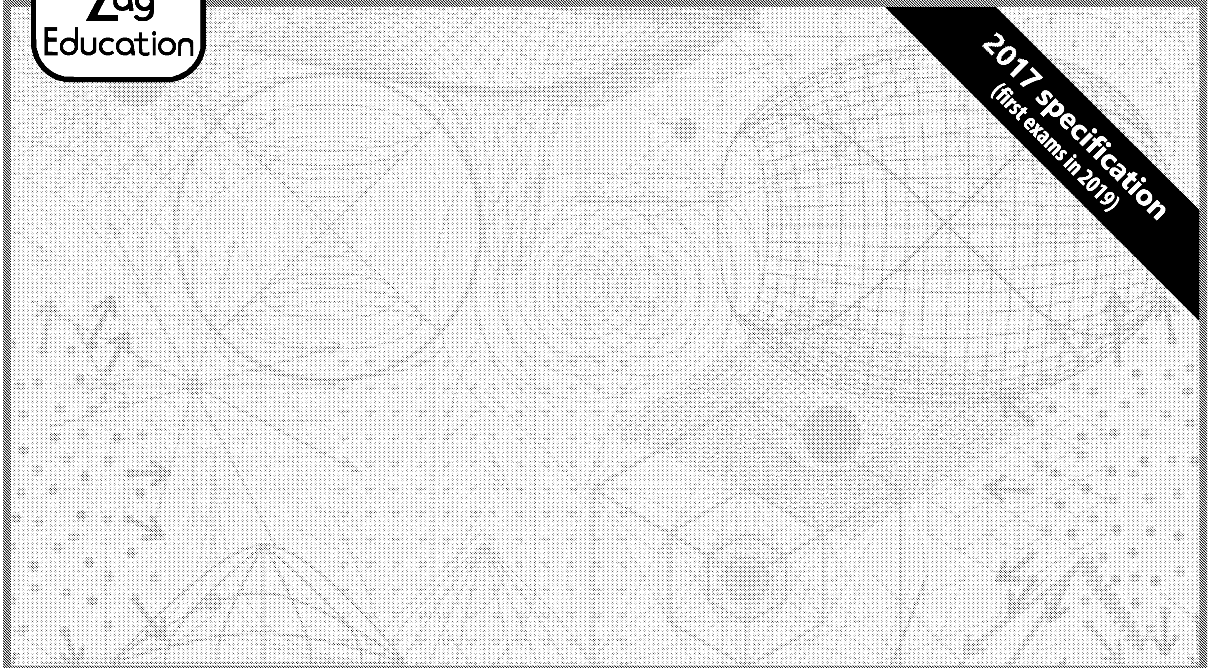
Zig Zag Education

Further Maths AS / A Level | Edexcel | 8FM0/9FM0



2017 specification (first exams in 2019)

Topic Tests: Expert Tests – Set A

A Level Edexcel Further Mathematics: Core Pure Mathematics: Part 1#

#Every topic of AS (8FM0) Core Pure Mathematics

Update v1.1, April 2024

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Contents

Teacher's Introduction

Content

This pack contains 6 expert level topic tests for A Level Edexcel Further Mathematics: Core Pure Mathematics.

Each test comes with fully worked solutions, containing tips, hints and technique boxes to help students on a particular question. Answers should be given to three significant figures unless specified in the question.

These topic tests have been **fully cross-referenced** to the Pearson textbook, *Edexcel AS and A level Further Mathematics Core Pure Mathematics Book 1/AS* (ISBN 978-1292183336), for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who are ready to go beyond the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and progresses the concepts covered.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Update v1.1, April 2024
Test 2.3a question and answer 1d) – Coefficient of x^2 corrected to 138

Cross-referencing Grid

Topic	Edexcel spec. points	Subtopics
Complex Numbers	2.1–2.7	Imaginary and complex numbers, multiplying complex numbers, complex conjugation, roots of quadratic equations, solving cubic and quartic equations, Argand diagrams, modulus and argument, modulus-argument form of complex numbers, loci and regions in the Argand diagram
Algebra and Functions	4.1–4.3	Sums of natural numbers, sums of squares and cubes, roots of polynomials, linear transformations of roots
Volumes of Revolution	5.1	Volumes of revolution with Cartesian equations, adding and subtracting volumes, modelling with volumes
Matrices	3.1–3.8	Matrices, matrix multiplication, determinants, inverting 2×2 and 3×3 matrices, solving systems of equations using matrices, linear transformations in two and three dimensions, reflections and rotations, enlargements and stretches, successive transformations, the inverse of a linear transformation
Proof by Induction	1.1	Proof by mathematical induction, proving divisibility results, proving statements involving matrices
Further Vectors	6.1–6.5	Equation of a line in three dimensions, equation of a plane in three dimensions, scalar product, calculating angles between lines and planes, points of intersection, finding perpendiculars

Further Vectors – Test A (80 mins)

Subtopics: Equation of a line in three dimensions, equation of a plane in three d calculating angles between lines and planes, points of intersection, find

1. The point $(1, p, 3)$ lies on the line with vector equation $\mathbf{r} = (6\mathbf{i} + 6\mathbf{j} + a$
   Find the value of the constants p and a

2. Three points are given by $A(-1, -7, 3)$, $B(1, 1, -1)$ and $C(2, -2, -2)$
   a) Show that A, B and C **are not** collinear.
   The point D has coordinates $(3, 9, -5)$
   b) Show that A, B, C and D **are** coplanar.

3. The lines l_1 and l_2 are given by the vector equations $l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix}$
   a) Show that the lines l_1 and l_2 intersect, and find the coordinates of t
   b) Show that the **acute** angle between l_1 and l_2 is 81.8° to **1 decimal**
   The line l_3 has Cartesian equation $\frac{x+1}{2} = \frac{y-4}{-2} = \frac{z-2}{-1}$. The lines l_1 a
   c) Find the **shortest distance** between the lines l_1 and l_3 in the form

4. A submarine searches for a shipwreck. Relative to some fixed point O
   a point S with position vector $(6\mathbf{i} + 4\mathbf{j} - 5.2\mathbf{k})$ km. The submarine is m
   a **straight line** from A, which has position vector $(16\mathbf{i} - 6\mathbf{j} - 4\mathbf{k})$ km, to
   $(-12\mathbf{i} + 15\mathbf{j} - 4\mathbf{k})$ km. The submarine's sonar detects the shipwreck if
   2 km of the shipwreck.
   a) Determine whether the submarine detects the shipwreck.
   b) Give one criticism of this model.

5. The line L has equation $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(6\mathbf{i} + \mathbf{j} + \mathbf{k})$. The plane Π has eq
   a) Find the coordinates of the **reflection** of the point $P = (1, 0, 1)$ in t
   L and Π intersect at the point $(-5, -1, 0)$
   b) Show that the point P lies on the line L, and hence find a vector eq
   the line L in the plane Π

6. A space probe is approaching Jupiter. Relative to some fixed point O,
   modelled as a straight line that passes through the points (49, 17, 8) an
   outside Jupiter's atmosphere, and where the unit of distance is thousan

   the atmosphere of Jupiter is modelled as a plane with vector equation

   probe will survive entry into Jupiter's atmosphere if the acute angle at
   and 60
   a) Find a vector equation for the plane modelling the edge of Jupiter's
   $\mathbf{r}.(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) = d$, where a, b, c and d are **integers**.
   b) Determine whether the probe will survive entry into Jupiter's atmo
   c) Give one criticism of the model.

Preview of Questions Ends Here

Solutions to Algebra and Functions – Test A

1. $x^4 - 4x^3 + 6x - 20 = 0$ has roots α, β, γ and δ

a) $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\dfrac{d}{a}$

$= -\dfrac{6}{1}$ **M1**

$= -6$ **A1**

b) $\alpha\beta\gamma\delta = \dfrac{e}{a}$

$= \dfrac{-20}{1}$ **M1**

$= -20$ **A1**

c) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta}{\alpha\beta\gamma\delta}$

$= \dfrac{-6}{-20}$ **M1**

$= \dfrac{3}{10}$ **A1**

d) $20x^4 + Px^3 + 132x^2 - 94x + 21 = 0$ has roots $\dfrac{1}{\alpha}+1$, $\dfrac{1}{\beta}+1$, $\dfrac{1}{\gamma}+1$ and $\dfrac{1}{\delta}+1$

$-\dfrac{P}{20} = \left(\dfrac{1}{\alpha}+1\right) + \left(\dfrac{1}{\beta}+1\right) + \left(\dfrac{1}{\gamma}+1\right) + \left(\dfrac{1}{\delta}+1\right)$

$= \left(\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}\right) + 4$

$= \dfrac{3}{10} + 4$ **M1**

$= \dfrac{43}{10}$

So $P = -20 \times \dfrac{43}{10} = -86$ **A1** **[8 Marks]**

2. a) **Show that $\sum_{r=1}^{n}\left(2r^3 + r\right) = \frac{1}{2}n(n+1)\left(n^2+n+1\right)$**

$\sum_{r=1}^{n}\left(2r^3 + r\right) = 2\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$

$= 2 \times \dfrac{1}{4}n^2(n+1)^2 + \dfrac{1}{2}n(n+1)$ **M1**

$= \dfrac{1}{2}n(n+1)\left[n(n+1)+1\right]$

$= \dfrac{1}{2}n(n+1)\left(n^2+n+1\right)$ **A1**

Tip: L…
try to…
where…
also a…
This is…
every…
attem…
result…

b) **Show that there is no positive integer n that satisfies $\sum_{r=1}^{n}\left(2r^3 + r\right) = \sum_{r=1}^{n^2} r$**

$\sum_{r=1}^{n^2} r = \dfrac{1}{2}n^2\left(n^2+1\right)$ **M1**

If $\sum_{r=1}^{n}\left(2r^3 + r\right) = \sum_{r=1}^{n^2} r$ then $\dfrac{1}{2}n(n+1)\left(n^2+n+1\right) = \dfrac{1}{2}n^2\left(n^2+1\right)$ **M1**

Divide both sides by $\frac{1}{2}n$ and expand to get:

$n^3 + 2n^2 + 2n + 1 = n^3 + n$

which rearranges to become:

$2n^2 + n + 1 = 0$

The discriminant of this polynomial is $b^2 - 4ac = 1^2 - 4 \times 2 \times 1 = -7 < 0$ so th

Hence, in particular, there is no positive integer *n* such that $\sum_{r=1}^{n}(2r^3 + r) = \sum$

3. $27x^3 + 81x^2 + 18x - 16 = 0$ has roots α, $\alpha - k$, and $\alpha + 2k$

Alter
27(x
then
$27x^3$

$$\alpha + \alpha - k + \alpha + 2k = -\frac{b}{a}$$

$$= -\frac{81}{27} \quad \textbf{M1}$$

$$= -3$$

So $3\alpha + k = -3$

This gives $k = -3 - 3\alpha$

$$\alpha(\alpha - k) + \alpha(\alpha + 2k) + (\alpha - k)(\alpha + 2k) = \frac{c}{a}$$

$$= \frac{18}{27} \quad \textbf{M1}$$

$$= \frac{2}{3}$$

Also, $\alpha(\alpha - k) + \alpha(\alpha + 2k) + (\alpha - k)(\alpha + 2k) = \alpha^2 - \alpha k + \alpha^2 + 2\alpha k + \alpha^2 + \alpha k - 2k$

$$= 3\alpha^2 + 2\alpha k - 2k^2$$

So $3\alpha^2 + 2\alpha k - 2k^2 = \frac{2}{3}$ **A1**

Substituting in $k = -3 - 3\alpha$ leads to:

$$3\alpha^2 + 2\alpha(-3 - 3\alpha) - 2(-3 - 3\alpha)^2 = 3\alpha^2 - 6\alpha - 6\alpha^2 - 18 - 36\alpha - 18\alpha^2$$

$$= -21\alpha^2 - 42\alpha - 18$$

So $-21\alpha^2 - 42\alpha - 18 = \frac{2}{3}$

Rearranging this gives: $21\alpha^2 + 42\alpha + \frac{56}{3} = 0$ **M1**

and so $\alpha = \dfrac{-42 \pm \sqrt{42^2 - 4 \times 21 \times 56/3}}{2 \times 21}$ **M1**

$$= \frac{-42 \pm \sqrt{196}}{42}$$

So $\alpha = -\frac{4}{3}$ or $\alpha = -\frac{2}{3}$ **A1**

If $\alpha = -\frac{4}{3}$ then $k = -3 - 3\alpha = -3 + 4 = 1$

Then the three roots are $-\frac{4}{3}$, $-\frac{7}{3}$ and $\frac{2}{3}$

but then $\alpha(\alpha - k)(\alpha + 2k) = \left(-\frac{4}{3}\right) \times \left(-\frac{7}{3}\right) \times \frac{2}{3} = \frac{56}{27} \neq -\frac{d}{a} = \frac{16}{27}$ **M1**

If $\alpha = -\frac{2}{3}$ then $k = -3 - 3\alpha = -3 + 2 = -1$

Then the three roots are $-\frac{2}{3}$, $\frac{1}{3}$ and $-\frac{8}{3}$

and then $\alpha(\alpha - k)(\alpha + 2k) = \left(-\frac{2}{3}\right) \times \frac{1}{3} \times \left(-\frac{8}{3}\right) = \frac{16}{27} = -\frac{d}{a}$

So the three roots are $-\frac{2}{3}, \frac{1}{3}$ and $-\frac{8}{3}$ **A1** **[8 Marks]**

4. a) $\sum_{r=1}^{n}(ar+b) = a\sum_{r=1}^{n}r + b\sum_{r=1}^{n}1$

$= a \times \frac{1}{2}n(n+1) + bn$ **M1**

We are told in the question that $\sum_{r=1}^{8}(ar+b) = 6$ and $\sum_{r=1}^{12}(ar+b) = 17$, so

$\frac{a}{2} \times 8 \times (8+1) + b \times 8 = 6$

$\frac{a}{2} \times 12 \times (12+1) + b \times 12 = 17$ **M1**

Simplifying these gives the simultaneous equations:

$36a + 8b = 6$ **(1)**

$78a + 12b = 17$ **(2)**

2(2) − 3(1)

$48a = 16$

So $a = \frac{16}{48} = \frac{1}{3}$ **A1**

Substituting this into **(1)** gives:

$36 \times \frac{1}{3} + 8b = 6$

$\therefore b = \frac{6 - \frac{1}{3} \times 36}{8} = -\frac{3}{4}$ **A1**

b) $\sum_{r=1}^{n}(ar+b) = \sum_{r=1}^{n}\left(\frac{1}{3}r - \frac{3}{4}\right)$

$= \frac{1}{3}\sum_{r=1}^{n}r - \frac{3}{4}\sum_{r=1}^{n}1$

$= \frac{1}{3} \times \frac{1}{2}n(n+1) - \frac{3}{4}n$ **M1**

$= \frac{1}{12}n(2(n+1) - 9)$

$= \frac{1}{12}n(2n-7)$ **A1**

c) $\sum_{r=1}^{120}(ar+b) = \frac{1}{12} \times 120 \times (2 \times 120 - 7)$ **M1**

$= 2330$ **A1** **[8 Marks]**

5. $36x^3 + px + 12 = 0$ has roots α, β and γ

a) $\alpha + \beta + \gamma = -\frac{b}{a}$

$= -\frac{0}{36}$

$= 0$ **A1**

b) $\alpha\beta\gamma = -\frac{d}{a}$

$= -\frac{12}{36}$ **M1**

$= -\frac{1}{3}$ **A1**

c) We are told in the question that $\gamma = 8\alpha$. Substituting this into the two equati[illegible]

$\alpha + \beta + \gamma = \alpha + \beta + 8\alpha = 9\alpha + \beta$, so $9\alpha + \beta = 0$ **(1)**

$\alpha\beta\gamma = \alpha\beta(8\alpha) = 8\alpha^2\beta$, so $8\alpha^2\beta = -\frac{1}{3}$ **(2)** **M1**

(1) gives $\beta = -9\alpha$. Substituting this into **(2)** gives:

$8\alpha^2 \times (-9\alpha) = -72\alpha^3 = -\frac{1}{3}$ **M1**

This simplifies to:

$\alpha^3 = \frac{1}{216}$

We are told in the question that the roots are real numbers, so $\alpha = \sqrt[3]{\frac{1}{216}} = \frac{1}{6}$

Hence $\beta = -9\alpha = -9 \times \frac{1}{6} = -\frac{3}{2}$

and $\gamma = 8\alpha = 8 \times \frac{1}{6} = \frac{4}{3}$ **A1**

d) [illegible] $\alpha\beta + \alpha\gamma + \beta\gamma$

$= \frac{1}{6} \times \left(-\frac{3}{2}\right) + \frac{1}{6} \times \frac{4}{3} + \left(-\frac{3}{2}\right) \times \frac{4}{3}$ **M1**

$= -\frac{73}{36}$

and so $p = 36 \times \left(-\frac{73}{36}\right) = -73$ **A1** **[11 Marks]**

6. $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ **M1**

$\sum_{r=1}^{7n} (2r - 18) = 2\sum_{r=1}^{7n} r - 18\sum_{r=1}^{7n} 1$

$= 2 \times \frac{1}{2} \times 7n(7n+1) - 18 \times 7n$ **M1**

$= 7n(7n - 17)$

So these sums are equal when $\frac{1}{6}n(n+1)(2n+1) = 7n(7n-17)$ **M1**

Dividing through by $\frac{1}{6}n$ and multiplying out gives:

$2n^2 + 3n + 1 = 294n - 714$

$\therefore 2n^2 - 291n + 715 = 0$ **A1**

So $n = \frac{291 \pm \sqrt{(-291)^2 - 4 \times 2 \times 715}}{2 \times 2}$ **M1**

$= \frac{291 \pm \sqrt{78961}}{4}$

$= \frac{291 \pm 281}{4}$

so that $n = \frac{5}{2}$ or $n = 143$ **A1**

We require n to be a positive integer, so $n = 143$ **A1** **[7 Marks]**

7. $x^4 - 6x^3 - 7x^2 - 2x + 14 = 0$ has roots α, β, γ and δ

a) $w^4 + pw^3 + 17w^2 - 10w + q = 0$ has roots $\alpha + 1$, $\beta + 1$, $\gamma + 1$ and $\delta + 1$

$$-p = (\alpha+1)+(\beta+1)+(\gamma+1)+(\delta+1)$$
$$= (\alpha+\beta+\gamma+\delta)+4$$

Alter... that x... into th... polyn... $w = \alpha$... $\delta +$ 1...

We have

$$\alpha+\beta+\gamma+\delta = -\frac{b}{a} = -\frac{-6}{1} = 6$$

So $-p = 6 + 4 = 10$ **M1**

Hence $p = -10$ **A1**

$$q = (\alpha+1)(\beta+1)(\gamma+1)(\delta+1)$$
$$= \alpha\beta\gamma\delta$$
$$+\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta$$
$$+\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta$$
$$+\alpha+\beta+\gamma+\delta$$
$$+1$$

We have

$$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{c}{a} = \frac{-7}{1} = -7$$

$$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = -\frac{d}{a} = -\frac{-2}{a} = 2$$

$$\alpha\beta\gamma\delta = \frac{e}{a} = \frac{14}{1} = 14 \quad \textbf{B1}$$

So $q = 14 + 2 - 7 + 6 + 1 = 16$ **A1**

b) $\alpha^2+\beta^2+\gamma^2+\delta^2 = (\alpha+\beta+\gamma+\delta)^2 - 2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)$

From part a), $\alpha + \beta + \gamma + \delta = 6$, and $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = -7$, so:

$$\alpha^2+\beta^2+\gamma^2+\delta^2 = 6^2 - 2\times(-7) \quad \textbf{M1}$$
$$= 50 \quad \textbf{A1}$$

[6 Marks]

Preview of Answers Ends Here