Further Maths A Level | Edexcel | 9FM0

2017 specification
(first exams in 2019)

Topic Tests: Challenge Tests – Set A

A Level Edexcel Further Mathematics: Core Pure Mathematics: Part 2#

#The remaining topics to cover the A Level (9FM0) Core Pure Mathematics

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Contents

Teacher's Introduction

Content

This pack contains 6 challenge level topic tests for A Level Edexcel Further Mathematics: Core Pure Mathematics 2.

Each test comes with fully worked solutions, containing tips, hints and technique boxes to help students on a particular question. Answers should be given to three significant figures unless specified in the question.

These topic tests have been **fully cross-referenced** to the Pearson textbook, *Edexcel A level Further Mathematics Core Pure Mathematics Book 2* (ISBN 978-1292183343), for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the challenge tests

These **challenge** tests have been designed to **stretch and challenge** your students. 50% of the marks come from questions similar in style to our fundamentals tests. These questions isolate and test the core skills in each topic. The other 50% of the marks come from questions of increased difficulty that progress and start to combine the concepts in the topic.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Cross-referencing Grid

Topic	Edexcel spec. points	Subtopics
Complex Numbers II	2.8–2.11	Exponential form of complex numbers, multiplying and dividing complex numbers, de Moivre's theorem, trigonometric identities, sums of series, roots of a complex number, solving geometric problems
Series	4.4–4.6	The method of differences, higher derivatives, Maclaurin series, series expansions of compound functions
Further Calculus	5.2–5.6	Improper integrals, the mean value of a function, differentiating inverse trigonometric functions, integrating with inverse trigonometric functions, integrating using partial fractions, volumes of revolution around the x-axis and the y-axis, volumes revolution of parametrically defined curves, modelling with volumes of revolution
Polar Coordinates	7.1–7.3	Polar coordinates and equations, sketching curves, area enclosed by a polar curve, tangents to polar curves
Hyperbolic Functions	8.1–5	Hyperbolic functions, inverse hyperbolic functions, identities and equations, differentiating hyperbolic functions, integrating hyperbolic functions
Differential Equations	9.1–9.9	First- and second-order differential equations, boundary conditions, modelling, simple harmonic motion, damped and forced harmonic motion, coupled first-order simultaneous differential equations

Further Calculus – Test A (55 mins)

Subtopics: Improper integrals, the mean value of a function, differentiating inverse trigo
inverse trigonometric functions, integrating using partial fractions, volumes of revoluti
volumes of revolution of parametrically defined curves, modelling with vo

1. a) Find $\int \frac{1}{4+x^2}\,dx$

   b) Find $\int \frac{1}{4+49x^2}\,dx$, giving your answer in the form $A\arctan(Bx)$
   constants to be found, and c is an arbitrary constant

2. By considering their behaviour at each limit, show that the following i

   a) $\int_4^{\infty} \frac{1}{\sqrt{x}}\,dx$ b) $\int_0^{\pi/2} \tan x\,dx$

3. Use the substitution $x = \frac{1}{5}\sin\theta$ to find $\int \frac{1}{\sqrt{1-25x^2}}\,dx$ in the form
   A and B are constants to be found, and c is an arbitrary constant.

4. Use **implicit differentiation** to show that $\frac{d}{dx}\arccos\left(\frac{1}{x}\right) = \frac{1}{x\sqrt{x^2-1}}$

5. Evaluate the improper integral $\int_0^{\infty} 3x^2 e^{-x^3}\,dx$

6. a) Find $\int_0^{\pi/2} \frac{\cos x}{(\sin x+1)^2}\,dx$

   b) Find the **exact mean value** of $f(x) = \frac{\cos x}{(\sin x+1)^2}$ over the interval

   c) Hence state the **exact mean value** of $f(x)+3$ over the interval $[0$

7. **Using partial fractions**, show that $\int \frac{20x-15}{(x^2+9)(x+6)}\,dx = P\ln\left|\frac{\sqrt{x^2+9}}{x+6}\right.$
   where P and Q are constants to be found, and c is an arbitrary constant

8. The diagram to the right shows the curve with equation $2y^2 = x\cos\left(\frac{x}{2}\right.$

   a) Find the x-coordinates of A and B

   b) Find the **exact** volume of the solid formed when the shaded region
   R is rotated 2π radians about the x-axis

9. The diagram to the right shows the curve C with parametric equations
   $x = \frac{400}{11+t}$, $y = \frac{1}{2}t+3$, $t \geq -6$. A table base is modelled as the solid of
   revolution formed when the finite region R bounded by C, the x-axis,
   the y-axis and the line $y = a$ is rotated 2π radians about the y-axis.
   Given that the units of x and y are centimetres, and that the table base
   has volume $15\,500\pi$ cm^3, find the **exact** value of a.

Preview of Questions Ends Here

Solutions to Polar Coordinates – Test A

1. a) $r = 3$
$\therefore r^2 = 9$
$\therefore x^2 + y^2 = 9$ **M1A1**

b) $r\sin\theta = 1$
$\therefore y = 1$ **M1A1**

c) $r - 2\sec\theta = 0$
$\therefore r = 2\sec\theta$
$\therefore r\cos\theta = 2$ **M1**
$\therefore x = 2$ **A1**

d) $r\tan\theta - r = 0$
$\therefore r\dfrac{\sin\theta}{\cos\theta} = r$
$\therefore r\sin\theta = r\cos\theta$ **M1**
$\therefore y = x$ **A1**

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Tip: T
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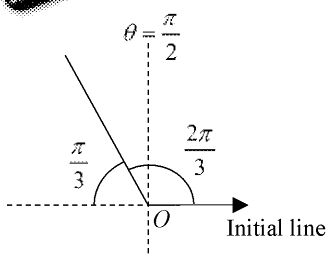
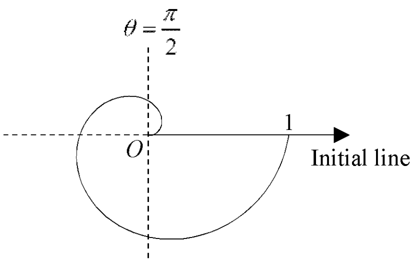
until i
involv
Then u
$r^2 = x$
$r\sin\theta$

2. a) π



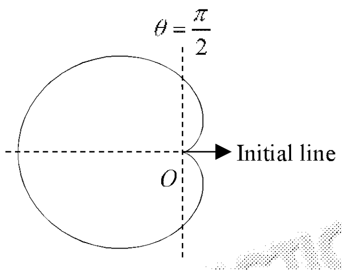
For a half-line starting at the origin
For a line that makes an angle of $\frac{\pi}{3}$

b) $r = \dfrac{\theta}{2\pi}$



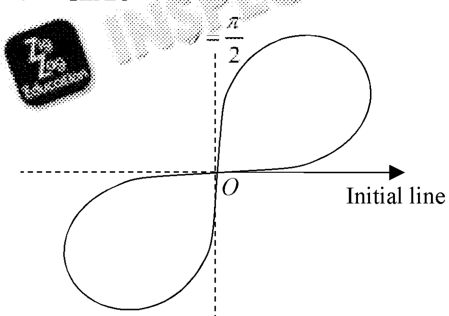
For a spiral that increases in radius
For a curve starting at the origin **A1**

c) $r = 1 - \cos\theta$



For cardioid shape **A1**
For correct orientation **A1**

d) $r^2 = \sin 2\theta$



For figure-of-eight shape **A1**
For correct orientation **A1**

[8

3. $r = 1 - \sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

a) $y = r\sin\theta = (1 - \sin\theta)\sin\theta = \sin\theta - \sin^2\theta$

$\frac{dy}{d\theta} = \cos\theta - 2\sin\theta\cos\theta = \cos\theta(1 - 2\sin\theta)$ **M1**

So $\frac{dy}{d\theta} = 0$ when $\cos\theta = 0$ or when $1 - 2\sin\theta = 0$ **M1**

$\cos\theta$ is never zero in the domain considered, so we are left with $1 - 2\sin\theta =$

$\therefore 2\sin\theta = 1$

$\therefore \sin\theta = \frac{1}{2}$

$\therefore \theta = \arcsin\left(\frac{1}{2}\right)$ **M1**

We want the solution in the range $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, so $\theta = \frac{\pi}{6}$

and so $r = 1 - \sin\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$

So the tangent to the curve is parallel to the initial line at the point with pola

Tip: T
initial

b) $x = r\cos\theta = (1 - \sin\theta)\cos\theta = \cos\theta - \cos\theta\sin\theta$

$\frac{dx}{d\theta} = -\sin\theta + \sin^2\theta - \cos^2\theta$ **M1**

$= -\sin\theta + \sin^2\theta - (1 - \sin^2\theta)$

$= 2\sin^2\theta - \sin\theta - 1$

So $\frac{dx}{d\theta} = 0$ when $2\sin^2\theta - \sin\theta - 1 = 0$ **M1**

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the in

This is a quadratic equation in $\sin\theta$ that factorises as:

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\therefore \sin\theta = -\frac{1}{2}$ or 1 **M1**

We want the solution in the range $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\sin\theta \neq 1$ in this interval, so:

$\theta = \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

and so $r = 1 - \sin\left(-\frac{\pi}{6}\right) = 1 + \frac{1}{2} = \frac{3}{2}$

So the tangent to the curve is perpendicular to the initial line at the point wi

[Marks]

4. $r = \sqrt{\theta}$

The region R is enclosed by the curve between $\theta = 0$ and $\theta = \frac{3\pi}{2}$, so:

Area $= \frac{1}{2}\int_0^{\frac{3\pi}{2}} (\sqrt{\theta})^2\, d\theta$ **M1M1**

Tip: T
curve
$\theta = \beta$
$\frac{1}{2}\int_\alpha^\beta$

$= \frac{1}{2}\int_0^{\frac{3\pi}{2}} \theta\, d\theta$

$= \frac{1}{2}\left[\frac{1}{2}\theta^2\right]_0^{\frac{3\pi}{2}}$ **M1**

$= \frac{1}{2}\left(\frac{1}{2}\times\left(\frac{3\pi}{2}\right)^2 - \frac{1}{2}\times 0\right)$

$= \frac{9}{16}\pi^2$ **A1** **[4 Marks]**

5. a) $x^2 + 2y^2 = 3$

$\therefore x^2 + y^2 + y^2 = 3$

$\therefore r^2 + (r\sin\theta)^2 = 3$ **M1**

$\therefore r^2(1+\sin^2\theta) = 3$

$\therefore r^2 = \dfrac{3}{1+\sin^2\theta}$ **A1**

b) $y = 2x + 1$

$\therefore r\sin\theta = 2r\cos\theta + 1$ **M1**

$\therefore r\sin\theta - 2r\cos\theta = 1$

$\therefore r(\sin\theta - 2\cos\theta) = 1$ **M1**

$\therefore r = \dfrac{1}{\sin\theta - 2\cos\theta}$ **A1**

c) $x^2 - y^2 = 1$

$\therefore r^2\cos^2\theta - r^2\sin^2\theta = 1$ **M1**

$\therefore r^2(\cos^2\theta - \sin^2\theta) = 1$

$\therefore r^2\cos 2\theta = 1$ **M1**

$\therefore r^2 = \sec 2\theta$ **A1**

Tip: Th… trigono… to turn… functio… angle…

[8 M…

6. $r = a - \sin\theta$

$$\text{Area} = \frac{1}{2}\int_0^{2\pi} (a-\sin\theta)^2\,d\theta \quad \textbf{M1}$$

$$= \frac{1}{2}\int_0^{2\pi} (a^2 - 2a\sin\theta + \sin^2\theta)\,d\theta$$

Tip: Re… formu… $\sin^2\theta$…

$$= \frac{1}{2}\int_0^{2\pi} \left(a^2 - 2a\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= \frac{1}{2}\left[a^2\theta + 2a\cos\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{2\pi} \quad \textbf{M1}$$

$$= \frac{1}{2}\left(a^2 \times 2\pi + 2a\cos 2\pi + \frac{1}{2}\times 2\pi - \frac{1}{4}\sin 4\pi - a^2\times 0 - 2a\cos 0 - \frac{1}{2}\times 0 + \frac{1}{4}\sin 0\right)$$

$$= \frac{1}{2}(2\pi a^2 + 2a + \pi - 2a)$$

$$= \left(a^2 + \frac{1}{2}\right)\pi \quad \textbf{A1}$$

We are told in the question that this area is $\frac{33}{2}\pi$, so $a^2 + \frac{1}{2} = \frac{33}{2}$ **M1**

$\therefore a^2 = \frac{33}{2} - \frac{1}{2} = 16$

So $a = \pm\sqrt{16} = \pm 4$, but we are told in the question that $a > 1$, so $a = 4$ **A1** **[5 M…**

7. $r = 1 + \cos\theta$

$y = r\sin\theta = (1 + \cos\theta)\sin\theta$

$\dfrac{dy}{d\theta} = -\sin^2\theta + (1+\cos\theta)\cos\theta$ **M1**

$= \cos\theta + \cos^2\theta - \sin^2\theta$

$= \cos\theta + \cos^2\theta - (1-\cos^2\theta)$

$= 2\cos^2\theta + \cos\theta - 1$

So $\dfrac{dy}{d\theta} = 0$ when $2\cos^2\theta + \cos\theta - 1 = 0$ **M1**

$\therefore (2\cos\theta - 1)(\cos\theta + 1) = 0$

$\therefore 2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$

$\therefore \theta = \arccos\left(\dfrac{1}{2}\right)$ or $\theta = \arccos(-1)$ **M1**

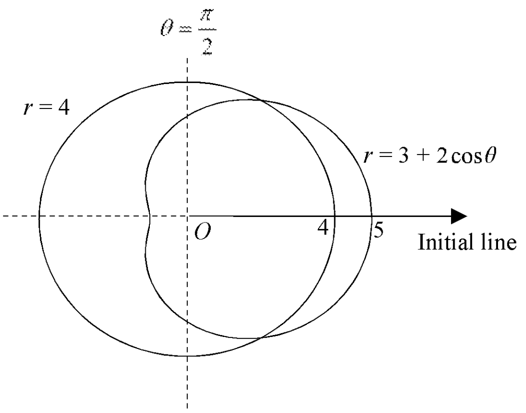
We are only conside... $\dfrac{\pi}{2}$, so the only solution we have is $\theta = \dfrac{\pi}{3}$

and s... $\cos\dfrac{\pi}{3} = \dfrac{3}{2}$ **A1**

So the tangent passes through the point $\left(\dfrac{3}{2}, \dfrac{\pi}{3}\right)$ and is parallel to the initial line, h...

$y = r\sin\theta = \dfrac{3}{2} \times \sin\dfrac{\pi}{3} = \dfrac{3\sqrt{3}}{4}$ **A1** **[5 Marks]**

8. a)



For curve ...
For correc...
For circle ...
For crossi...

b) The curves cross where $3 + 2\cos\theta = 4$ **M1**

$\therefore 2\cos\theta = 1$

$\therefore \theta = \arccos\left(\dfrac{1}{2}\right)$ **M1**

So for $0 \le \theta \le 2\pi$, $\theta = \dfrac{\pi}{3}$ or $\theta = \dfrac{5\pi}{3}$ **A1**

Since both points lie or ... 4, the coordinates of the crossing poin...

[8 Marks]

Preview of Answers Ends Here