Further Maths

A Level | Edexcel | 9FM0

2017 specification
(first exams in 2019)

Topic Tests: Expert Tests – Set B

A Level Edexcel Further Mathematics: Core Pure Mathematics: Part 2#

#The remaining topics to cover the A Level (9FM0) Core Pure Mathematics

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Content

This pack contains 6 expert level topic tests, which together form 'Set B' in a paired range of tests for A Level Edexcel Further Mathematics: Core Pure Mathematics 2.

Each test comes with fully worked solutions, containing tips, hints and technique boxes to help students on a particular question. Answers should be given to three significant figures unless specified in the question.

These topic tests have been **fully cross-referenced** to the Pearson textbook, *Edexcel A level Further Mathematics Core Pure Mathematics Book 2* (ISBN 978-1292183343), for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these expert tests, the relevant times are the fifth and sixth times listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who are ready to go beyond the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and progresses the concepts covered.

Cross-referencing Grid

Topic	Edexcel spec. points	Subtopics
Complex Numbers II	2.8–2.11	Exponential form of complex numbers, multiplying and dividing complex numbers, de Moivre's theorem, trigonometric identities, sums of series, nth roots of a complex number, solving geometric problems
Series	4.4–4.6	The method of differences, higher derivatives, Maclaurin series, series expansions of compound functions
Further Calculus	5.2–5.6	Improper integrals, the mean value of a function, differentiating inverse trigonometric functions, integrating with inverse trigonometric functions, integrating using partial fractions, volumes of revolution around the x-axis and the y-axis, volumes of revolution of parametrically defined curves, modelling with volumes of revolution
Polar Coordinates	7.1–7.3	Polar coordinates and equations, sketching curves, area enclosed by a polar curve, tangents to polar curves
Hyperbolic Functions	8.1–5	Hyperbolic functions, inverse hyperbolic functions, identities and equations, differentiating hyperbolic functions, integrating hyperbolic functions
Differential Equations	9.1–9.9	First- and second-order differential equations, boundary conditions, modelling, simple harmonic motion, damped and forced harmonic motion, coupled first-order simultaneous differential equations

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Complex Numbers II		**Series**	
1.1a	40	2.1a	30
1.1b	40	2.1b	30
1.2a	50	2.[illegible]	55
1.2b	50	2.2b	55
1.3a	[illegible]	2.3a	65
1.[illegible]	70	2.3b	65
[illegible]oordinates		**Hyperbolic Functions**	
4.1a	55	5.1a	40
4.1b	55	5.1b	40
4.2a	60	5.2a	50
4.2b	60	5.2b	50
4.3a	75	5.3a	65
4.3b	75	5.3b	65

Subtopics: Hyperbolic functions, inverse hyperbolic functions, identities and equations, d integrating hyperbolic functions

1. Differentiate each of the following with respect to x:

   a) $\sinh 5x$ b) $\frac{1}{3}\tanh 3x$ c) $x\cosh 2x$

2. Find: a) $\int \frac{6}{\sqrt{x^2-1}}\,dx$ b) $\int \frac{2}{\sqrt{x^2+9}}$ c

3. On the **same diagram**, sketch the graphs of $y = \cosh x$ and $y = 5 + \cos$

4. Solve the equation 3 [illegible] $+ 5\sinh x = 3$ for **real** values of x

5. Sketch the graph of $y = \operatorname{cosech} x$

6. a) By writing $\operatorname{artanh} x = u$, show that $u = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, $|x| < 1$

   b) Given that $\operatorname{artanh} x - \operatorname{artanh} y = \ln\sqrt{7}$, show that $y = \frac{4x-3}{4-3x}$

7. Use the substitution $x = \sqrt{\cosh u}$ to show that $\int \frac{2}{x^3\sqrt{x^4-1}}\,dx = \frac{\sqrt{x^4-}}{x^2}$

8. Use the substitution $x = \frac{1}{2}(3 + 3\cosh u)$ to find $\int \frac{1}{\sqrt{x^2-3x}}\,dx$

9. Given that $4\cosh x + 6\sinh x = R\sinh(x+\alpha)$, $R > 0$, find the **exact** va α to **3 significant figures**.

10. Find the equation of the tangent to $y = \operatorname{arsinh}\left(\frac{x}{3}\right)$ at the point where x in the form $y = px + q + \ln r$, where p, q and r are **exact real numbers**.

11. The sketch to the right shows the region R bounded by the curves $y = 1$ and $y = 7\cosh x$

    a) Find the exact **coordinates** of the two points where the curves intersect.

    b) Find the area of the shaded region R, giving your answer in the form $y = a\ln b + c$, where a, b and c are integers.

Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Solutions to Complex Numbers II – Test B

1. a) $x=\left(2\left(\cos\frac{\pi}{8}+\mathrm{i}\sin\frac{\pi}{8}\right)\right)^6=2^6\left(\cos\left(6\times\frac{\pi}{8}\right)+\mathrm{i}\sin\left(6\times\frac{\pi}{8}\right)\right)$

$=64\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right)$ by de Moivre's theorem **A1**

$y=\left(6\left(\cos\frac{\pi}{6}+\mathrm{i}\sin\frac{\pi}{6}\right)\right)^3=6^3\left(\cos\left(3\times\frac{\pi}{6}\right)+\mathrm{i}\sin\left(3\times\frac{\pi}{6}\right)\right)$

$=216\left(\cos\frac{\pi}{2}+\mathrm{i}\sin\frac{\pi}{2}\right)$ by de Moivre's theorem **A1**

b) $\frac{x}{y}=\frac{64\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right)}{216\left(\cos\frac{\pi}{2}+\mathrm{i}\sin\frac{\pi}{2}\right)}$

This is of the form $\frac{z_1}{z_2}$, with $z_1=64\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right)$ and $z_2=216\left(\cos\frac{\pi}{2}\right.$

So $\frac{z_1}{z_2}=\frac{64e^{\frac{3\pi}{4}\mathrm{i}}}{216e^{\frac{\pi}{2}\mathrm{i}}}$ **M1**

$=\frac{64}{216}e^{\mathrm{i}\left(\frac{3\pi}{4}-\frac{\pi}{2}\right)}=\frac{8}{27}e^{\frac{\pi}{4}\mathrm{i}}$ **M1**

$=\frac{8}{27}\left(\cos\frac{\pi}{4}+\mathrm{i}\sin\frac{\pi}{4}\right)$ **M1**

$=\frac{4\sqrt{2}}{27}+\frac{4\sqrt{2}}{27}\mathrm{i}$ **A1** **[6 Marks]**

Techni... nume... the fo... modul... to fin...

2. a) **Show that $\cos 4\theta\equiv 8\cos^4\theta-8\cos^2\theta+1$**

$(\cos\theta+\mathrm{i}\sin\theta)^4=\cos 4\theta+\mathrm{i}\sin 4\theta$ by de Moivre's theorem **M1**

$(\cos\theta+\mathrm{i}\sin\theta)^4=\cos^4\theta+{}^4C_1\cos^3\theta\times\mathrm{i}\sin\theta+{}^4C_2\cos^2\theta\times(\mathrm{i}\sin\theta)^2+{}^4C_3\cos\ldots$

$=\cos^4\theta+4\mathrm{i}\cos^3\theta\sin\theta+6\mathrm{i}^2\cos^2\theta\sin^2\theta+4\mathrm{i}^3\cos\theta\sin^3\theta+\ldots$

$=\cos^4\theta+4\mathrm{i}\cos^3\theta\sin\theta-6\cos^2\theta\sin^2\theta-4\mathrm{i}\cos\theta\sin^3\theta+\sin\ldots$

Techni... side u... and si...

Equating real parts:

$\cos 4\theta\equiv\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$

$\equiv\cos^4\theta-\sin^2\theta\left(6\cos^2\theta-\sin^2\theta\right)$

$\equiv\cos^4\theta-\left(1-\cos^2\theta\right)\left(6\cos^2\theta-\left(1-\cos^2\theta\right)\right)$ **M1**

$\equiv\cos^4\theta+\left(\cos^2\theta-1\right)\left(7\cos^2\theta-1\right)$

$\equiv\cos^4\theta+\left(7\cos^4\theta-8\cos^2\theta+1\right)$

$\equiv 8\cos^4\theta-8\cos^2\theta+1$ **A1**

Tip: W... sin θ w... save s...

Techni... cos² θ...

b) **Show that $\tan 4\theta\equiv\frac{4\tan\theta-4\tan^3\theta}{8-8\sec^2\theta+\sec^4\theta}$**

Equating imaginary parts from a):

$\sin 4\theta\equiv 4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta$ **A1**

$\tan 4\theta\equiv\frac{\sin 4\theta}{\cos 4\theta}\equiv\frac{4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta}{8\cos^4\theta-8\cos^2\theta+1}$ **M1**

$\equiv\frac{\frac{1}{\cos^4\theta}\left(4\cos^3\theta\sin\theta-4\cos\theta\sin^3\theta\right)}{\frac{1}{\cos^4\theta}\left(8\cos^4\theta-8\cos^2\theta+1\right)}$ **M1**

$=\frac{4\tan\theta-4\tan^3\theta}{8-8\sec^2\theta+\sec^4\theta}$ **A1** **[9 Marks]**

3. a) **Given that $z = \cos\theta + \mathrm{i}\sin\theta$, use the results $z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$ and $z^n +$**

$\sin^4\theta\cos^2\theta \equiv \frac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2)$

For $n = 1$, $z - \frac{1}{z} = 2\mathrm{i}\sin\theta$ and $z + \frac{1}{z} = 2\cos\theta$

So $\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2 = (2\mathrm{i}\sin\theta)^4 (2\cos\theta)^2 = 64\sin^4\theta\cos^2\theta$ **M1**

Also $\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2 = \left(z^2 - 2z\left(\frac{1}{z}\right) + \left(-\frac{1}{z}\right)^2\right)^2 \left(z^2 + 2z\left(\frac{1}{z}\right) + \left(\frac{1}{z}\right)^2\right)$ **M1**

$= \left(z^4 - 2z^2 + 1 - 2z^2 + 4 - \frac{2}{z^2} + 1 - \frac{2}{z^2} + \frac{1}{z^4}\right)\left(\ldots \frac{1}{z^2}\right)$ **M1**

$= \left(z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}\right)\left(z^2 + 2 + \frac{1}{z^2}\right)$

$\ldots + 2 \ldots 4z^4 - 8z^2 - 4 + 6z^2 + 12 + \frac{6}{z^2} - 4 - \frac{8}{z^2} - \frac{4}{z^4} + \frac{1}{z^2} + \frac{2}{z^4} + \frac{1}{z^6}$

$\ldots - 2z^4 - z^2 + 4 - \frac{1}{z^2} - \frac{2}{z^4} + \frac{1}{z^6} = \left(z^6 + \frac{1}{z^6}\right) - 2\left(z^4 + \frac{1}{z^4}\right) - \left(z^2 + \frac{1}{z^2}\right) + 4$

$= 2\cos 6\theta - 2(2\cos 4\theta) - (2\cos 2\theta) + 4 = 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$ by

$\therefore 64\sin^4\theta\cos^2\theta \equiv 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$

$\therefore \sin^4\theta\cos^2\theta \equiv \frac{1}{64}(2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4)$

$\equiv \frac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2)$ **A1**

Tech... side ... and ...

b) $\int_0^{\frac{2\pi}{3}} \sin^4\theta\cos^2\theta \,\mathrm{d}\theta = \int_0^{\frac{2\pi}{3}} \frac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2)\,\mathrm{d}\theta$

$= \left[\frac{1}{192}\sin 6\theta - \frac{1}{64}\sin 4\theta - \frac{1}{64}\sin 2\theta + \frac{\theta}{16}\right]_0^{\frac{2\pi}{3}}$ **M1**

$= \frac{1}{192}\sin\frac{12\pi}{3} - \frac{1}{64}\sin\frac{8\pi}{3} - \frac{1}{64}\sin\frac{4\pi}{3} + \frac{1}{16}\times\frac{2\pi}{3} - 0$ **M1**

$= 0 - \frac{\sqrt{3}}{128} + \frac{\sqrt{3}}{128} + \frac{\pi}{24} = \frac{\pi}{24}$ **A1[9 Marks]**

4. a) Let $w = -8\sqrt{2} + 8\mathrm{i}\sqrt{2}$, so $|w| = \sqrt{\left(-8\sqrt{2}\right)^2 + \left(8\sqrt{2}\right)^2} = \sqrt{256} = 16$ **M1**

$\arg w = \pi - \arctan\left(\frac{8\sqrt{2}}{8\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ **M1**

$\therefore w = 16\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$ ...

Tip: ... this ... form ...

b) $z^4 = -8\sqrt{2} + 8\mathrm{i}\sqrt{2} = 16\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$

$= 16\left(\cos\left(\frac{3\pi}{4} + 2k\pi\right) + \mathrm{i}\sin\left(\frac{3\pi}{4} + 2k\pi\right)\right)$ **M1**

Tech... $\cos\theta$ ... $\sin\theta$ ... integ...

Also $z^4 = (r(\cos\theta + \mathrm{i}\sin\theta))^4 = r^4(\cos 4\theta + \mathrm{i}\sin 4\theta)$ by de Moivre's theore...

So $16 = r^4 \therefore r = 2$ and $\frac{3\pi}{4} + 2k\pi = 4\theta$ **M1**

Tech... and ... side ... differ...

For $k = 0$, $\theta = \frac{3\pi}{16}$, so $z_1 = 2\mathrm{e}^{\mathrm{i}\frac{3\pi}{16}}$ **A1**

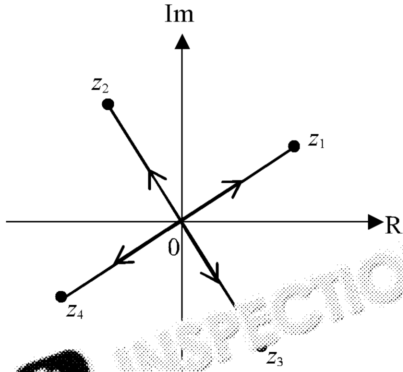
For $k = 1$, $\theta = \dfrac{11\pi}{16}$, so $z_2 = 2e^{i\frac{11\pi}{16}}$ **A1**

For $k = -1$, $\theta = -\dfrac{5\pi}{16}$, so $z_3 = 2e^{-i\frac{5\pi}{16}}$ **A1**

For $k = -2$, $\theta = -\dfrac{13\pi}{16}$, so $z_4 = 2e^{-i\frac{13\pi}{16}}$ **A1**

Alte… $k = 2$, … you w… to pro… interv…

c)



A1 **[11 Marks]**

5. a) **Show that $A - iB = \dfrac{2}{2 + e^{3i\theta}}$**

$A = 1 - \frac{1}{2}\cos 3\theta + \frac{1}{4}\cos 6\theta - \frac{1}{8}\cos 9\theta + \ldots$ and $B = \frac{1}{2}\sin 3\theta - \frac{1}{4}\sin 6\theta + \frac{1}{8}\sin 9\theta$…

So $A - iB = \left(1 - \frac{1}{2}\cos 3\theta + \frac{1}{4}\cos 6\theta - \frac{1}{8}\cos 9\theta + \ldots\right) - i\left(\frac{1}{2}\sin 3\theta - \frac{1}{4}\sin 6\theta + \frac{1}{8}\sin\ldots\right.$

$= 1 - \frac{1}{2}(\cos 3\theta + i\sin 3\theta) + \frac{1}{2^2}(\cos 6\theta + i\sin 6\theta) - \frac{1}{2^3}(\cos 9\theta + i\sin 9\theta\ldots$

$= 1 - \frac{1}{2}e^{3i\theta} + \frac{1}{2^2}e^{6i\theta} - \frac{1}{2^3}e^{9i\theta} + \ldots$ **M1**

This is an infinite geometric series with $a = 1$ and $r = -\frac{1}{2}e^{3i\theta}$

So $S_\infty = \dfrac{1}{1 - \left(-\frac{1}{2}e^{3i\theta}\right)} = \dfrac{1}{1 + \frac{1}{2}e^{3i\theta}}$ **M1**

Tech… sum o… geom… and c…

$= \dfrac{2}{2 + e^{3i\theta}}$ **A1**

b) **Show that $A = \dfrac{4 + 2\cos 3\theta}{5 + 4\cos 3\theta}$, and find a corresponding expression for B**

Tech… and c… deno…

$\dfrac{2}{2 + e^{3i\theta}} = \dfrac{2}{2 + \cos 3\theta + i\sin 3\theta}$

$= \dfrac{2}{2 + \cos 3\theta + i\sin 3\theta} \times \dfrac{2 + \cos 3\theta - i\sin 3\theta}{2 + \cos 3\theta - i\sin 3\theta} = \dfrac{4 + 2\cos 3\theta - 2i\sin 3\theta}{(2 + \cos 3\theta)^2 - i^2\sin^2 3\theta}$

$= \dfrac{4 + 2\cos 3\theta - 2i\sin 3\theta}{4 + 4\cos 3\theta + \cos^2 3\theta + \sin^2 3\theta}$ **M1**

Hint: … $\cos^2 3$…

$= \dfrac{4 + 2\cos 3\theta - 2i\sin 3\theta}{5 + 4\cos 3\theta}$ **M1**

$\therefore A = \text{Re}\left(\dfrac{2}{2 + e^{3i\theta}}\right) = \dfrac{4 + 2\cos 3\theta}{5 + 4\cos 3\theta}$ **A1**

Similarly, $B = -\text{Im}\left(\dfrac{2}{2 + e^{3i\theta}}\right) = \dfrac{2\sin 3\theta}{5 + 4\cos 3\theta}$ **A1** **[9 Marks]**

6. $z = 2 + 2\mathrm{i}\sqrt{3}$ so $|z| = \sqrt{2^2 + \left(2\sqrt{3}\right)^2} = \sqrt{16} = 4$ and $\arg z = \arctan\left(\frac{2\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\therefore z = 4\mathrm{e}^{\frac{\pi}{3}\mathrm{i}}$ **M1**

Let $w = r\mathrm{e}^{\mathrm{i}\theta}$

So $\frac{z^3}{2w} = \frac{4^3}{2r}\mathrm{e}^{\left(3\left(\frac{\pi}{3}\right)-\theta\right)\mathrm{i}}$

$\left|\frac{z^3}{2w}\right| = \frac{4^3}{2r} = \frac{64}{2r} = \frac{32}{r}$ and $|z| = 4$ **M1**

Tech… theor… cube … argu… argu…

We are told that these are equal; therefore, $\frac{32}{r} = 4 \therefore r = 8$ **M1**

So $\frac{z^3}{2w} = 4\mathrm{e}^{\left(3\left(\frac{\pi}{3}\right)-\theta\right)\mathrm{i}}$

$\mathrm{Re}\left(\frac{z^3}{2w}\right) = 4\cos\left(3\times\frac{\pi}{3} - \theta\right) = 4\cos(\pi-\theta)$ **M1**

Tech… compl… and … argu…

We are told that this is equal to $-2\sqrt{2}$, so $\cos(\pi-\theta) = \frac{-\sqrt{2}}{2}$, i.e. $\pi - \theta = \frac{3\pi}{4}$ or $\frac{5…}{4}$

This gives $\theta = \frac{\pi}{4}$ or $-\frac{\pi}{4}$ since $-\pi < \theta \le \pi$ **M1**

$\therefore w = 8\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$ or $8\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}$ **A1** **[6 Marks]**

7. $\frac{8\sqrt{3}-8\mathrm{i}}{1-\mathrm{i}\sqrt{3}} = \frac{8\sqrt{3}-8\mathrm{i}}{1-\mathrm{i}\sqrt{3}} \times \frac{1+\mathrm{i}\sqrt{3}}{1+\mathrm{i}\sqrt{3}} = \frac{8\sqrt{3}+24\mathrm{i}-8\mathrm{i}-8\mathrm{i}^2\sqrt{3}}{1-3\mathrm{i}^2}$ **M1**

$= \frac{16\sqrt{3}+16\mathrm{i}}{4} = 4\sqrt{3}+4\mathrm{i}$ **A1**

$\left|4\sqrt{3}+4\mathrm{i}\right| = \sqrt{\left(4\sqrt{3}\right)^2+4^2} = \sqrt{64} = 8$ and $\arg\left(4\sqrt{3}+4\mathrm{i}\right) = \arctan\left(\frac{4}{4\sqrt{3}}\right) = \frac{\pi}{6}$

$\therefore 4\sqrt{3}+4\mathrm{i} = 8\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}$ **M1**

So $\left(\frac{8\sqrt{3}-8\mathrm{i}}{1-\mathrm{i}\sqrt{3}}\right)^n = \left(4\sqrt{3}+4\mathrm{i}\right)^n = 8^n\mathrm{e}^{\frac{n\pi}{6}\mathrm{i}}$

$= 8^n\left(\cos\frac{n\pi}{6} + \mathrm{i}\sin\frac{n\pi}{6}\right)$ **M1**

This is real and negative when $\sin\frac{n\pi}{6} = 0$ and $\cos\frac{n\pi}{6} < 0$

Alter… multi… retur…

So we need $\frac{n\pi}{6} = k\pi$, where k is an odd integer

For $k = 1$, $n = 6$, $\sin\pi = 0$ and $\cos\pi = -1$, so the number is real and negative $\therefore n = …$

[… Marks]

8. For this triangle, one vertex is $V_1 = \left(5, 2\sqrt{3}\right)$ and its centre is $\left(2, \sqrt{3}\right)$

So consider $V_1' = \left(3, \sqrt{3}\right)$, which represents $2\sqrt{3}\mathrm{e}^{\frac{\pi}{6}\mathrm{i}}$ **M1**

Tech… equil… at the… point … trans…

The cube roots of unity are 1, ω and ω^2, where $\omega = \mathrm{e}^{\frac{2\pi}{3}\mathrm{i}}$

So the other vertices of the translated triangle are given by:

$V_2' = 2\sqrt{3}\mathrm{e}^{\mathrm{i}\left(\frac{2\pi}{3}+\frac{\pi}{6}\right)} = 2\sqrt{3}\mathrm{e}^{\frac{5\pi}{6}\mathrm{i}}$, i.e. $\left(-3, \sqrt{3}\right)$ **M1A1**

Tech… vertic… by 2…

$V_3' = 2\sqrt{3}\mathrm{e}^{\mathrm{i}\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)} = 2\sqrt{3}\mathrm{e}^{\frac{3\pi}{2}\mathrm{i}}$, i.e. $\left(0, -2\sqrt{3}\right)$ **M1A1**

So $V_2 = \left(-1, 2\sqrt{3}\right)$ **A1**

$V_3 = \left(2, -\sqrt{3}\right)$ **A1** **[7 Marks]**

Preview of Answers Ends Here