) (2, 6) (4, 3) (9, 1) (7, 8) (0, 5) (1, 0)

### Non-random sampling

For a specific question of the form ‘How does X affect Y?’ you should take a **non-random** sample.

#### Systematic sampling

This is when you take samples at set intervals to answer a question about a change in a population. For example, to answer the question ‘Does the distance from the river affect the distribution of plant species seen?’ you could take samples every 1 m in a line from the river’s edge.

#### Stratified sampling

This type of sampling divides a population into different categories and samples from each category in proportion to its size. For example, if there were two men for every one woman in a room of 30 people, to get a representative sample you should sample 10 men and five women because 10 : 5 is in the ratio 2 : 1. This method is more accurate than random sampling but ensures that individuals are neither over- nor under-represented.

### Recording results

You should keep a record of your results in a rough table when sampling. Tables should have a clear title, where column headings are labelled with units and (where relevant) data should be entered to the same number of significant figures or decimal places throughout.

The **independent variable** (the one which you are changing) should be placed in the first column, and the **dependent variable** (the one which you are measuring) should go in the second column.

Table 1

Species	Frequency
B. ...	...
T. ...	...
R. ...	...
T. ...	...
R. ...	...

### Simpson's index of diversity

You can compare biodiversity in different areas on a scale of 0 to 1 using **Simpson's index of diversity**.

$$D = 1 - \left( \sum \left( \frac{n}{N} \right)^2 \right)$$

where  $n$  is the number of individuals of a certain species, and  $N$  is the **total** number of individuals.

Simpson's index of diversity takes into account **species richness** and **species evenness** a habitat has, and whether they are equal in number of individuals or whether there is a more equal distribution of species will have a value closer to 1.

#### WORKED EXAMPLE

Calculate Simpson's index of diversity for the data in the table.

Total number of individuals,  $N = 6 + 9 + 7 + 13 + 2 = 65$

$$D = 1 - \left( \left( \frac{6}{65} \right)^2 + \left( \frac{9}{65} \right)^2 + \left( \frac{7}{65} \right)^2 + \left( \frac{13}{65} \right)^2 + \left( \frac{2}{65} \right)^2 \right)$$
$$= 1 - 0.345...$$
$$= 0.655 \text{ (3 s.f.)}$$

Species name
Daisy
Dandelion
Butterfly
Clover
Rose

### Producing diagrams

Like tables, diagrams should also have a clear title, as well as axes labelled with units and values that are easy to plot. The type of diagram you produce will depend upon the type of data.

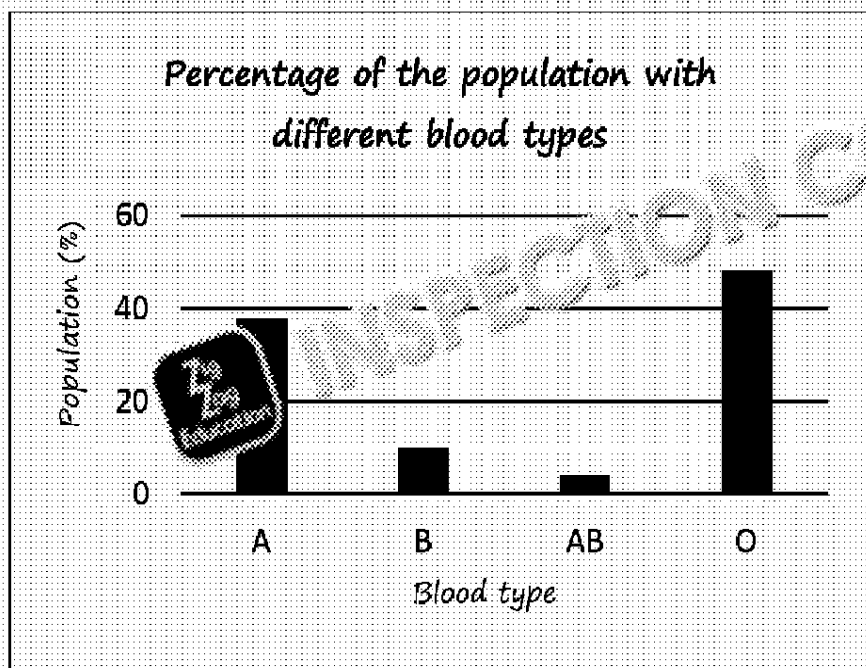
#### Bar charts

These are the type of charts you are probably most familiar with. They are used for qualitative data (data which fits into categories, or quantitative data that can only take integer values), e.g. number of siblings.

The bars are all the same width and should not touch each other.

#### WORKED EXAMPLE

Draw a bar chart to represent the data in the table to the right.



Blood type
A
B
AB
O

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# Histograms

For **continuous** data (values that can take any number within a range, e.g. height), used with touching bars.

Consider a group of 30 students whose heights are measured to the nearest centimetre and recorded in the table to the right.

The end points in the classes are important to consider. If the first two groups were simply written as 135–150 and 150–160, it would be unclear which group a student with a height of 150 cm should fall into. When plotting a histogram from this table, you also need to consider that the values were **rounded** to the nearest centimetre. This means that the end points for each class are actually half a centimetre different in each case, and it is these which should be plotted on the histogram.

Height
135
150
160
170
180

You may also notice that not all the classes are the same width here. Unequal widths mean that the **area** is proportional to frequency rather than the height. To ensure that this is the case, the histogram is always plotted as **frequency density**. This is calculated as:

$$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$$

In the example above, the first group would be represented by a bar of width 15 and height 0.2. The fifth group would be represented by a bar of width 5 and height  $3 \div 5 = 0.6$ . Both have a frequency of 3 and have the same area.

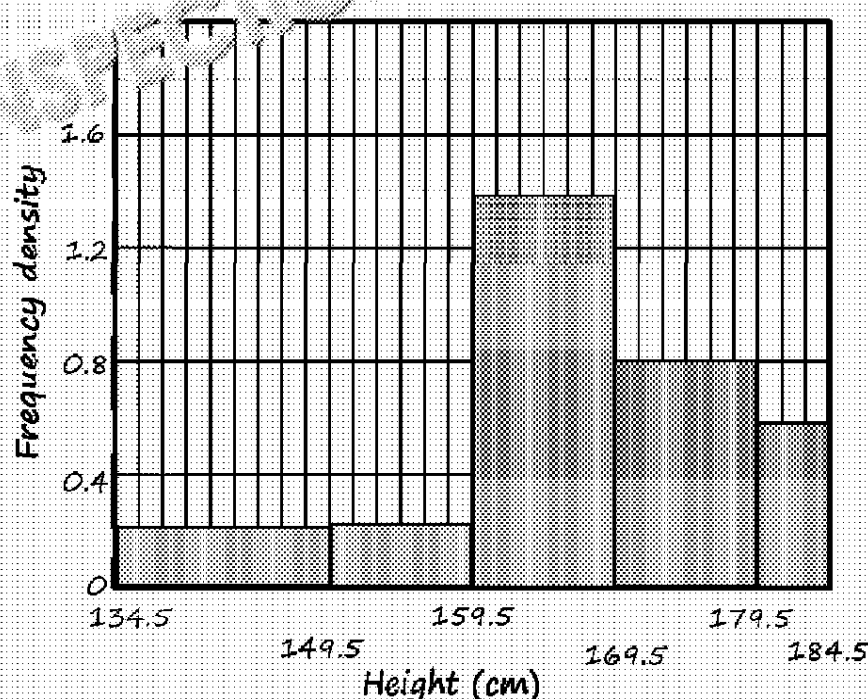
When plotting a histogram, it is usually helpful to rewrite the table with the correct columns for class width and frequency density.

## WORKED EXAMPLE

Draw a histogram to represent the data in the table above.

Rewrite the table as:

Height (nearest cm)	Frequency	Class width	Frequency density
$134.5 \leq h < 149.5$	3	15	$3 \div 15 = 0.2$
$149.5 \leq h < 159.5$	2	10	$2 \div 10 = 0.2$
$159.5 \leq h < 169.5$	14	10	$14 \div 10 = 1.4$
$169.5 \leq h < 179.5$	8	10	$8 \div 10 = 0.8$
$179.5 \leq h < 184.5$	3	5	$3 \div 5 = 0.6$



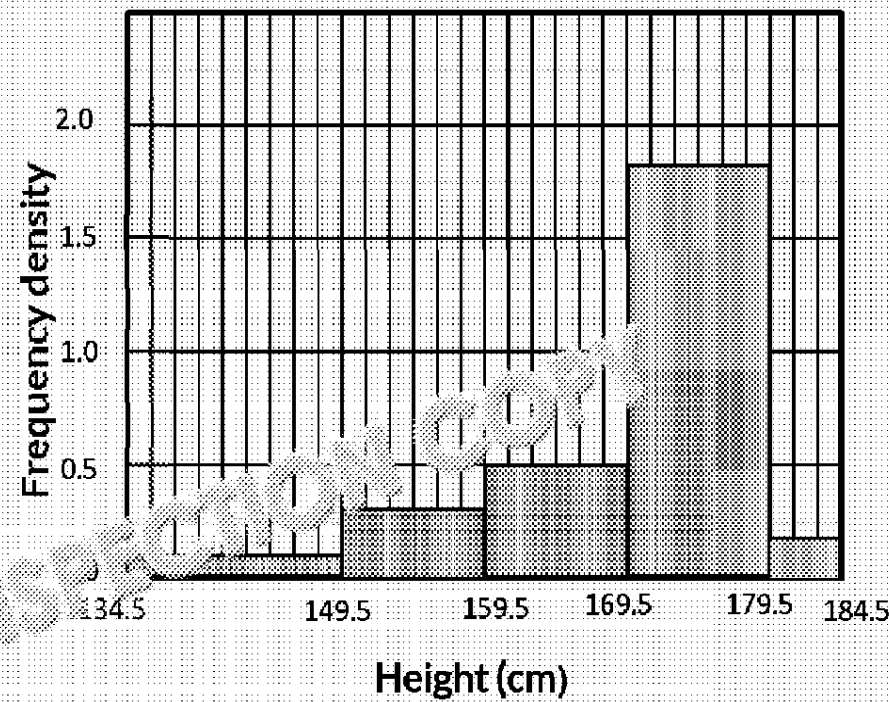
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Once you know the relationship between frequency density, class width and frequency, you can find the frequency of classes by rearranging the equation and working backwards.

WORKED EXAMPLE

Calculate the number of people whose height falls into the category  $159.5\text{ cm} \leq h < 169.5\text{ cm}$ .



Frequency density for the class is 0.5, and class width is 10  
Rearrange the equation frequency density = frequency ÷ class width to:  
frequency = frequency density × class width  
So, frequency =  $0.5 \times 10 = 5$  people

PRACTICE QUESTIONS

- 1. Sharon is taking a sample of pea plants from among 450 tall varieties and 150 short varieties. She wants to sample 20 plants in total. How many tall plants and how many short plants should she take for her stratified sample?
- 2. Calculate Simpson's index of diversity for the data in the following table using the formula:

$$D = 1 - \left( \sum \left( \frac{n}{N} \right)^2 \right)$$

Species name	Number of individuals
Woodlouse	22
Worm	9
Ant	74
Spider	1

- 3. Draw a histogram to represent the data in the table below:

Age (in next year)	Frequency
$16 \leq a < 21$	45
$21 \leq a < 25$	26
$25 \leq a < 35$	20
$35 \leq a < 55$	30
$55 \leq a < 85$	6

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# 17. CORRELATION

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## LEARNING OUTCOME

Understand and identify correlation using scatter diagrams.

## THEORETICAL OVERVIEW

Correlation describes the relationship between two variables. It can come in different forms and can be demonstrated using scatter diagrams.

### Linear correlation

The easiest type of correlation to spot is **linear correlation**, in which the points roughly follow a straight line of best fit.

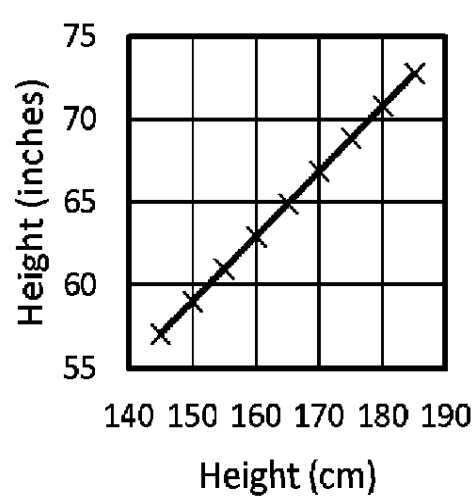
#### Positive and negative correlation

A line with a **positive** gradient going up from left to right indicates **positive** correlation (i.e. an increase in  $x$  correlates with an increase in  $y$ ). A line with a **negative** gradient going down from left to right indicates **negative** correlation (i.e. an increase in  $x$  correlates with a decrease in  $y$ ).

#### Strength of correlation

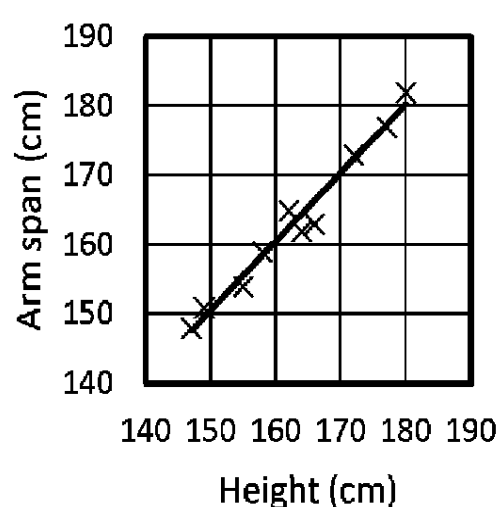
Strength of correlation has nothing to do with the gradient of the line, but how close the points are to the line. If all points are exactly on the line, this is **perfect** correlation, regardless of the angle which the line takes. Points that are very close to the line demonstrate **strong** correlation, and those that vary highly from the line demonstrate **weak** correlation. You may decide that some variables show **no correlation** to each other if there is no discernible pattern.

Human height in centimetres and inches



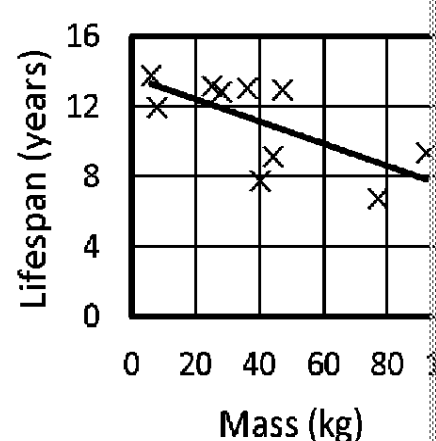
Perfect positive correlation

Human height and arm span



Strong positive correlation

Dog mass and average lifespan



Weak negative correlation

### Quadratic correlation

Other forms of correlation are possible, such as **quadratic** correlation, but you will rarely have to identify these in your Biology course.

Quadratic correlation shows a changing relationship between two variables, where after a point the correlation changes from positive to negative or vice versa. For example, in the graph to the right, there is an **optimum** sodium intake per day with the gradient of the line being positive before it is reached and negative afterwards.

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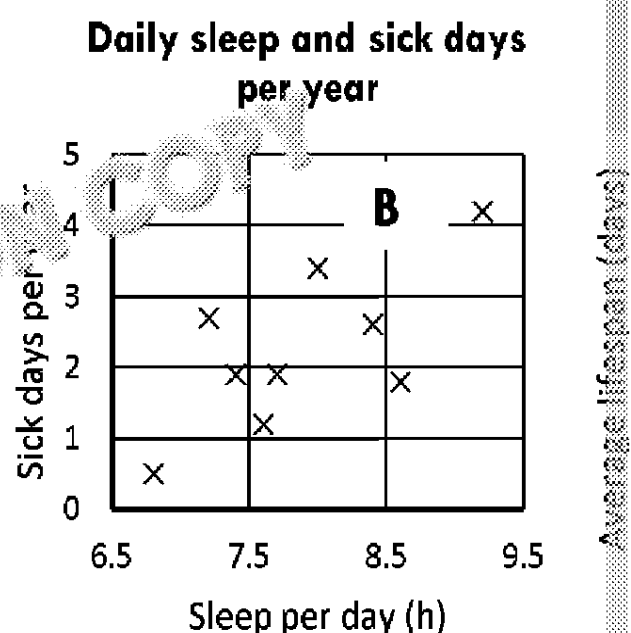
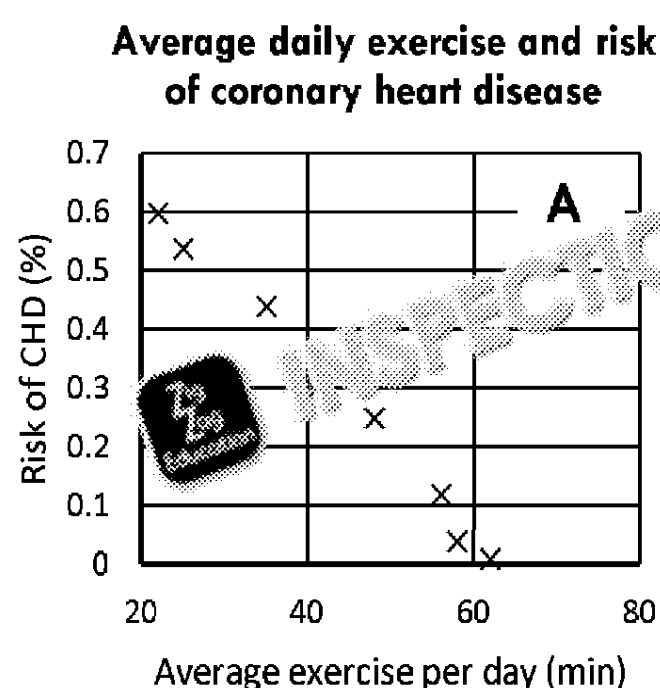


## Correlation and causation

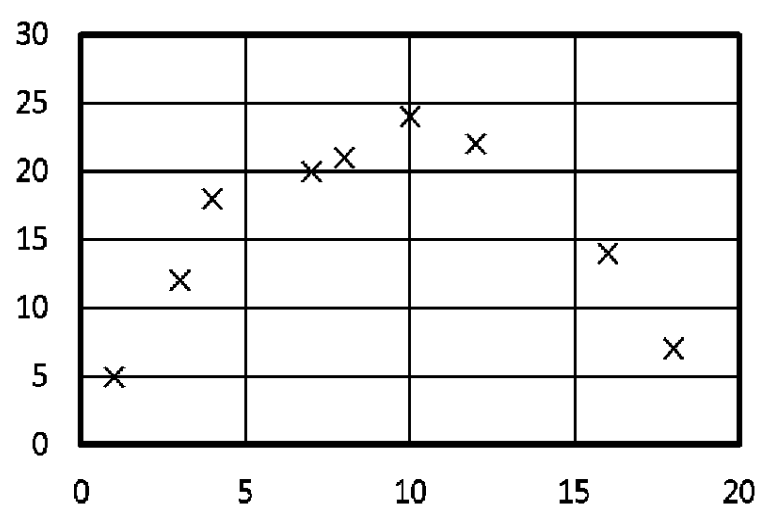
An important thing to remember is that **correlation does not imply causation**. This means that if B increases when A increases, it does not mean that A causes B. It also does not mean that we should hypothesise that the two are related, but it could be that a third variable, C, is involved or just that the correlation is a coincidence.

## PRACTICE QUESTIONS

1. For the following graphs, state which, if any, show:



- Perfect correlation
  - Strong negative correlation
  - No correlation
  - Weak positive correlation
  - That the independent variable causes a change in the dependent variable
2. Suggest what sort of correlation is shown in the following graph:



# 18. STANDARD DEVIATION AND RANGE

## LEARNING OUTCOME

Be comfortable with measures of dispersion, including standard deviation and range.

## THEORETICAL OVERVIEW

### Range

The range of a data set shows the values over which the data spreads. It is calculated by subtracting the smallest value from the largest value.

### WORKED EXAMPLE

The range of the data set 7, 9, 2, 11, 3, 5, 4 is:

$$11 - 2 = 9$$



### Standard deviation

Standard deviation ( $s$ ) measures the spread of data about the mean.

A small standard deviation means that most points do not deviate much from the mean. A large standard deviation indicates that the data encompasses a greater variety of values.

Take a look at the two sets of data below:

99, 100, 102, 99, 98, 101, 100, 99, 103, 99

86, 103, 88, 95, 113, 100, 92, 97, 105, 94

Both sets of data have a mean of 100, but it is obvious that the data on the right is more spread out from the mean.

This is represented by their standard deviations. The data on the left has  $s = 1.56$ , while the data on the right has  $s = 14.6$ .

Standard deviation measures the **dispersion** of a data set, but it is less affected by outliers than the range.

For example, the data set 11, 12, 13, 14, 15 has range  $15 - 11 = 4$  and  $s = 1.41$ . If the outlier 25 is added, the range increases by 10 to 15, but the standard deviation only increases by just over 3 to  $s = 4.65$ .

### Calculating standard deviation

Standard deviation is calculated using the formula:

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

where  $\bar{x}$  is the mean value and  $n$  is the number of data points.

Squaring the difference between each value and the mean removes the discrepancy of whether the difference was positive or negative, and accentuates the effect of each difference (by increasing numbers greater than 1 and decreasing numbers less than 1).

Dividing the sum of these values by the number of points minus 1 finds the average. Taking the square root returns the units to the correct value by undoing the squaring that happened in the previous step.

$n - 1$

Note that you divide by  $n - 1$  rather than by  $n$  when you are taking a sample. This is known as Bessel's correction. If you were ever sampling a large population then you could use  $n$ .

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## WORKED EXAMPLE

Find the standard deviation of the following data:

49, 50, 47, 50, 51, 45, 62, 46

The mean is  $\bar{x} = \frac{49 + 50 + 47 + 50 + 51 + 45 + 62 + 46}{8} = 50$

$n = 8$  so  $n - 1 = 7$

Now find the difference between each data point and the mean:

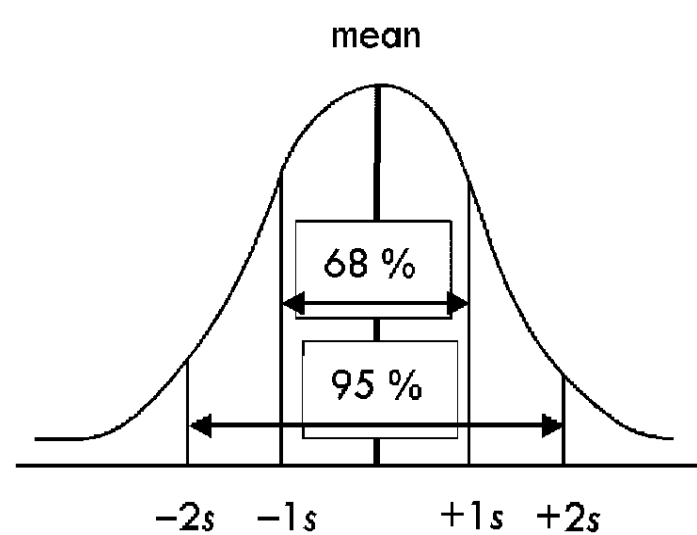
-1, 0, -3, 0, +1, -5, +12, -4

The differences squared are:

1, 0, 9, 0, 1, 25, 144, 16

So  $s = \sqrt{\frac{1 + 0 + 9 + 0 + 1 + 25 + 144 + 16}{7}} = 5.29$  (3 s.f.)

For data that is **normally distributed** (i.e. symmetrical about the mean), 68 % of the deviation of the mean, or 1 standard deviation, and a majority (95 %) is within two standard deviations.



## PRACTICE QUESTIONS

- For the following data, find the range:
  - 0.7, 1.6, 0.2, 1.4, 0.5, 0.8, 0.8, 1.0
  - 24, 98, 77, 17, 54, 101, 33, 65
- For the following data, find the standard deviation to two decimal places:
  - 0.08, 0.12, 0.24, 0.31, 0.22, 0.07, 0.15, 0.13
  - 526, 523, 517, 530, 524, 521, 520



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# 19. STATISTICAL TESTS

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## LEARNING OUTCOME

Be comfortable with choosing and performing statistical tests.

## THEORETICAL OVERVIEW

After collecting data from a sample, you may decide to perform a **statistical test**.

This chapter will focus on four types of statistical test that you can use to tell whether a result is **significant**, i.e. whether your results are more likely to have occurred because something is **actually** happening in the population, rather than being **simply** due to **chance** alone.

### Hypotheses

All statistical tests require **two** hypotheses.

- Null hypothesis,  $H_0$ :** There is no statistical significance in the results. Any difference between the two groups is due to **chance**.
- Alternative hypothesis,  $H_1$ :** Framed around the question you are asking, e.g. 'Individuals from group A are **different** from individuals from group B.'

After performing the test, you will **reject** one of these hypotheses based on the evidence. If the test result exceeds the **critical value** or not.

Note that neither of the hypotheses can be **accepted** because experimental evidence is never **perfect** and theory on its own.

### Critical values

#### Degrees of freedom

You will first need to know the **degrees of freedom** you are using for the test you are performing. It is related to the sample size, and is usually  $n - 1$ , where  $n$  is the sample size. So if you collect eight data points, the degrees of freedom will be 7 for most tests.

#### Significance level

A significance level is used to describe how **confident** you can be that your results are significant. The most common significance level used is  $p = 0.05$ .

If your results are significant at the 0.05 significance level, this means that there is a 5% chance that the results happened due to random chance. In other words, the confidence level is 1 - 0.05 = 0.95.

If you are not directly given the critical value for the test you are performing, you will be able to look it up in a critical value table. Simply find the correct row and column based on the significance level and degrees of freedom you are using, and use the value at their intersection.

If the result of your test is **greater than** the critical value, the result is considered **significant**, and you reject the null hypothesis. Alternatively, if the result of your test is **less than** the critical value, you fail to reject the null hypothesis and instead reject the alternative hypothesis.

## WORKED EXAMPLE

Determine whether to reject the null hypothesis for a result of 2.92 at the 0.05 significance level with five degrees of freedom.

From the table above, the critical value is 2.57.

$2.92 > 2.57$ , so the result is significant at the 0.05 significance level because the test result is greater than the critical value. Therefore, the null hypothesis should be rejected.

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### Chi-squared test

The chi-squared test is used to see whether **observed** frequencies in a population **match** expected frequencies. You might use this to check ratios in genetic crosses.

The test is carried out using the following formula:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

where  $f_o$  represents an observed frequency and  $f_e$  represents an expected frequency.

You generally need each frequency to be greater than 5 for this test to work with any accuracy.

Critical

De

#### WORKED EXAMPLE

In a genetic cross involving pea plants, offspring are expected to have black fur, brown fur or white fur in a ratio 1 : 2 : 1. 40 offspring were sampled.

Six have black fur, 27 have brown fur and seven have white fur.

Perform a chi-squared test at the 0.05 significance level using the table above and

$H_0$ : There is no significant difference between the observed and expected frequencies

$H_1$ : There is a significant difference between the observed and expected frequencies

The expected frequencies are 10 black : 20 brown : 10 white.

Calculate the differences between observed and expected values, square each value and divide by the expected value.

Fur colour	Expected ( $f_e$ )	Observed ( $f_o$ )	$f_e - f_o$	$(f_e - f_o)^2 / f_e$
Black	10	6	4	1.60
Brown	20	27	-7	2.45
White	10	7	3	0.90

So  $\chi^2 = 1.60 + 2.45 + 0.90 = 4.95$

The significance level is 0.05 and the degrees of freedom is  $3 - 1 = 2$ , so the critical value is 5.99.  $4.95 < 5.99$ , so we fail to reject the null hypothesis.

### Paired t-test

A paired t-test is used for paired data that has some repeated measure, e.g. data from the same individual at two different times. It generally compares a continuous variable before and after a change for each individual to determine the effect of a drug on a particular aspect of an individual's health.

For this test, you have to assume that the values you are measuring are **normally distributed**.

The test finds the mean difference between the before and after values for each individual and uses the standard deviation of these differences to calculate a result based on the following formula:

$$t = \frac{\bar{d}\sqrt{n}}{s_d}$$

where  $\bar{d}$  is the mean difference,  $n$  is the number of pairs of data points, and  $s_d$  is the standard deviation of the differences.

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WORKED EXAMPLE

A new drug, Acclimitase, is designed to lower the heart rate of a patient who takes it. The data below shows the resting heart rate (RHR) of 12 patients before taking the drug every day for two weeks.

	001	002	003	004	005	006	007	008
RHR before (bpm)	130	142	129	147	118	123	127	128
RHR after (bpm)	118	125	126	142	120	121	116	123

Perform a paired t-test at the 0.05 significance level to investigate the effect of the drug. The critical value for 11 degrees of freedom is 1.80.

$H_0$ : There is no significant difference between the resting heart rate of patients before and after taking drug Acclimitase.

$H_1$ : The resting heart rate of patients is lower after taking the drug Acclimitase for two weeks. Calculate the difference between RHR before and after, and find the mean and standard deviation of the differences.

	001	002	003	004	005	006	007	008
Difference in RHR (d)	12	17	3	5	-2	2	11	5

$\bar{d} = 90 \div 12 = 7.5$

$s_d = \sqrt{\frac{359}{12}} = 5.47$

So  $t = \frac{7.5 \times \sqrt{12}}{5.47} = 4.75$

The critical value at  $p = 0.05$  and 11 degrees of freedom is 1.80.  
 $4.75 > 1.80$ , so we reject the null hypothesis.

Student's t-test

A Student's t-test (sometimes called an **unpaired t-test**) is similar to the test above, but it compares the mean of a continuous variable in two **different, independent groups**. This type of test is used to determine whether there are inherent differences in two populations of similar types. You have to assume that the data you are testing is **normally distributed** and that the standard deviation in each group is the same.

This test uses the following formula:

$$t = \frac{|\bar{x}_A - \bar{x}_B|}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

where  $\bar{x}_A$  and  $\bar{x}_B$  are the mean of group A and the mean of group B,  $s_A^2$  and  $s_B^2$  are the standard deviation of group A and the standard deviation of group B, and  $n_A$  and  $n_B$  are the number of data points in group A and the number of data points in group B.

The modulus (the top of the fraction) makes the difference positive regardless of whether the difference is positive or negative to begin with.

The degrees of freedom in this case is the total of  $n - 1$  **for each group**. You could also say it is the sample size minus 2 (i.e.  $(n_A - 1) + (n_B - 1)$ ). The sample size for each group should be at least 30, but it doesn't need to be exactly equal.

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WORKED EXAMPLE

A biologist kept cockatiels at home when she was a child, some of which were born in Europe and some which were born in Australia. She kept a record of how long each of them lived and wanted to investigate if there is any difference in lifespan between the cockatiels born on different continents. The table below shows the lifespan of her eight cockatiels.

Lifespan (months)	
European cockatiels	Australian cockatiels
161	146
159	152
143	137

Perform an unpaired t-test at the 0.05 significance level to investigate whether the lifespans of cockatiels born on different continents have different lifespans.

Use the table on the first page of this chapter to find the critical value.

$H_0$ : There is no significant difference between the lifespans of the cockatiels born on different continents.

$H_1$ : The cockatiels born on different continents have different lifespans.

The mean for each group is  $\bar{x}_A = 154.3$  and  $\bar{x}_B = 145$ .

The standard deviation for each group is  $s_A = 9.87$  and  $s_B = 7.55$ .

So 
$$t = \frac{|\bar{x}_A - \bar{x}_B|}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} = 1.30$$

The significance level is 0.05 and the degrees of freedom is  $6 - 2 = 4$ , so the critical value is 2.78.  $1.30 < 2.78$ , so we fail to reject the null hypothesis.

Spearman's rank

Spearman's rank is used to identify **linear correlation** between any two variables. It is used when the data are collected from several individuals and ordered to produce ranks for each variable.

Each individual receives a rank for both of these variables depending on the position in the ordered list. If two individuals have the same value, they are given a rank halfway between the two ranks (e.g. if two individuals have the same value and are ranked 3 and 4, they are given a rank of 3.5). Generally we assume there will be no tied ranks for the data).

WORKED EXAMPLE

Rank the following data for mass (to the nearest kilogram) and arm span (to the nearest centimetre) for six individuals, from least to greatest.

	Chloe	Zahid	Fiona	Rachel	John	Vivienne
Mass (nearest kg)	44	56	50	50	61	48
Mass rank	1	5	3.5	3.5	7	2
Arm span (nearest cm)	156	162	162	172	178	152
Arm span rank	1	7	3	6	8	2

Both Fiona and Rachel have the same mass, so they are given the rank 3.5 to represent the average of the two ranks. Similarly, both Fiona and Rachel should have the third and fourth rank for this variable.

Spearman's rank finds the mean difference in rank between individuals and uses the following formula to calculate the test statistic:

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

where  $d$  is the difference in rank and  $n$  is the number of pairs of data points.

The degrees of freedom for this test is  $n - 2$ .

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A **low** average difference in rank indicates that the variables are **positively** correlated (with correlation close to 1), whereas a **high** average difference in rank suggests that the variables are **negatively** correlated (with correlation close to -1). Values that are close to zero show little or no correlation.

Degrees of freedom
3
4
5
6
7
8
9
10
Critical value

WORKED EXAMPLE

Perform a Spearman's rank test at the 0.05 significance level, using the critical value table above, to see whether mass and arm span are correlated. Use the worked example above to see whether mass and arm span are correlated.

	Chloe	Zahid	Fiona	Rachel	John
Mass (nearest kg)	52	56	50	50	61
Mass rank	1	5	3.5	3.5	7
Arm span (nearest cm)	156	174	162	172	178
Arm span rank	2	7	3	6	8

$H_0$ : There is no significant correlation between mass and arm span.  
 $H_1$ : Mass and arm span are correlated.  
Calculate the difference in rank for each person, and the difference in rank squared.

	Chloe	Zahid	Fiona	Rachel	John
Difference in rank (d)	-1	-2	0.5	-2.5	-1
$d^2$	1	4	0.25	6.25	1

$n = 8$ , and  $n^2 - 1 = 8^2 - 1 = 63$   
So  $r_s = 1 - \frac{6(1 + 4 + 0.25 + 6.25 + 1 + 1 + 16 + 1)}{8 \times 63} = 0.939$   
The significance level is 0.05 and the degrees of freedom is  $8 - 2 = 6$ , so the critical value is 0.771.  
 $0.939 < 0.771$ , so we reject the null hypothesis.

PRACTICE QUESTIONS

For the following questions, use the critical value tables provided above at a 0.05 significance level.

1. A group of seven subjects test a new diet programme for a week. Their mass is recorded before starting the programme, and after one week of being on the programme:

	Adam	Teresa	Suhail	Veronica	James
Mass before (kg)	121.1	95.3	102.4	98.7	84.2
Mass after (kg)	120.4	93.4	102.2	96.4	84.1

Carry out a paired  $t$ -test to investigate whether subjects lose weight after undertaking the programme for one week.

2. Carry out a Spearman's rank test for the following data to investigate whether cholesterol levels are correlated:

	A	B	C	D	E
Daily salt intake (g)	2.4	1.9	2.7	2.8	2.1
Blood cholesterol ( $\text{mg dl}^{-1}$ )	198	231	210	199	165

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3. In flies, the dominant allele R causes red eyes, while the recessive allele r is for white eyes. Similarly, the dominant allele W is for long wings, and the recessive allele w is for vestigial wings. A genetic cross is carried out between two flies heterozygous for both these traits. Predict the eye colour and wing type of the offspring.

The offspring are expected to have the phenotypes red eyes, long wings : red eyes, vestigial wings : white eyes, long wings : white eyes, vestigial wings in the ratio 9 : 3 : 3 : 1.

In the 80 offspring of the flies, 56 had red eyes and long wings, 10 had red eyes and vestigial wings, 10 had white eyes and long wings, and six had white eyes and vestigial wings.

Carry out a chi-squared test to investigate whether there was a significant difference between the expected and observed frequencies for eye colour and wing type of the flies.

4. A gardener wanted to test two different types of fertiliser on his sunflowers. He planted 10 of them and inorganic fertiliser on half of them. After the first six weeks they were measured. The height of each plant at the end of the six weeks is shown in the table.

Height (cm)	
Organic fertiliser	Inorganic fertiliser
78	75
65	84
74	76
68	80

Carry out a Student's *t*-test to investigate whether the fertiliser had an effect on the height of sunflower plants.

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# 20. CONSTRUCTING GRAPHS

## LEARNING OUTCOME

Use experimental data to plot representative graphs and draw a line of best fit.

## THEORETICAL OVERVIEW

The following example will walk you through the steps of constructing a graph. This is using a potometer to measure the position of an air bubble over time.

**independent variable**  
the variable chosen by the person doing the experiment

Time (s)	Position (cm)
0	2.30
30	3.30
60	4.55
90	5.35
120	6.40
150	7.85
180	9.10

**dependent variable**  
the variable which depends on the independent variable

### 1. Choosing the axes

The independent variable (usually in the left column of a table) goes on the x-axis, which in this case is time, as you have **chosen** the times at which to measure the position. As the position **depends** on the time at which it is measured, position is the **dependent variable**, and so goes on the y-axis.

When labelling your axes, always make sure you write the **units** of each variable.

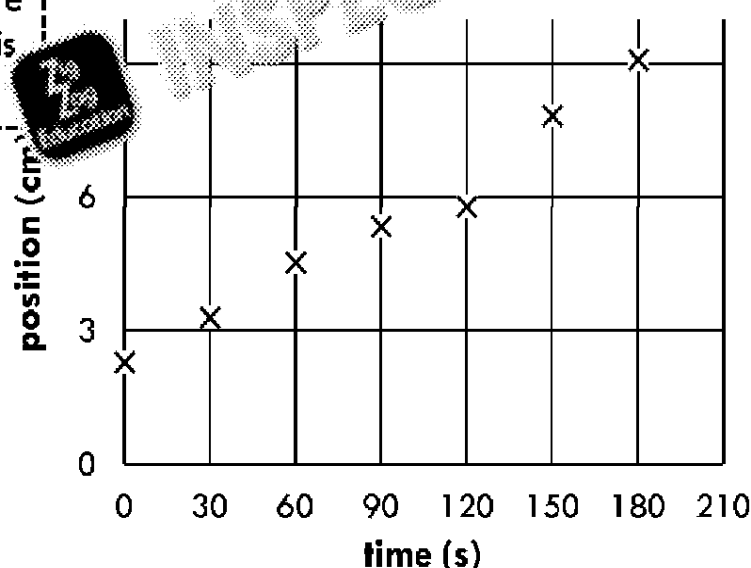
### 2. Choose a scale for each axis

- The scale must be regular.
- The data must cover at least half the page.
- All of the data must fit on the scale!
- Each large square should be a round number.

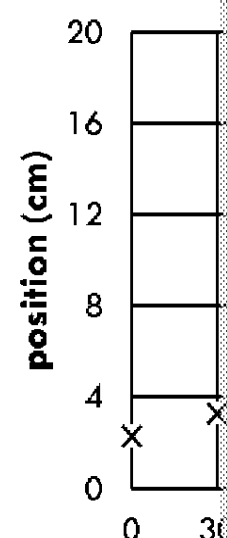
#### Bad examples:

Divisions are difficult to judge because the major grid lines are at every 3 cm on the y-axis.

e.g. where on here is 8 cm?



Poor use of the y-axis



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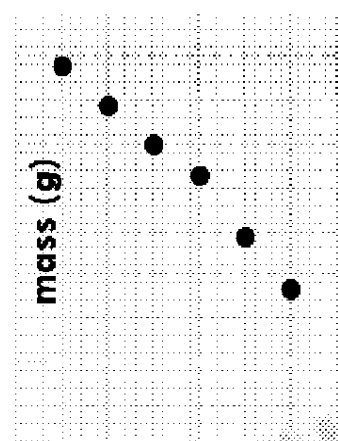
### 3. Plot the points

To plot a point, imagine two lines coming from the x-axis and y-axis at the correct values. The point where the two imaginary lines cross is where you plot your data point.

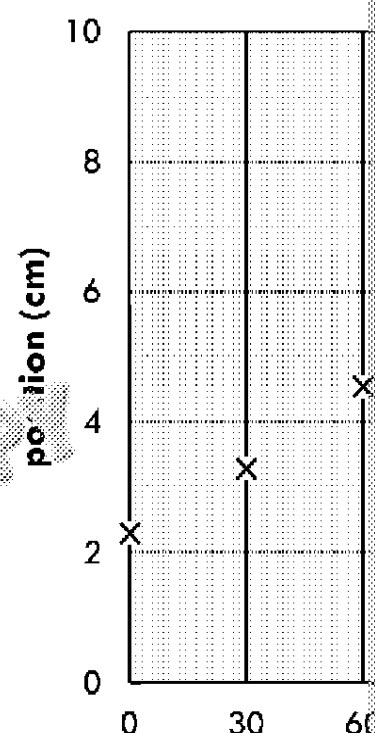
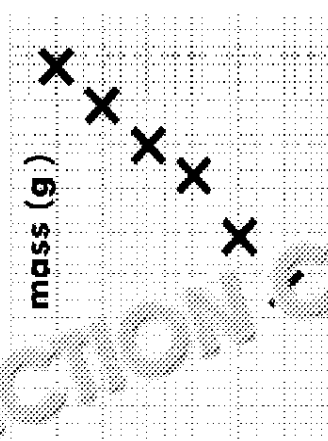
Use a small  $\times$  to represent each data point.

Examples of bad points plotted on graphs:

dots are unclear



crosses are too big

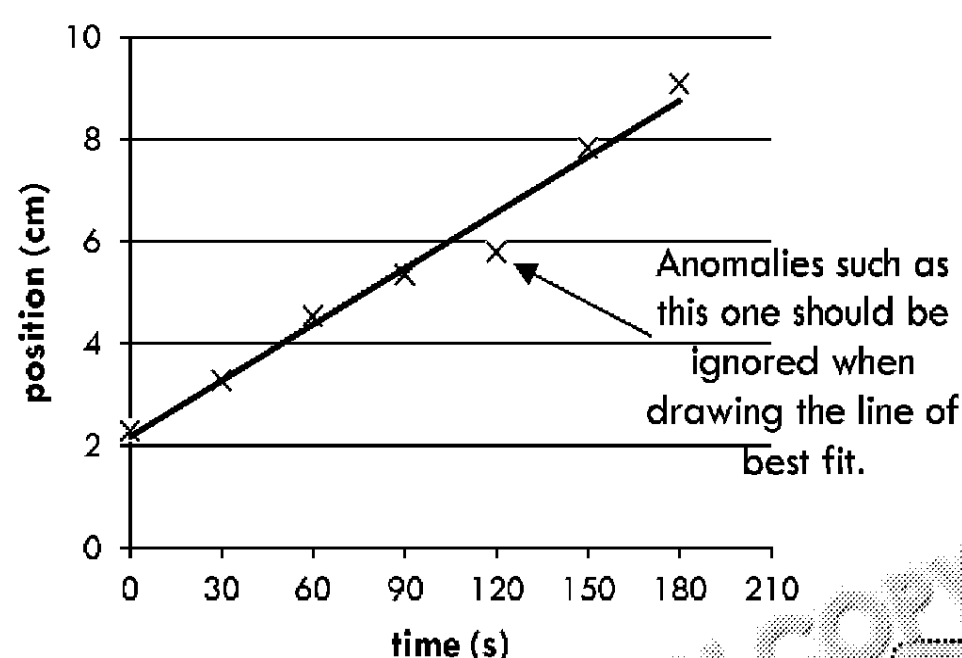


### 4. Draw a line or curve of best fit

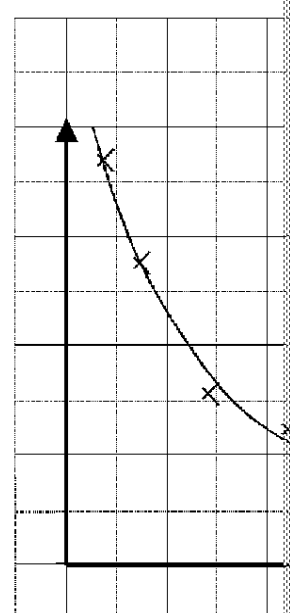
Depending on the data being plotted, a curved or straight best-fit line is used to show the trend. Look at the data to judge which is appropriate. Sometimes two straight lines may be needed. **Do not do dot-to-dot.**

You should aim to have an equal number of data points on each side of the line of best fit.

**Straight line** – use a ruler to draw through the points



**Curved line** – use a smooth free-hand curve



A curve can be tricky to draw. It is a temptation to go for 'sketchy' double lines.

Lines do not have to go through the origin (0, 0) but sometimes it makes sense for the timing how far something travels in a given time, you know it hasn't travelled anywhere.

You may occasionally want to **extrapolate** using your line of best fit. This is when you use the line of best fit to estimate a value larger or smaller than the data you recorded. Be careful with extrapolation where the points are all close to the line, but any extrapolation is into unknown territory. You don't know if the trend would change beyond the data you have gathered.

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PRACTICE QUESTIONS

1. Using graph paper, plot graphs for the data and draw an accurate line of best fit.
- a) A calibration curve for reducing sugar concentration against absorbance.

Concentration ( $\text{mol dm}^{-3}$ )	Absorbance (au)
0	0.98
0.2	0.85
0.4	0.76
0.6	0.60
0.8	0.43
1.0	0.37

- b) An experiment monitoring the amount of  $\text{CO}_2$  produced by a reaction over time.

Time (s)	Volume of $\text{CO}_2$ ( $\text{cm}^3$ )
0	0.0
10	46.5
20	62.3
30	66.3
40	77.2
50	79.3

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# 21. ANALYSING GRA

## LEARNING OUTCOME

Read data values from graphs, predict the shapes of linear graphs, calculate the y-  
best fit, and calculate the rate of change at a given point on a curved line of best fit

## THEORETICAL OVERVIEW

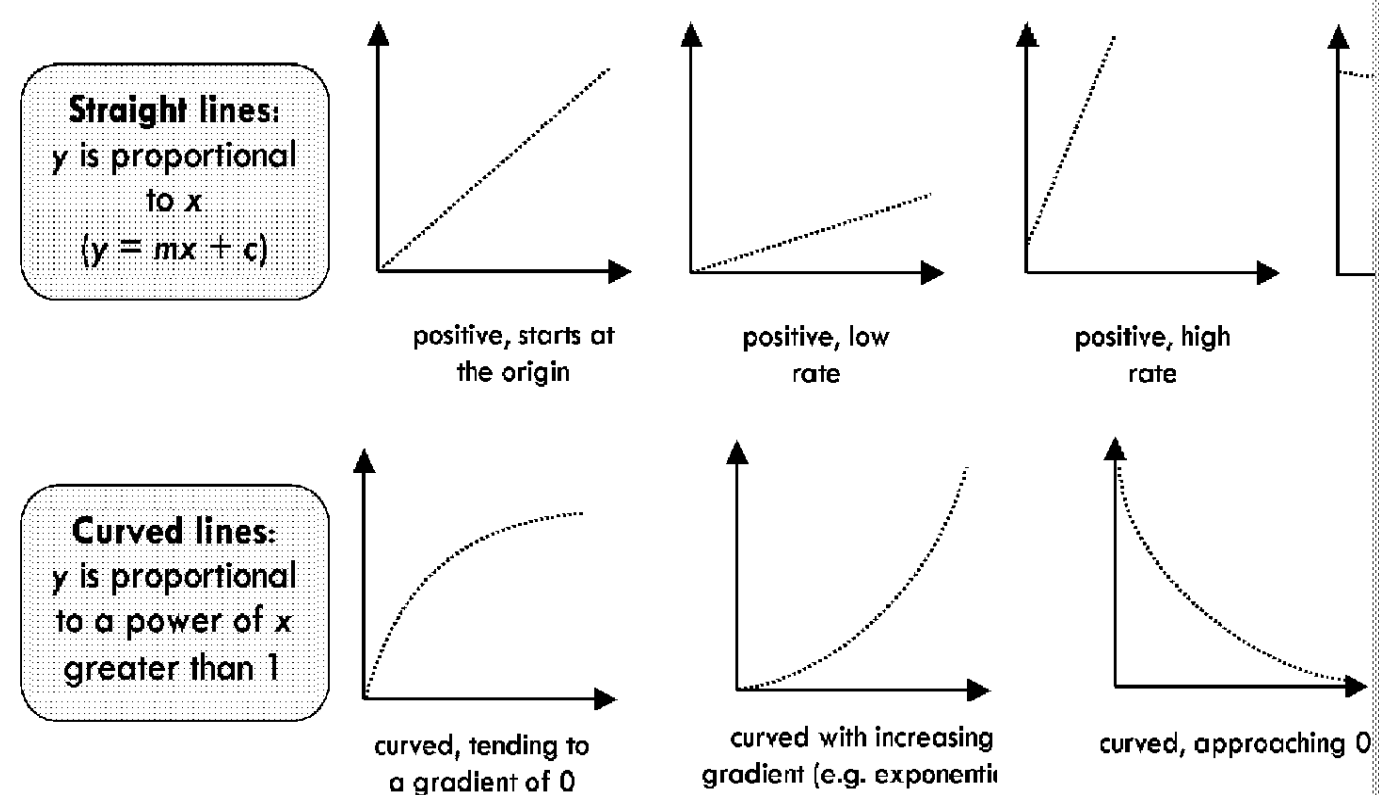
### Reading data from a graph

Reading data from a graph is very similar to plotting data on a graph.  
To find the mass at a certain time, imagine lines going directly up from the  
time (x-axis), and directly across from the line of best fit. The place where  
the imaginary line crosses the y-axis gives the mass.

For example, to find the mass at 25 s, draw an imaginary line up from  
25 s, and across to the line of best fit. The mass at 25 s is 0.74 g.

### Slopes of graphs

The slope of the line can tell you how quickly the concentration changes.  
In other words, it tells you the **rate** at which the concentration is changing.  
A steeper slope shows a higher rate. It can also indicate the relationship  
between x and y. Linear relationships produce straight lines, and other  
relationships (e.g. exponential) produce curved lines.



### Calculating the gradient

To obtain a value for this rate of change, you have to calculate the gradient of the  
you choose two points on the line, and find the difference between the two y-values.

$$\text{gradient} = \frac{\text{difference in } y}{\text{difference in } x}$$

The two points should be far apart, but within the data range.

### WORKED EXAMPLE

The gradient of the graph in the top right of the page is:

$$\begin{aligned} \text{gradient} &= \frac{\text{change in mass}}{\text{change in time}} = \frac{\text{mass}_2 - \text{mass}_1}{\text{time}_2 - \text{time}_1} \\ &= \frac{0.50 - 0.79}{49 - 20} \\ &= -0.010 \text{ g s}^{-1} \end{aligned}$$

When you have calculated the gradient, you need to decide whether it should be positive or negative.

Upwards slope = positive  
x leads to increase in y

Downwards slope = negative  
(increase in x leads to decrease in y)

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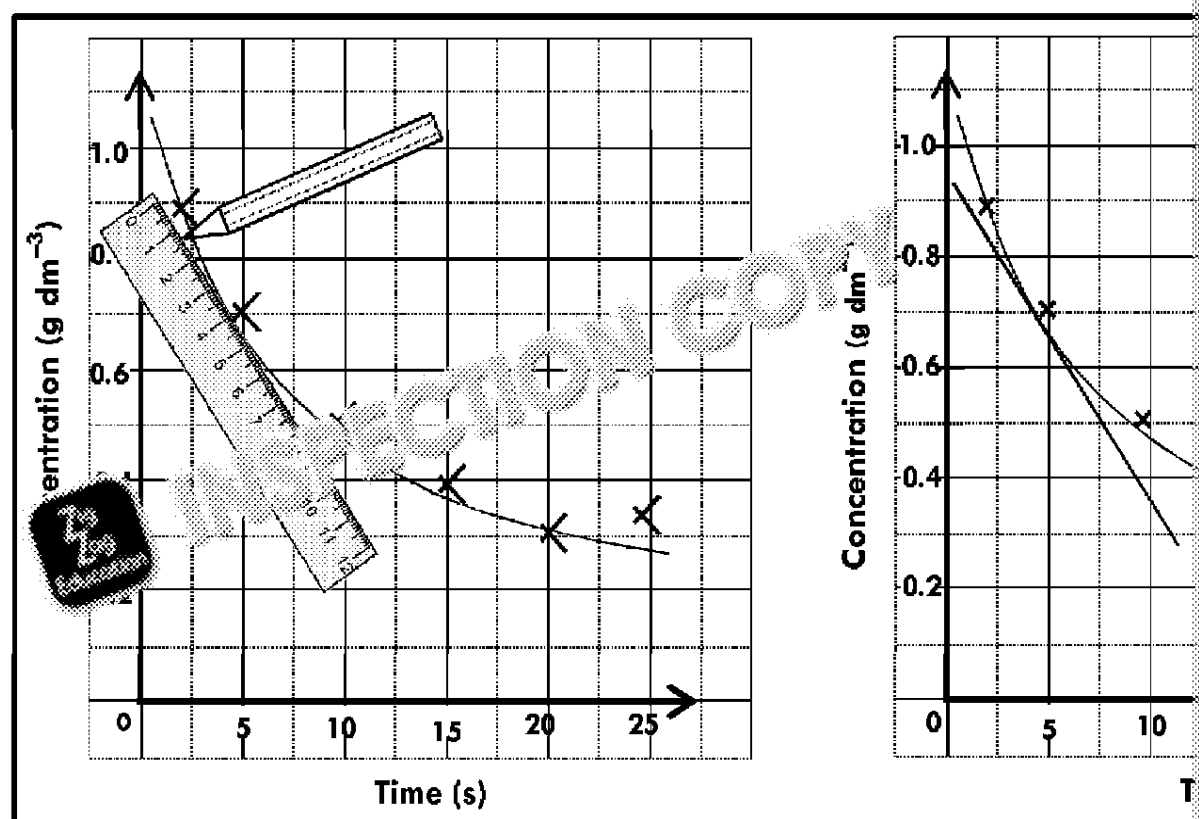


## Calculating rate of change from a curved graph

If you have a curved line of best fit, the gradient is different at different points on

To calculate the gradient at a point on the curve, you can draw a tangent to the curve only once. To do this, position a ruler on the curve so that it touches the

The tangent has the same gradient as the curve at the point where the tangent touches the gradient of the line as normal.



## Changing gradients

In many graphs, the steepness of the gradient tells you how fast the reaction is occurring.

As the reaction slows down, the gradient changes. Later in the experiment, the slope is less steep.

For this graph, we can compare the two gradients to see how much the reaction has slowed down.

## y-intercept

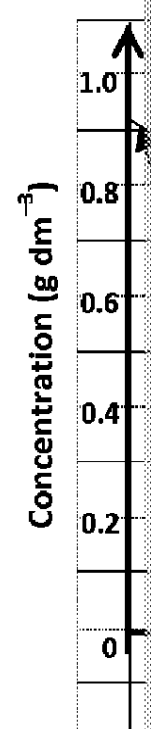
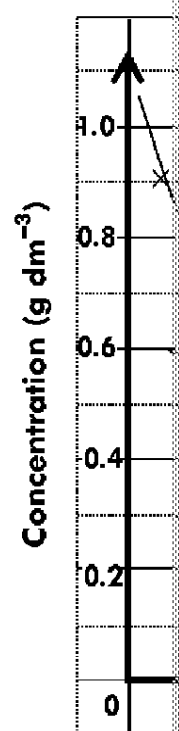
The **y-intercept** of a line is the point at which the line crosses the y-axis. In order to work this out, you need to **extrapolate** back from the line of best fit to the axis.

The y-intercept gives you the y-value when the x-value is 0. In this graph, the y-axis tells you the concentration (g dm<sup>-3</sup>) at the beginning of the experiment (x = 0). In concentration graphs, it will tell you the **initial concentration**.

Sometimes, it is possible to read the y-intercept straight from the graph. In this graph, you can see that the line of best fit crosses the y-axis at approximately 0.92 g dm<sup>-3</sup>.

This means that the concentration was 0.92 g dm<sup>-3</sup> before the experiment started.

In other cases, you may need to find the intersection of two lines. This is simply any point at which the two lines cross, and you can read off these points as normal.

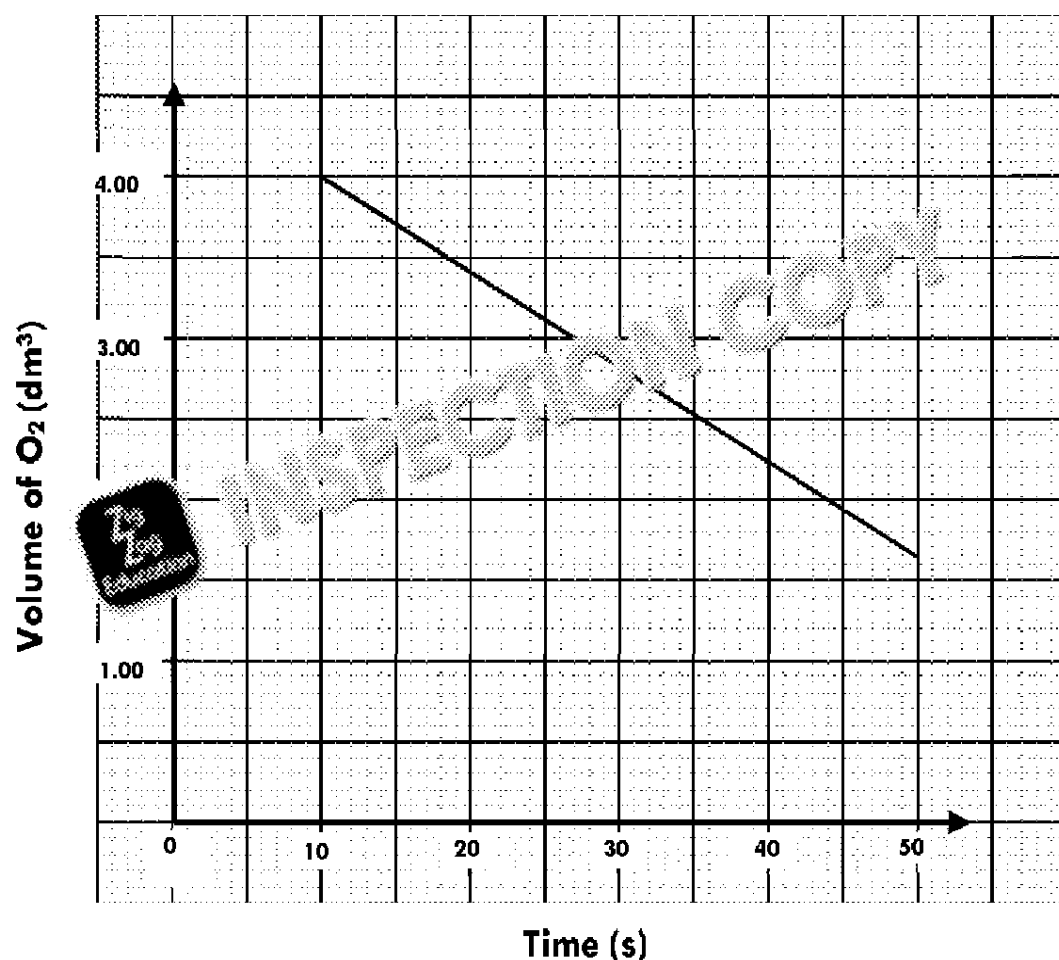


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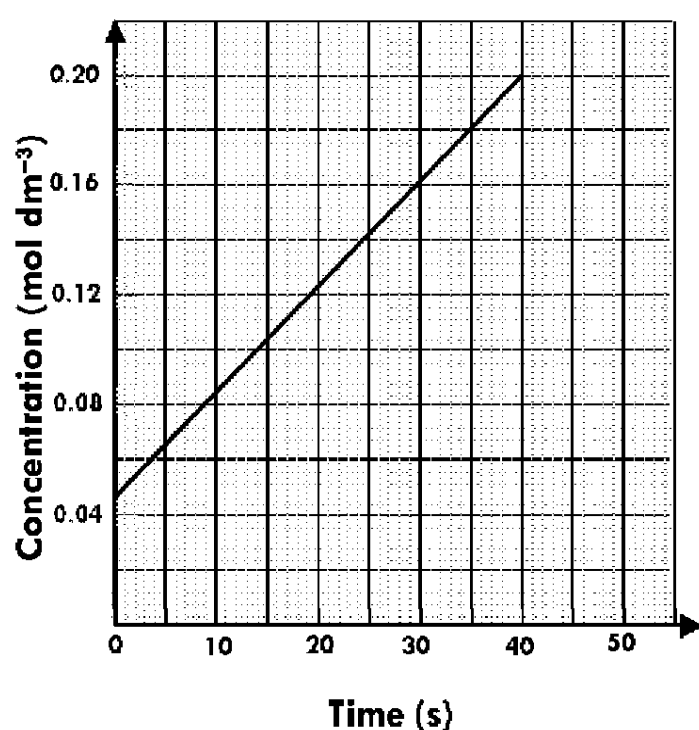


## PRACTICE QUESTIONS

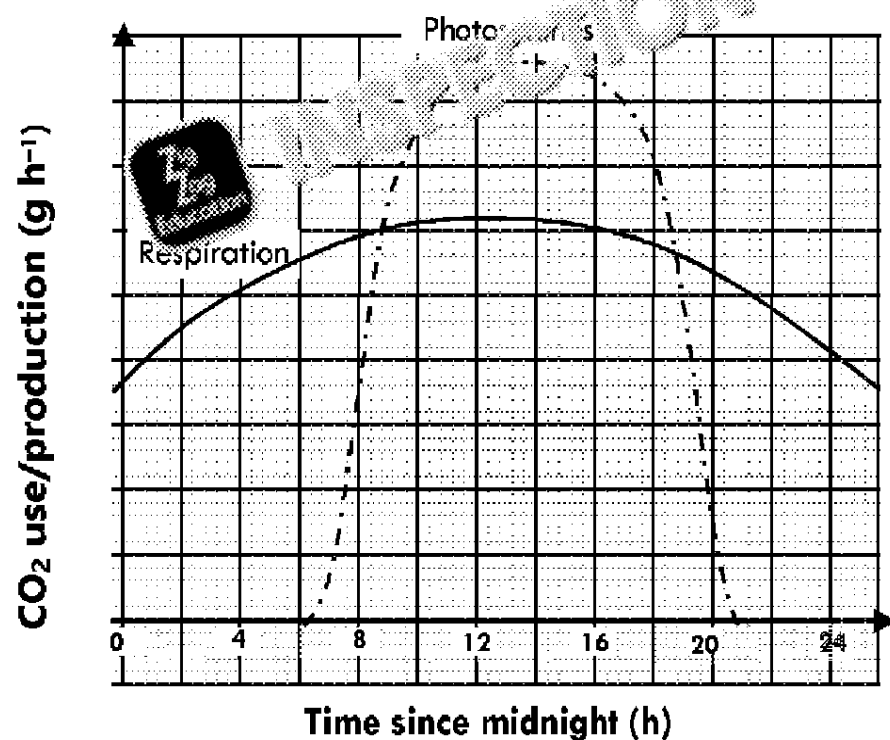
- Sketch a graph of substrate concentration against rate of reaction for an enzyme which is in excess.
- For the following graph, find:
  - The volume at 20 s
  - The volume at 35 s
  - The time when the volume is  $3.00 \text{ dm}^3$



- Calculate the gradient and y-intercept of the following graph:



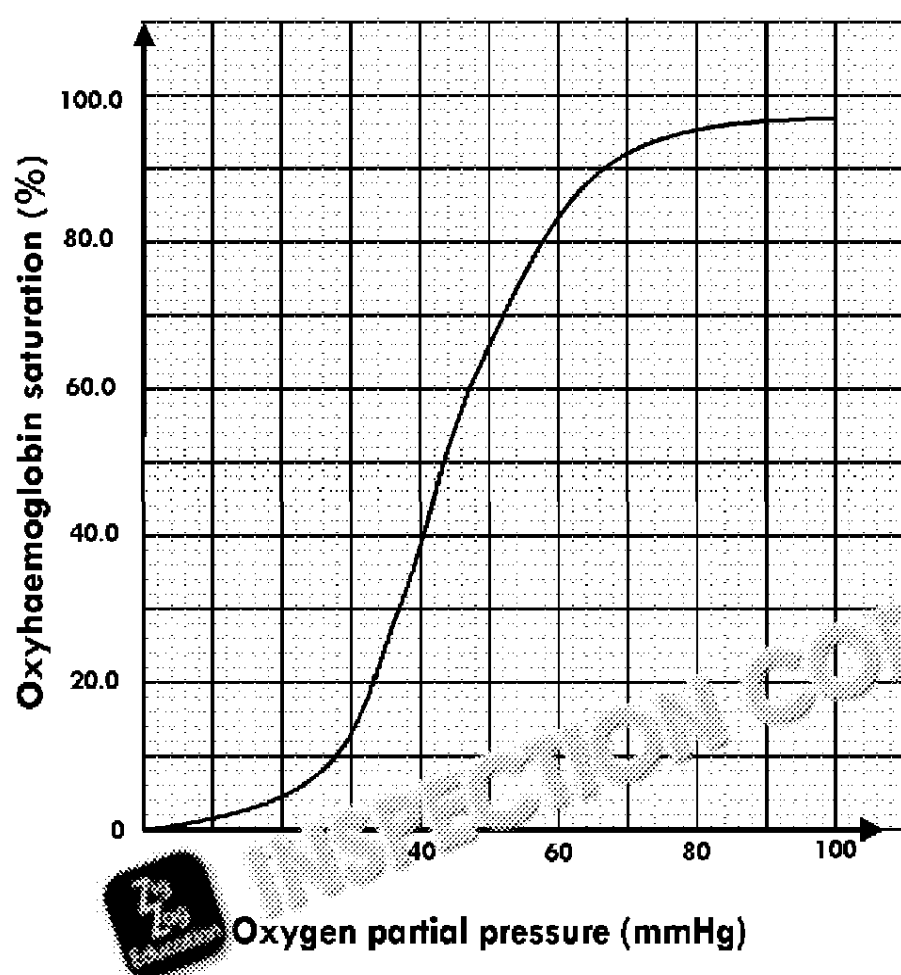
- Find the times of the intersections of the two graphs below:



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5. Calculate the gradient of the tangent at an oxygen partial pressure of 60 mmHg



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## 22. SURFACE AREA AND VOLUME

### LEARNING OUTCOME

Be able to calculate the circumference of circles and the surface area and volume of regular shapes.

### THEORETICAL OVERVIEW

To approximate the area and volume of structures in Biology, it is useful to be able to calculate the surface area and volume of regular shapes.

#### Circle

The circumference of a circle is given by:

$$C = \pi \times d$$

$$\text{or } C = \pi \times (2 \times r)$$

and the area of a circle is given by:

$$A = \pi r^2$$

$$\text{or } A = \pi \left( \frac{d}{2} \right)^2$$

where  $r$  is the radius,  $d$  is the diameter and  $\pi$  is the irrational number 3.14159...

#### Square/rectangle

The area of a rectangle is given by:

$$A = b \times h$$

The special case for this is the case of the square, when  $b = h$

The area is then given by:

$$A = b \times b$$

$$A = b^2$$

#### Surface area

Of a rectangular prism is  $2(hb + bd + hd)$

Of a cylinder is  $2\pi r(r + l)$

Of a sphere is  $4\pi r^2$

#### Volume

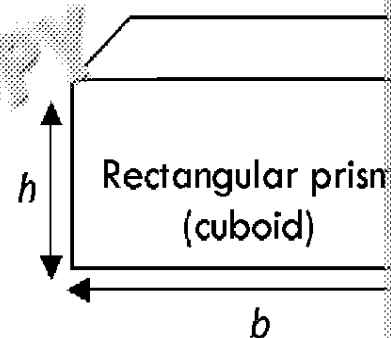
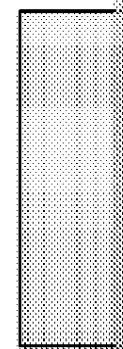
The general equation for the volume of any shape is given by:

$$V = A \times h$$

where  $A$  is the area of one of the faces of the shape and  $h$  is the height.

**NB** The equation for face area depends on the shape; for a cylinder, for example, the equation for the area of a circle, whereas the face area for a rectangular block is the area of a rectangle.

The equation can be applied to any prism.



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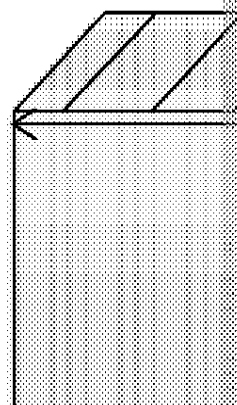
## Cylinder

If we apply the general equation to the cylinder we obtain:

$$V = (\pi r^2) \times h$$

## Rectangular block

$$V = (b \times d) \times h$$



## Sphere

$$V = \frac{4}{3} \pi r^3$$

### WORKED EXAMPLES

- Find the exact surface area of a cylinder with radius 2 mm and length 30 mm.

Surface area of a cylinder is  $2\pi r(r + l)$   
 $= 2 \times \pi \times 2 \times (2 + 30) = 4\pi \times 32 = 128\pi \text{ mm}^2$

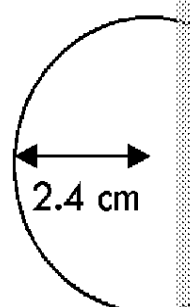
- Find the volume of a spherical cell with radius 5  $\mu\text{m}$  (to three significant figures).

Volume of a sphere is  $V = \frac{4}{3} \pi r^3$   
 $= \frac{4}{3} \pi \times 5^3 = 524 \mu\text{m}^3$  (3 s.f.)

### PRACTICE QUESTIONS

Give your answers to three significant figures:

- Find the circumference of the circle to the right.
- For the rectangular prism to the right, find:
  - the surface area
  - the volume
- Find the surface area of a spherical virus with radius 15 nm.
- Find the volume of a cylinder with radius 0.2 mm and height 7 mm.



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# APPENDIX – USING A CALCULATOR

## LEARNING OUTCOME

Use your calculator to make calculations involving powers, standard form, exponent

## THEORETICAL OVERVIEW

### Powers

Powers mean that a number is multiplied by itself.

For example:

$$4^3 = 4 \times 4 \times 4 = 64$$

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$$

To calculate  $2^5$ , write:



$$2 \times^y 5 =$$

### Roots

Roots are the opposite of powers.

$$\sqrt[3]{125} = 5 \text{ because } 5 \times 5 \times 5 = 125$$

To calculate  $\sqrt[3]{64}$ , write:

$$3 \sqrt[3]{64} =$$

### Logarithms

A logarithm is the opposite of '10 to the power of'. It tells you how many times you multiply 10 by itself to get a certain number.

$$\log 1000 = 3$$

This is because  $10^3 = 10 \times 10 \times 10 = 1000$

To find the log of 1000 on a calculator, type:

$$\log 1000 =$$

### e

e is a number which often comes up in mathematics and nature. It is a number, similar to pi, which has a set value of 2.71828 to four decimal places.

The  $e^x$  button works in the same way as the power button, so to calculate  $e^2$ , you

$$e^x 2 =$$

### Natural logarithms

Some equations may include  $\ln$ , which is called the natural logarithm (which is the logarithm to the base e, rather than the base 10). To find the natural logarithm of 8, type:

$$\ln 8 =$$

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## Standard form

Standard form is a way of representing numbers, especially very large or very small numbers.

To write  $5 \times 10^7$ , type:

**5** **10<sup>x</sup>** **7** **=**

### WORKED EXAMPLE

The radius of a cell, which can be modelled as a sphere, is given by:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

where  $r$  is the radius in  $\mu\text{m}$ ,

$V$  is the volume in  $\mu\text{m}^3$ ,

$\pi$  has the value 3.14 to three significant figures.

Calculate  $r$  when  $V = 779.428$ .

To calculate this type:

Some functions may require a **SHIFT** key, e.g. **SHIFT** **√** to calculate the cube root.

**SHIFT** **√** **(** **3** **)** **×** **7** **7** **9** **4** **×** **8** **)** **÷** **(** **4** **)** **×**

which should give an answer of 12.3.

## PRACTICE QUESTIONS

1. Calculate the following:

- $3^5 - 3^4$
- $\log 10\,000\,000$
- $(2.95 \times 10^7) \div (1.41 \times 10^8)$
- $\ln(e^{4^2})$
- $e^{4^2} - 3^2$
- $\log 27$
- $9^8 - 3^2$
- $(6.41 \times 10^{-6}) \div (1.23 \times 10^{-7})$
- $\log(4.5) - e^7$
- $3^2 + e^2 + (\log(3))^2$

2. The pH scale makes use of the equations  $\text{pH} = -\log(\text{concentration of } \text{H}^+)$  and  $\text{concentration of } \text{H}^+ = 10^{-\text{pH}}$ . Use these equations to find:

- the concentration of  $\text{H}^+$  for a solution with a concentration of  $0.00469 \text{ mol dm}^{-3}$
- the concentration of  $\text{H}^+$  for a solution with a pH of 6.3

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# DIAGNOSTIC TEST

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1.
  - a) Write 0.00248 in standard form.
  - b) Write the number  $7.035 \times 10^2$  in full.
  - c) Round 9.5754 to three significant figures.
  - d) A reaction starts at 23 °C and ends at 37.4 °C. Write the temperature change to three significant figures.
2.
  - a) Convert 2 minutes 35 seconds into seconds.
  - b) Convert 0.24 m into mm.
3.
  - a) Convert 0.15 dm<sup>3</sup> into cm<sup>3</sup>.
  - b) Convert 500 ml into dm<sup>3</sup>.
  - c) Convert 64 s<sup>-1</sup> into ms<sup>-1</sup>.
  - d) Convert 3 g cm<sup>-3</sup> into mg dm<sup>-3</sup>.
4.
  - a) How many significant figures does the number 0.0007692 have?
  - b) Write 149.572 to four significant figures.
5.
  - a) Write the ratio 4 : 12 in the form x:1.
  - b) Write the fraction  $\frac{2}{5}$  as a decimal.
  - c) Write the percentage 80 % as a fraction in its simplest form.
6.
  - a) How many times larger than 6 is 30?
  - b) Calculate the percentage yield of a reaction with actual yield 4.68 g and theoretical yield 6.0 g.
  - c) A 120 cm<sup>3</sup> solution contains 3.40 g of enzyme. What is the mass of enzyme per cm<sup>3</sup> of solution?
7.
  - a) Calculate the mean of the following values: 2.5, 2.6, 2.7, 2.6
  - b) Calculate the median of the following values: 3, 8, 4, 6, 9
  - c) Identify the mode of the following values: 5, 10, 12, 8, 5, 9, 11, 7, 6, 5, 10
8.
  - a) Rearrange the equation  $PVR = TV \times BR$  to make BR the subject of the formula.
  - b) Find the size of the image when magnification =  $\times 400$ , and the size of the object is 0.05 mm.
9.
  - a) Rearrange the equation  $y = \frac{x+8}{4}$  to make x the subject of the formula.
  - b) An experiment with three measurements has a mean result of 15.45 g. Measurement 1 = 15.40, and measurement 2 = 13.60. Determine the value of measurement 3.
10. Rearrange the formula  $V = \frac{4\pi r^3}{3}$  to find the value of r when  $V = 16.0$  mm<sup>3</sup>.
11.
  - a) Sketch a graph to show the relationship between Rate and Water potential. Rate  $\propto$  Water potential.
  - b) For the expression Rate  $\propto x^2$ , describe the effect on the rate if x is tripled.
12. In a distance change of 17.85 cm measured on a ruler with 0.1 cm divisions, state an uncertainty.
13.
  - a) Calculate the percentage uncertainty in reading of 3.45 g on an analogue balance with a resolution of 0.01 g.
  - b) Calculate the percentage change when a value increases from 6 to 9.
14.
  - a) Use a calculator to find the value of  $\log(15\ 000)$ .
  - b) Find the value of a in  $\log(a) = 2.2$ .
15.
  - a) If  $P(X) = 0.5$  and  $P(Y) = 0.2$  are independent events, calculate  $P(X \text{ and } Y)$ .
  - b) If  $P(A) = 0.15$ , A and B are mutually exclusive events, and only A or B can occur, calculate  $P(A \text{ or } B)$ .
  - c) If event C succeeds with probability 0.8, what is the probability that event C succeeds twice in a row?

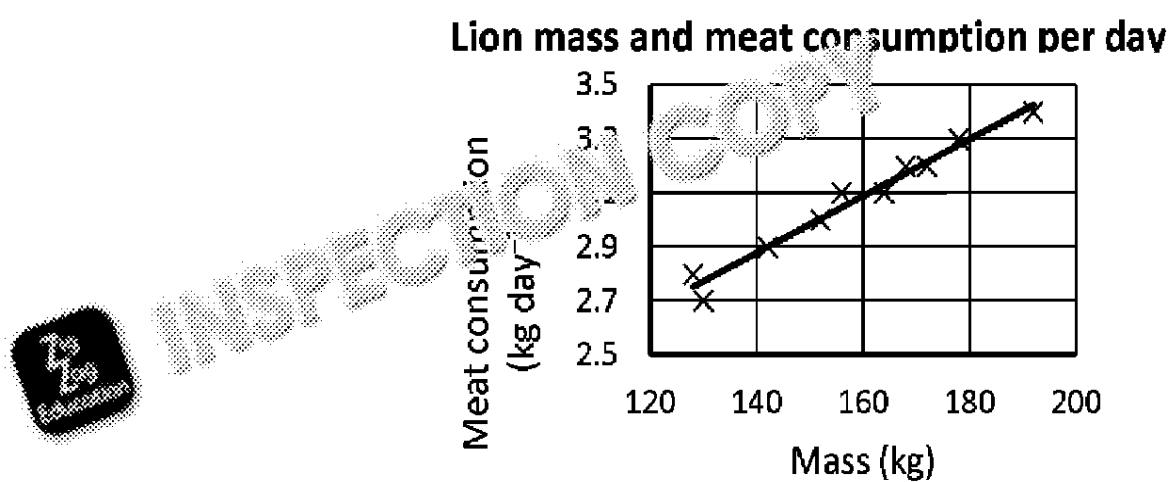
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16. a) What type of sample would you use to estimate the total population size  
b) Draw a histogram for the following data:

Wing length (nearest mm)	Frequency
$35 \leq l < 65$	6
$65 \leq l < 85$	50
$85 \leq l < 95$	72
$95 \leq l < 105$	85
$105 \leq l < 120$	15

17. a) What sort of correlation is shown by the following graph?



- b) Sketch a scatter graph of eight points with perfect negative correlation.
18. a) Find the range of: 36, 31, 27, 34, 35, 21, 19  
b) Find the standard deviation to three significant figures of: 79, 76, 81, 75
19. a) What degrees of freedom should be used for carrying out a Spearman's data points?  
b) Which statistical test should be used to investigate whether there is a difference per female wolf between wolves in captivity and wild wolves?
20. a) Plot a graph of the following data. Add a trend line.

Time (s)	Volume (cm <sup>3</sup> )
0	39.8
60	36.2
120	32.5
180	29.2
240	20.9

- b) Plot a graph of the following data. Add a trend line.



Time (s)	Concentration (g cm <sup>-3</sup> )
20	0.133
40	0.122
60	0.288
80	0.403
100	0.365
120	0.613
140	0.860
160	1.393
180	2.340
200	3.952

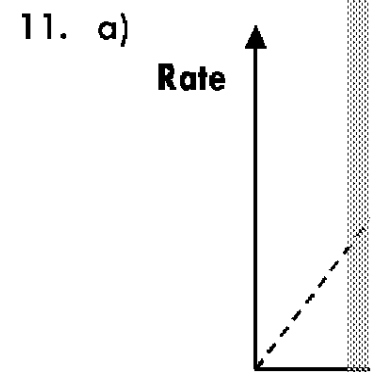
21. a) Find the gradient of the line of best fit for the graph in 20 a.  
b) Find the gradient of the tangent at 130 s for the graph in 20b.
22. a) What is the exact circumference of a circle with radius 2 cm?  
b) What is the surface area (to three significant figures) of a cylinder with radius 2 cm and height 5 cm?  
c) What is the volume of a rectangular prism with height 12 cm, base 7 cm and length 10 cm?

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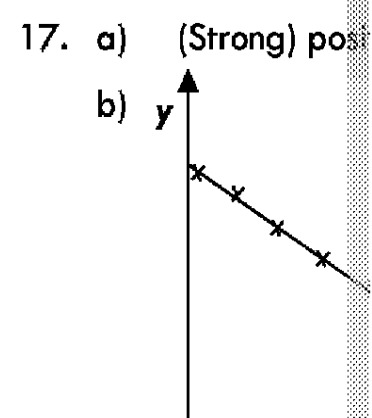
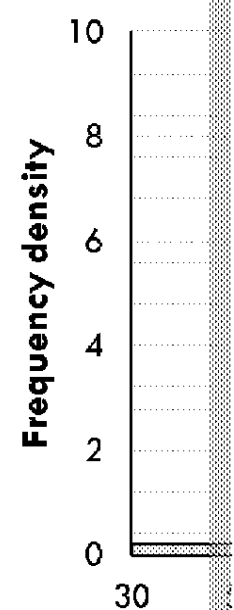


# DIAGNOSTIC TEST ANSWERS

1. a)  $2.48 \times 10^{-3}$   
b) 703.5  
c) 9.58  
d)  $37.4 - 23 = 14.4$   
 $= 14^\circ\text{C}$  (2 s.f.)
2. a) 155 s  
b)  $0.24 \times 10^3 = 240$  mm
3. a)  $0.15 \times 10^3 = 150$  cm<sup>3</sup>  
b)  $500 \div 10^3 = 0.5$  dm<sup>3</sup>  
c)  $64 \div 10^3 = 0.064$  ms<sup>-1</sup>  
d)  $3 \times 10^3 \times 10^3 = 3\,000\,000$  g dm<sup>-3</sup>  
 $= 3 \times 10^6$  g dm<sup>-3</sup>
4.  49.6
5. a) 3.5 : 1  
b) 0.4  
c)  $\frac{4}{5}$
6. a)  $\frac{30}{6} = 5$   
b)  $\frac{4.68}{6.50} \times 100 = 72\%$   
c)  $\frac{3.4}{120} \times 77 = 2.18$  g
7. a)  $\frac{2.5 + 2.6 + 2.7 + 2.6}{4} = 2.6$   
b) 6  
c) 5
8. a)  $BR = \frac{PVR}{TV}$   
b) Image =  $0.35 \times 400$   
 $= 140$  mm  
(= 14 cm)
9. a)  $x = 4y - 8$  OR  $= 4(y - 2)$   
b)  $\frac{15.40 + 13.60 + x}{3} = 15.45$   
 $15.40 + 13.60 + x = 46.35$   
 $x = 17.35$
10.   
$$= \sqrt{\frac{\frac{3V}{\pi} \times 16}{4\pi}}$$
  
 $= 1.56$  mm



- b) Multiplied by
12. Uncertainty in a re  
Uncertainty in the  
 $= 0.05 \times 2 = 0.1$   
So distance chang
13. a) Percentage  
 $= 0.29\%$  (2  
b) 50 % increas
14. a) 4.18  
b)  $10^{2.2} = 158$
15. a)  $0.5 \times 0.2 =$   
b)  $1 - 0.15 = 0$   
c)  $0.8 \times 0.8 = 0$
16. a) Random samp  
b)



18. a)  $36 - 19 = 17$   
b)  $\bar{x} = 78.5$   
 $s = 2.56$  (3 s.f.)

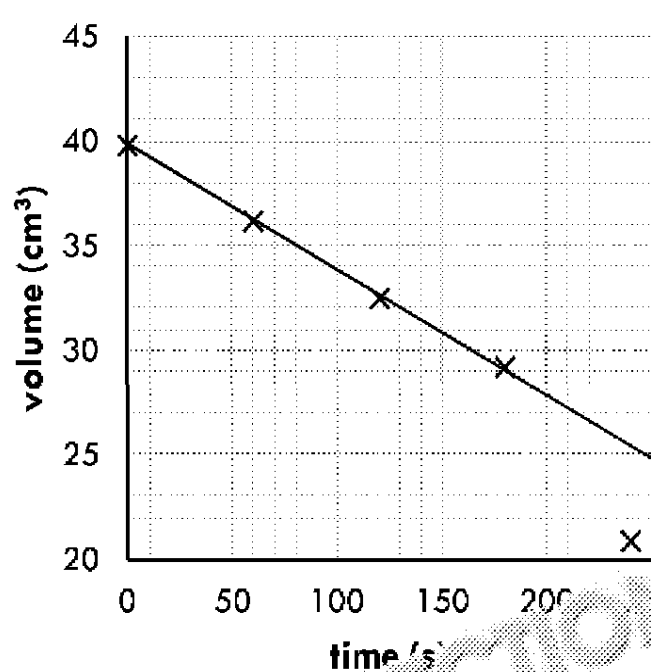
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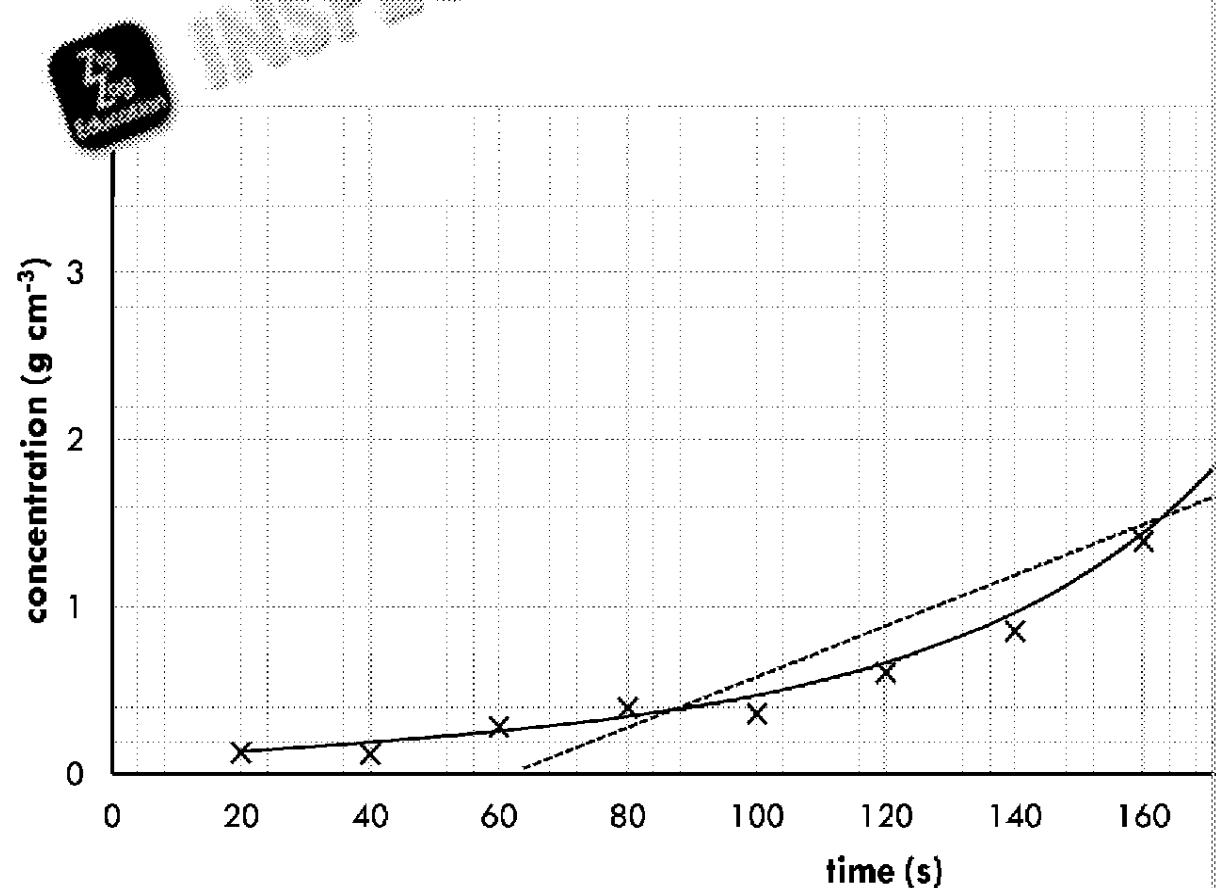


19. a) 10  
b) Unpaired/Student's *t*-test

20. a)



b)



21. a)  $\text{gradient} = \frac{40 - 25}{250 - 0}$   
 $= 0.06 \text{ cm}^3 \text{ s}^{-1}$   
b)  $\text{Gradient} = \frac{2.42 - 0}{240 - 80} = 0.0151 \text{ g cm}^{-3} \text{ s}^{-1}$  (allow between 0.0131 and 0.0171)
22. a)  $2\pi r = 2 \times \pi \times 2 = 4\pi \text{ cm}$   
b)  $2\pi r(r + l) = 2 \times \pi \times 10 \times (10 + 40) = 20\pi \times 50 = 3140 \text{ cm}^2$  (3 s.f.)  
c)  $b d h = 7 \times 3 \times 12 = 252 \text{ cm}^3$

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4. a) 11.63 ha  
b)  $7.74 \times 10^{-1}$  fo

5.  $\frac{9400 \times 12}{100} = 1128 =$

2. a)  $m = 25 \times 10^3 = 25\,000$   
b)  $m = 0.004 \times 10^6 = 4\,000$   
c)  $m = 360 \times 10^{-3} = 360\,000$
3. a)  $d = 75.2 \div 10^3 = 0.0752$   
b)  $d = 3800 \times 10^{-3} = 3.8$   
c)  $t = 520 \div 10^3 = 0.52$   
d)  $d = \frac{6 \times 10^8}{10^9} = 0.6$   
e)  $t = 0.0042 \times 10^6 = 4\,200$

3. a)  $v = 5 \times 1000 =$   
b)  $v = 0.02 \times 100 =$   
c)  $v = 140 = 140$

4.  $2.45 \times 60 = 147$  be

2. a) 34 000 kJ ha<sup>-1</sup>  
b) 16 000 kJ ha<sup>-1</sup>  
c) 34 100 kJ ha<sup>-1</sup>

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## 5. Fractions, Percentages and Ratios

- $65 \div 100 = 0.65$
  - $1 \div 4 = 0.25$
  - $0.2 \div 100 = 0.002$
  - $0.8 \div 3.2 = 0.25$
  - $11 \div 12 = 0.917$
- $3 \div 7 \times 100 = 42.9 \%$
  - $6 \div 19 \times 100 = 31.6 \%$
  - $9 \div 10 \times 100 = 90.0 \%$
  - $2 \div 9 \times 100 = 22.2 \%$
  - $24 \div 100 \times 100 = 24.0 \%$
- $1/5$
  - $11/10$
  - $3/4$
  - $9/10$
  - $3/20$
- $3 \div (3 + 1) = 3/4$
  - $1 \div 4 \times 100 = 25 \%$
- $150 : 120 = 1.25 : 1$
- $7/20$

## 6. Scaling Quantities

- $(0.28 \div 14) \times 50 = 1.0 \text{ g}$
  - $(0.28 \div 4) \times 10 = 0.7 \text{ g}$
  - $(0.28 \div 4) \times 2 = 0.04 \text{ g}$
- $(1.20 \div 1.50) \times 1 = 0.80 \text{ dm}^3$
  - $(1.20 \div 1.50) \times 0.075 = 0.060 \text{ dm}^3$
  - $(1.20 \div 1.50) \times 2.6 = 2.1 \text{ dm}^3$
  - $(1.20 \div 1.50) \times 0.85 = 0.68 \text{ dm}^3$
- percentage yield
  - theoretical yield
- $\frac{0.23}{0.18} = 1.28 \text{ times longer}$
- $\frac{4.9 - 3.5}{3.5} \times 100 = 40 \%$

## 7. Calculating Means, Medians and Modes

- $\frac{1+2+3+4+5+6+7+8}{8} = 4.5$
  - $\frac{6+4+3+9}{4} = 5.5$
  - $\frac{6.5+6.2+6.6+6.9}{4} = 6.55$
  - $\frac{230+300+290+310+250}{5} = 276$
  - $\frac{0.04+0.07+0.05+0.01}{4} = 0.043 \text{ (2 s.f.)}$
- 3, 5, 6, 6, 8  
Median = 6; Mode = 6
  - 7.4, 8.7, 8.9, 9.2  
Median = 9.2; Mode = 8.9
  - 64, 71, 72, 74, 75  
Median = 72; Mode = 74
  - 0, 2, 2, 4, 4, 4, 8  
Median = 4; Mode = 4

## 8. Using Equations I – Rearranging Simple Equations

- concentration of substance moved = rate  $\times$  reaction time
- $x = 2R$
  - $x = aB/y$
  - $x = 3$
- volume of  $\text{CO}_2$  consumed =  $\frac{\text{volume of } \text{CO}_2 \text{ produced}}{\text{RQ}}$
  - volume of  $\text{CO}_2$  produced =  $\text{RQ} \times \text{volume of } \text{O}_2 \text{ consumed}$
- $R_f = 3.5 \div 4.2 = 0.833$
  - $16 \div 0.2 = 80 \text{ m}$
- $8 \div 0.2 = \times 40$

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## 9. Using Equations II – Equations with +, −, × and ÷

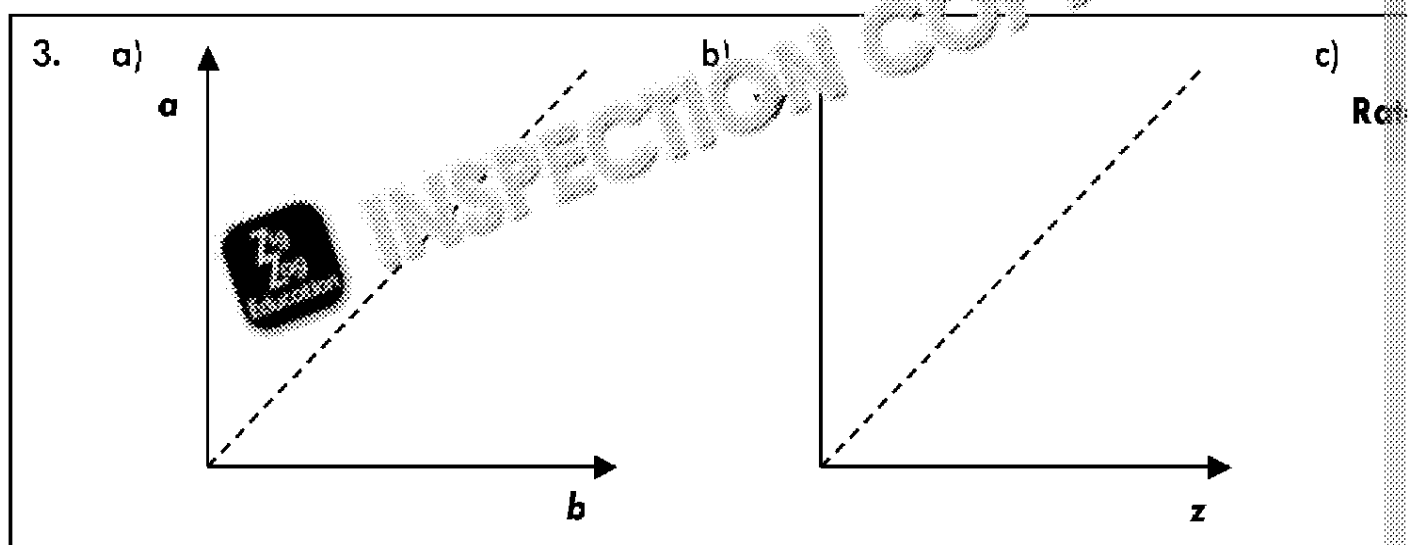
- $x = 3y + 5$
  - $x = \frac{y-8}{10}$
  - $x = \frac{y-c}{m}$
  - $x = \frac{3}{y}$
  - $x = 1 - y$
  - $x = \frac{y+2}{2}$  or  $x = \frac{y}{2} + 1$
  - $x = \frac{1}{8y}$
  - $x = \frac{y-3}{y}$  or  $x = \frac{y}{y-3}$
- $\text{Mean} = \frac{x + 6.1 + 8.6 + 7.2}{4}$   
 $x + 6.1 + 8.6 + 7.2 = 28.8$   
 $x = 28.8 - (6.1 + 8.6)$   
 $x = 6.9 \text{ cm}^3$
- $\text{Mean} = \frac{(12.0 - 220.0) + (-208.0 + x - 216.5)}{4}$   
 $x = -628.5 - (208.0 + 216.5 - 12.0)$   
 $x = -6.5 \text{ kPa}$

## 10. Using Equations III – Equations with Powers and Roots

- $3^2 + 2^3 = 17$
  - $c = \frac{2^3}{2^2} = 2$
  - $c^2 = 9$   
 $c = 3 \text{ (or } -3)$
  - $c = \frac{b^3}{a^3}$   
 $= \frac{2^3}{3^3} = \frac{8}{27}$
  - $c = \sqrt{a^2b^3 - a^2 - a^3}$   
 $= \sqrt{3^22^3 - 3^2 - 3^3}$   
 $= \sqrt{36}$   
 $= 6 \text{ (or } -6)$
  - $c = \sqrt[3]{\frac{a^3 + b^2 + 1}{b^2}}$   
 $= \sqrt[3]{\frac{3^3 + 2^2 + 1}{2^2}}$   
 $= \sqrt[3]{8}$   
 $= 2$
- $x = \sqrt{3y}$
  - $x = \sqrt{\frac{4\pi}{y}}$
  - $x = 9y^2$
  - $x = \sqrt[3]{\frac{y}{8z}}$  or  $x = \sqrt[3]{\frac{y}{8z}}$

## 11. Mathematical Symbols

- $3 \text{ cm}^3 > 1 \text{ cm}^3$
  - $2800 \text{ mg} < 3000 \text{ mg}$
  - $5000 \gg 0.001$
  - rate of diffusion  $\propto$  temperature
- Doubles (both sides increase proportionally)
  - Halves (both sides decrease proportionally)
  - Halves (AB does not cancel out the 3)
  - Doubles (both sides double)
  - Quarters (if A is halved, B must quarter. D must double)
  - Triples (C triples, B must triple to stay the same)



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## 12. Uncertainty I

1. a)  $56.0 \pm 0.5 \text{ cm}^3$   
 b)  $12.0 \pm 0.1 \text{ cm}$  (uncertainty is  $0.05 \times 2$  as two readings are taken)  
 c)  $18 \pm 1 \text{ }^\circ\text{C}$   
 d)  $9.50 \pm 0.10 \text{ cm}^3$   
 e)  $2 \times 0.01 \text{ g} = 0.02 \text{ g}$   
 $5.25 - 4.50 = 0.75 \text{ g}$   
 $0.75 \pm 0.02 \text{ g}$   
 f)  $22.0 \pm 0.5 \text{ }^\circ\text{C}$
2. a)  $\frac{26.2 + 25.8 + 26.0}{4}$   
 b)  $\pm 0.3 \text{ }^\circ\text{C}$
3.  $\frac{78.2 + 69.5 + 74.1 + 76.2}{4}$   
 $(78.2 - 69.5) \div 2 =$   
 $74.5 \pm 4.35 \text{ cm}^3$

## 13. Uncertainty II

1. a)  $\frac{0.5}{24.5} \times 100 = 2.0 \%$   
 b)  $\frac{0.1}{37.9} \times 100 = 0.26 \%$   
 c)  $\frac{0.15}{23} \times 100 = 0.65 \%$   
 d)  $\frac{1}{127} \times 100 = 0.79 \%$   
 e)  $\frac{5}{542} \times 100 = 0.92 \%$   
 f)  $\frac{0.05}{0.18} \times 100 = 28 \%$

b)

Initial reading ( $^\circ\text{C}$ )
Final reading ( $^\circ\text{C}$ )
Change ( $^\circ\text{C}$ )
Change (%)

Mean change =

c)

Initial reading ( $^\circ\text{C}$ )
Final reading ( $^\circ\text{C}$ )
Change (mm)
Change (%)

Mean change =

2. a)

	1	2	3
Initial reading ( $\text{cm}^3$ )	50.00	50.00	50.00
Final reading ( $\text{cm}^3$ )	66.05	68.15	65.90
Change ( $\text{cm}^3$ )	16.05	18.15	15.90
Change (%)	32.10	36.30	31.80

$$\text{Mean change} = \frac{32.1 + 36.3 + 31.8}{3} = 33.4 \text{ \% increase}$$

## 14. Logarithms

1. a) 1259  
 b) 5.146  
 c) -3.301  
 d) 0.8395  
 e) -5.000  
 f) 3.000
2. a) 10 000  
 b)  $7.943 \times 10^6$   
 c) -2.699  
 d) 0.8722  
 e) -4.246

3. a)

Days since start	0	20	30	40	50
Number of flies	46	230	617	910	1567
ln (number of flies)	0.6021	1.663	2.362	2.959	3.195

- b) Variance is a large range  
 It makes the values easier to compare / plot on a graph

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# 15. Understanding Simple Probability

- 1
- $0.2 \times 0.2 = 0.04$

3.

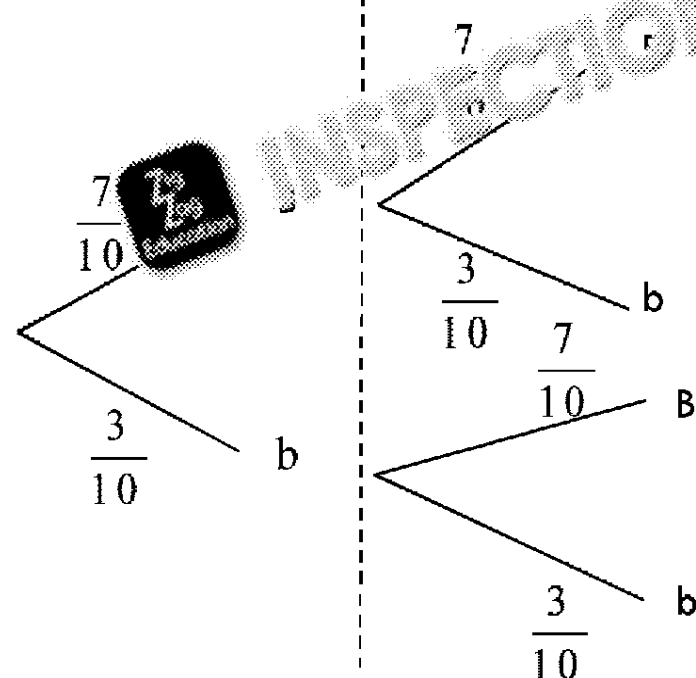
Parent	R	rr
Rr	R	rr
	r	Rr

$P(\text{round}) = \frac{3}{4}$

- For every five individuals, genotypes are BB, BB, BB, Bb, bb so  $P(B) = \frac{7}{10}$  and  $P(b) = \frac{3}{10}$

Allele from parent 1

Allele from parent 2



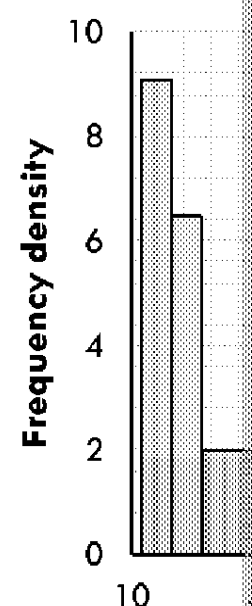
The Bb genotype is the heterozygous genotype, so find  $P(Bb \text{ or } bB) = \frac{7}{10} \times \frac{3}{10} + \frac{3}{10} \times \frac{7}{10} = \frac{21}{100} + \frac{21}{100} = \frac{42}{100} = \frac{21}{50}$

- Let the probability of a tall plant be  $p$  and the probability of a dwarf plant be  $q$ .  $P(xx) = q^2 = 0.16$ . So  $P(x) = q = \sqrt{0.16} = 0.4$ .  $p + q = 1$  so  $p = 1 - 0.4 = 0.6$ .  $P(Xx) = 2pq = 2 \times 0.6 \times 0.4 = 0.48$

# 16. Sampling, Frequency Diagrams and Histograms

- $450 : 150 = 3 : 1$  tall : dwarf  
For a sample of 20, 15 tall and five dwarf are needed
- $1 - \left( \left( \frac{22}{111} \right)^2 + \left( \frac{9}{111} \right)^2 + \left( \frac{74}{111} \right)^2 + \left( \frac{6}{111} \right)^2 \right) = 0.507$  (3 s.f.)

3.



# 17. Correlation

- a) No correlation  
b) A  
c) C  
d) B  
e) None
- Quadratic correlation

# 18. Standard Deviation and Range

- a)  $1.6 - 0.2 = 1.4$   
b)  $101 - 17 = 84$
- a)  $\bar{x} = 0.165$ ;  $s = 0.0002$   
b)  $\bar{x} = 523$ ;  $s = 100$

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19. Statistical Tests

1.  $H_0$ : There was no significant weight loss in the subjects after one week of the dieting programme.  
 $H_1$ : The subjects lost weight after one week on the dieting programme.

$$\bar{d} = \frac{0.7 + 1.9 + 0.2 + 2.3 - 0.1 + 2.4 + 0.9}{7} = 1.19$$

$$s_d = \sqrt{\frac{6.17}{6}} = 1.01$$

$$t = \frac{1.19\sqrt{7}}{1.01} = 3.11$$

Significance level is 0.05

Degrees of freedom = 7 - 1 = 6

So critical value is 2.45

3.11 > 2.45 so we reject the null hypothesis

2.  $H_0$ : There is no significant correlation between daily salt intake and blood cholesterol levels.  
 $H_1$ : Daily salt intake and blood cholesterol levels are correlated.

	A	B	C	D	E	F	G	H	I
Daily salt rank	4	1	5	6	2	9	7	3	8
Cholesterol rank	3	9	6	4	1	7	5	2	8
d	1	-8	-1	2	1	2	2	1	0
d <sup>2</sup>	1	64	1	4	1	4	4	1	0

$$r_s = 1 - \frac{6(1 + 64 + 1 + 4 + 1 + 4 + 4 + 1 + 0)}{9(9^2 - 1)} = 0.333$$

Significance level is 0.05

Degrees of freedom = 9 - 2 = 7

So critical value is 0.679

0.333 < 0.679 so we fail to reject the null hypothesis

3.  $H_0$ : There is no significant difference between the expected and observed frequencies of colour and wing type.  
 $H_1$ : The expected frequencies of colour and wing type differ from the observed frequencies.

Phenotype	$f_e$
Red eye, long wing	45
Red eye, vestigial wing	15
White eye, long wing	15
White eye, vestigial wing	5

$$\chi^2 = 2.69 + 1.67 = 4.36$$

Significance level is 0.05

Degrees of freedom = 3

So critical value is 7.81

4.36 < 7.81 so we fail to reject the null hypothesis

4.  $H_0$ : There is no significant difference between the plant growth and the plants grown in the shade.  
 $H_1$ : There is a significant difference between the plant growth and the plants grown in the shade.

$$\bar{x}_A = 71.25, \bar{x}_B = 78.75$$

$$s_A = 5.85, s_B = 5.85$$

$$t = \frac{71.25 - 78.75}{\sqrt{\frac{5.85^2}{4} + \frac{5.85^2}{4}}} = -2.10$$

Significance level is 0.05

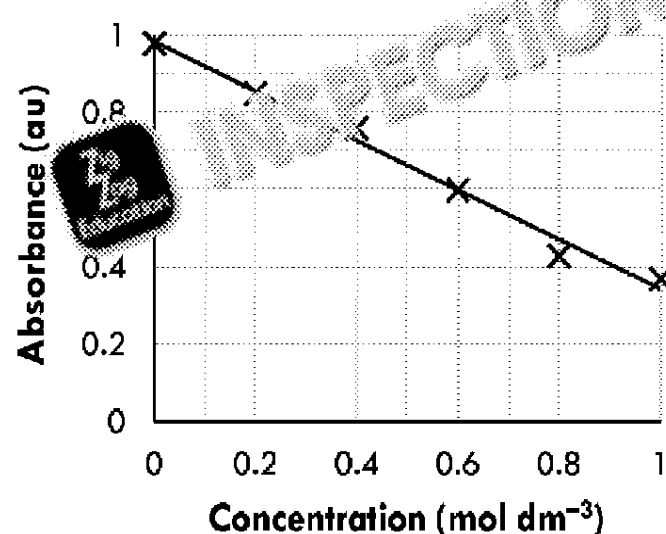
Degrees of freedom = 8

So critical value is 2.45

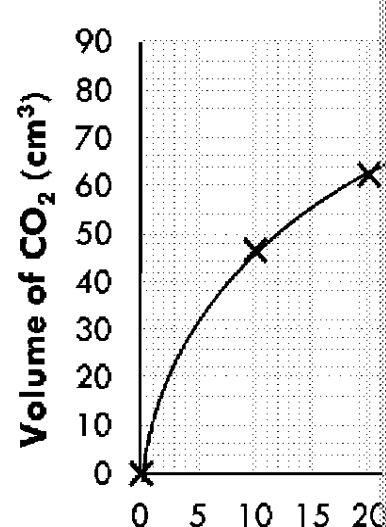
2.10 < 2.45 so we fail to reject the null hypothesis

20. Constructing Graphs

1. a)



- b)

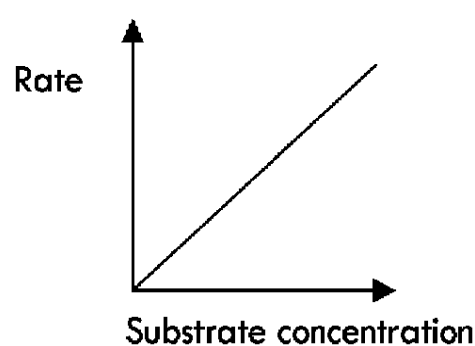


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## 21. Analysing Graphs

1.



$$3. \text{ Gradient} = \frac{0.2 - 0.048}{40 - 0}$$

$$y\text{-intercept} = 0.048$$

4. 08:48 (allow 08:40 to 08:56)  
(allow 18:40 to 18:56)

$$5. \frac{100 - 16}{73 - 0} = 1.15 \% \text{ min}^{-1}$$

2. a)  $3.40 \text{ dm}^3$   
b)  $2.50 \text{ dm}^3$   
c)  $27 \text{ s}$

## 22. Surface Area and Volume of Shapes

$$1. 2\pi r = 2 \times \pi \times 2.4 = 15.1 \text{ cm}$$

$$b) hbd = 8 \times 5 \times 4 = 160$$

$$2. a) 2(hb + bd + hd) + 4\pi r^2$$
  

$$= 2(5 \times 4 + 4 \times 8 + 8 \times 5) + 4\pi \times 2^2$$
  

$$= 160 + 100.66 = 260.66$$

$$3. 4\pi r^2 = 4 \times \pi \times 15^2 = 2826.9$$

$$4. \pi r^2 h = \pi \times 0.2^2 \times 7 = 0.88$$

## Appendix - Using a Calculator

1. a) 162  
b) 7  
c) 0.209 (3 d.p.)  
d) 16  
e) 1096.6  
f) 1.431 (3 d.p.)  
g) 0.1111 (4 s.f.)

- h) 52.114 (3 d.p.)  
i) -1096.0  
j) 16.617

2. a)  $\text{pH} = -\log(0.005) = 2.33$   
b) concentration =  $5.01 \times 10^{-7}$

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