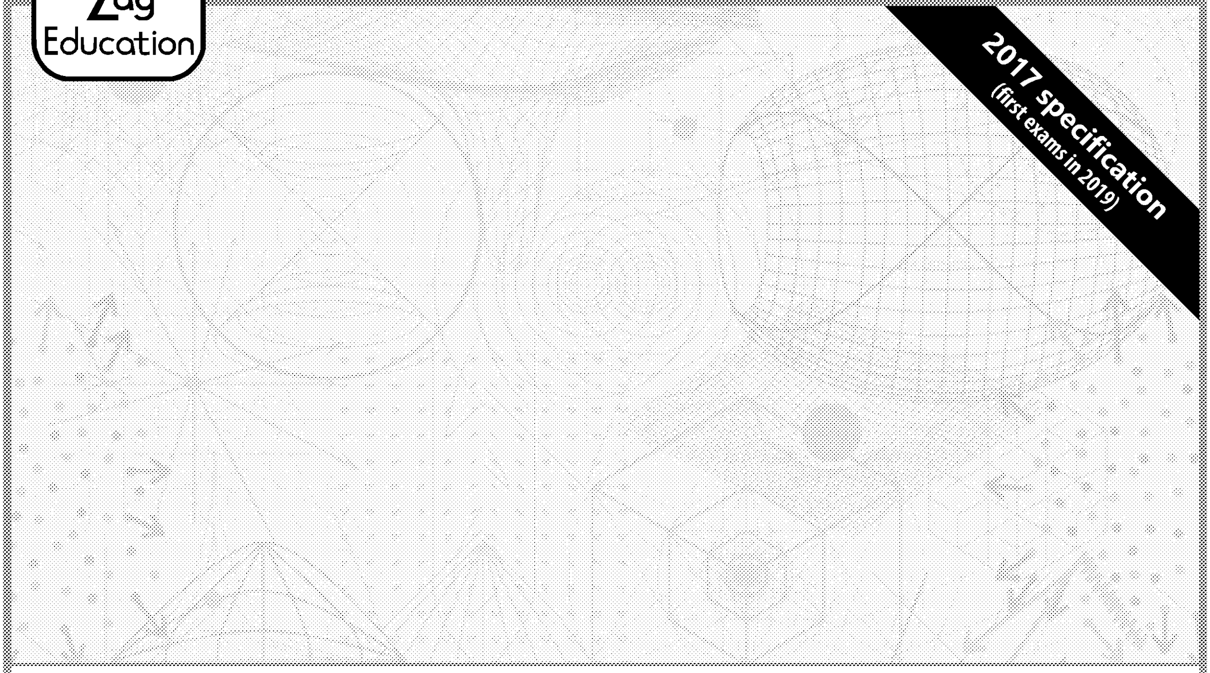
Further Maths AS / A Level | Edexcel | 8FM0/9FM0



Topic Tests: Fundamentals Tests – Set B

For AS / A Level Year 1 Edexcel Core Pure Further Mathematics

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Tests

- Test 1.1b – Complex Numbers
- Test 2.1b – Algebra and Functions
- Test 3.1b – Volumes of Revolution
- Test 4.1b – Matrices
- Test 5.1b – Proof by Induction
- Test 6.1b – Further Vectors

Solutions

Teacher's Introduction

Content

This pack contains 6 fundamentals level topic tests, which together form 'Set B' in a paired range of tests for the Edexcel Core Pure Further Mathematics AS / Year 1 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson textbook for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

Each test comes with fully worked solutions, containing helpful tips, hints and technique boxes for students struggling on a particular question. Answers should be given to three significant figures unless specified in the question.

About the fundamentals tests

These **fundamentals** tests focus on isolating and testing the core skills of each topic. The questions are designed to use simple numbers and contexts **so that students can show what they can do,** and to allow you to easily identify any weaknesses.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these fundamentals tests, the relevant times are the first two listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

For students who are ready to go beyond the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and progresses the concepts covered.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

Cross-referencing Grid

Topic	Edexcel spec. points	Subtopics
Complex Numbers	2.1–2.7	Imaginary and complex numbers, multiplying complex numbers, complex conjugation, roots of quadratic equations, solving cubic and quartic equations, Argand diagrams, modulus and argument, modulus-argument form of complex numbers, loci and regions in the Argand diagram
Algebra and Functions	4.1–4.3	Sums of natural numbers, sums of squares and cubes, roots of polynomials, linear transformations of roots
Volumes of Revolution	5.1	Volumes of revolution with Cartesian equations, adding and subtracting volumes, modelling with volumes
Matrices	3.1–3.8	Matrices, matrix multiplication, determinants, inverting 2×2 and 3×3 matrices, solving systems of equations using matrices, linear transformations in two and three dimensions, reflections and rotations, enlargements and stretches, successive transformations, the inverse of a linear transformation
Proof by Induction	1.1	Proof by mathematical induction, proving divisibility results, proving statements involving matrices
Further Vectors	6.1–6.5	Equation of a line in three dimensions, equation of a plane in three dimensions, scalar product, calculating angles between lines and planes, points of intersection, finding perpendiculars

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Complex Numbers		**Algebra and Functions**	
1.1a	45	2.1a	40
1.1b	45	2.1b	40
1.2a	40	2.2a	55
1.2b	40	2.2b	55
1.3a	[illegible]5	2.3a	70
1.3b	55	2.3b	70
Matrices		**Proof by Induction**	
4.1a	40	5.1a	35
4.1b	40	5.1b	35
4.2a	50	5.2a	55
4.2b	50	5.2b	55
4.3a	50	5.3a	55
4.3b	50	5.3b	55

Subtopics: Sums of natural numbers, sums of squares and cubes, roots of polynomials

1. Evaluate the following sums, showing your reasoning in each case:
   a) $\sum_{r=1}^{4}(r+2)$
   b) $\sum_{r=1}^{36} r$
   c) $\sum_{r=1}^{48}(4r+3)$
   d) $\sum_{r=64}^{128} r$

2. Find a[…]es[…] for $\sum_{r=1}^{n} r$ in terms of n

3. Given that $\sum_{r=1}^{n}(ar+5) = n(2n+7)$ for all $n \geq 1$, find the value of the co[…]

4. Show that $\sum_{r=1}^{n-1} r^2 = \frac{1}{6}n(An-1)(Bn-1)$ for some integers A and B, whe[…]

   You may use the formula $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

5. Evaluate $\sum_{r=1}^{19} r^3$ using the formula $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

6. The quadratic equation $ax^2 + bx + c = 0$ has roots $\alpha = 2$ and $\beta = 5$
   Find **integer** values for a, b and c

7. The cubic equation $3x^3 - 9x^2 + 6x - 2 = 0$ has roots α, β and γ. **Witho[…]**
   a) $\alpha + \beta + \gamma$
   b) $\alpha\beta + \alpha\gamma + \beta\gamma$
   c) $\alpha\beta\gamma$

8. The quadratic equation $(k-2)x^2 - (k+2)[…]+[…] 0$ has roots α and β. G[…]

9. The quartic equati[…] x + $8x^2 - x + 2 = 0$ has roots α, β, γ and δ[…]
   equati[…]n[…]
   a) α[…]+ $\alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$
   b) $\alpha\beta\gamma$ + $\alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

10. The cubic equation $x^3 + 3x^2 + 4x + 2 = 0$ has roots α, β and γ. These ro[…]
    $\alpha\beta + \alpha\gamma + \beta\gamma = 4$, and $\alpha\beta\gamma = -2$. **Without solving the equation**:
    a) find a cubic equation that has roots 2α, 2β and 2γ
    b) find a cubic equation that has roots $\alpha - 1$, $\beta - 1$ and $\gamma - 1$

Preview of Questions Ends Here

Solutions to Proof by Induction – Test B

1. $\sum_{r=1}^{n}(2r+1)=n(n+2)$

   a) **Show that the statement is true when $n = 1$**

   When $n = 1$ the left-hand side is $\sum_{r=1}^{1}(2r+1)=2\times1+1=3$ **A1**

   While the right-hand side is $1 \times (1 + 2) = 3$ **A1**
   And so the statement is true when $n = 1$

   b) **Given that $\sum_{r=1}^{98}(2r+1)=98\times100$, show that $\sum_{r=1}^{99}(2r+1)=99\times101$**

   $$\sum_{r=1}^{99}(2r+1)=\sum_{r=1}^{98}(2r+1)+(2\times99+1) \quad \textbf{M1}$$
   $$=98\times100+199$$
   $$=9\,999$$
   $$=99\times101 \quad \textbf{A1}$$

   [4 Marks]

2. **Prove by induction that $\sum_{r=1}^{n}r=\frac{1}{2}n(n+1)$ for all positive integers n**

   When $n = 1$: $\sum_{r=1}^{1}r=1$ and $\frac{1}{2}n(n+1)=\frac{1}{2}\times1\times2=1$ **B1**
   So the statement is true for $n = 1$

   Assume the statement is true for $n = k$, so $\sum_{r=1}^{k}r=\frac{1}{2}k(k+1)$ **M1**

   When $n = k + 1$:
   $$\sum_{r=1}^{k+1}r=\sum_{r=1}^{k}r+(k+1)$$
   $$=\frac{1}{2}k(k+1)+(k+1) \quad \textbf{M1}$$
   $$=\frac{1}{2}(k+1)(k+2)$$
   $$=\frac{1}{2}(k+1)((k+1)+1) \quad \textbf{A1}$$

   So if the statement is true when $n = k$, then it is true for $n = k + 1$
   Since it is true for $n = 1$, by the principle of mathematical induction it is true for all

   Tech… by ind… first p… the st… prove… $n = 1$…

   Tech… has b… induct… statem… when… that th… $n = k$…

   Tech… with a… princi…

3. **Prove by induction that $\sum_{r=1}^{n}(3r^2-r)=n^2(n+1)$ for all positive integers n**

   When $n = 1$: $\sum_{r=1}^{1}(3r^2-r)=3\times1^2-1=2$ and $n^2(n+1)=1^2\times(1+1)=2$ **B1**
   So the statement is true for $n = 1$

   Assume the statement is true for $n = k$, so $\sum_{r=1}^{k}(3r^2-r)=k^2(k+1)$ **M1**

   When $n = k + 1$:
   $$\sum_{r=1}^{k+1}(3r^2-r)=\sum_{r=1}^{k}(3r^2-r)+3(k+1)^2-(k+1)$$
   $$=k^2(k+1)+3(k+1)^2-(k+1) \quad \textbf{M1}$$
   $$=(k+1)(k^2+3(k+1)-1)$$
   $$=(k+1)(k^2+3k+2) \quad \textbf{A1}$$
   $$=(k+1)(k+1)(k+2)$$
   $$=(k+1)^2((k+1)+1) \quad \textbf{A1}$$

   So if the statement is true when $n = k$, then it is true for $n = k + 1$
   Since it is true for $n = 1$, by the principle of mathematical induction it is true for all

4. Alice has not carried out the inductive step, i.e. she has failed to prove the statemen assumed it is true when $n = k$ **B1** **[1 Mark]**

5. **Prove by induction that, for all positive integers n, $f(n) = 5^n + 3$ is divisible by 4**

When $n = 1$: $f(1) = 5^1 + 3 = 8 = 4 \times 2$ **B1**

So the statement is true for $n = 1$

Assume the statement is true for $n = k$, so $f(k) = 5^k + 3$ is divisible by 4 **M1**

$f(k+1) = 5^{k+1} + 3 = 5 \times 5^k + 3$

So $f(k+1) - f(k) = (5 \times 5^k + 3) - (5^k + 3)$ **M1**

$= 5 \times 5^k - 5^k$

$= 5^k(5-1) = 4 \times 5^k$ **A1**

Hence $f(k+1) = f(k) + 4 \times 5^k$ **A1**

Since $f(k)$ is divisible by 4 and 4×5^k is divisible by 4, their sum is divisible by 4

So if the statement is true for $n = k$ then it is true for $n = k + 1$

Since it is true for $n = 1$, by the principle of mathematical induction it is true for all

Altern
step y
$f(k +$
and
$f(k +$

Tip: I
by λ t

6. **Prove by induction that $\mathbf{M}^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ for every positive integer n**

$\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

When $n = 1$: left-hand side $= \mathbf{M}^1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and right-hand side $= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ **B1**

So the statement is true for $n = 1$

Assume the statement is true for $n = k$, so $\mathbf{M}^k = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ **M1**

$\mathbf{M}^{k+1} = \mathbf{M}^k\mathbf{M}$

$= \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ **M1**

$= \begin{pmatrix} 1\times1 + k\times0 & 1\times1 + k\times1 \\ 0\times1 + 1\times0 & 0\times1 + 1\times1 \end{pmatrix}$ **A1**

$= \begin{pmatrix} 1 & k+1 \\ 0 & 1 \end{pmatrix}$ **A1**

So if the statement is true for $n = k$, then it is true for $n = k + 1$

Since it is true for $n = 1$, by the principle of mathematical induction it is true for ev