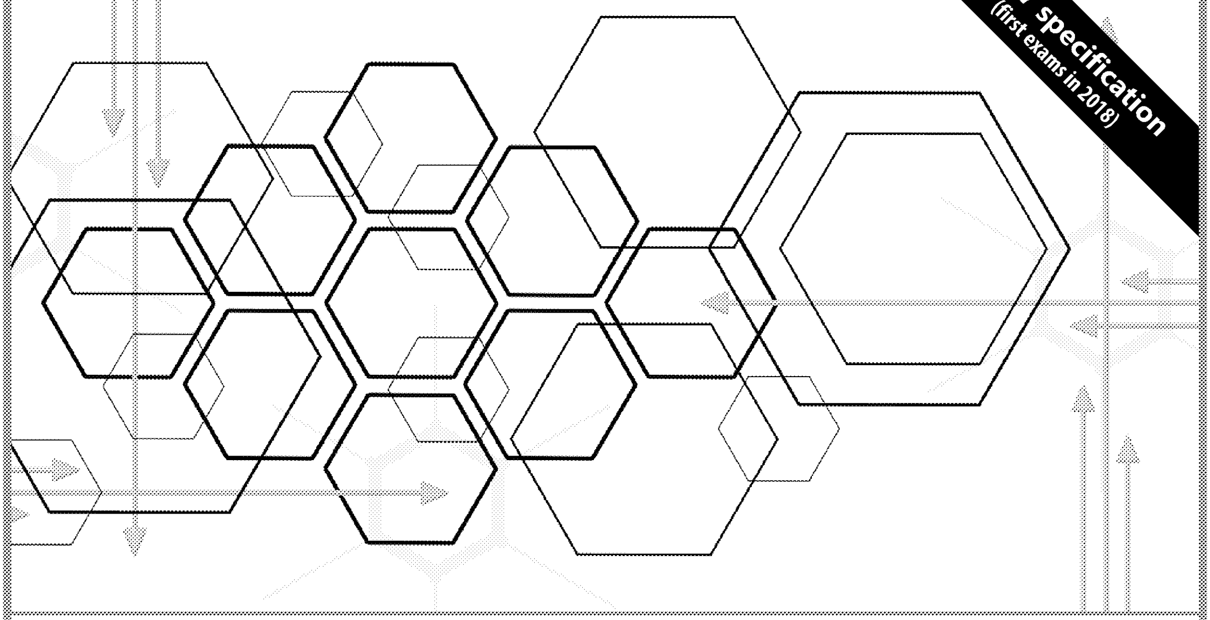
Maths

A Level | OCR A | H240

2017 specification
(first exams in 2018)



Topic Tests: Expert Tests – Set A

For A Level Year 2 OCR A

Statistics and Mechanics

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POD 8966

Contents

Teacher's Introduction

Content

This pack contains 6 expert level topic tests and solutions for the OCR A Applied Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests. In particular, students should be comfortable using the statistical functions on their calculator.

The large data set

As part of their assessment, students will be tested on data from the **large data set** provided by OCR. This data set contains data on workers' commutes and the age structure of the England & Wales population from two years in various locations. Familiarity with the large data set is assumed in these topic tests, but a copy of it is not needed to take the tests themselves.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Cross-referencing Grid

Topic	OCR A spec. points	Sub-topics
Regression and Correlation	2.02c–e, 2.05a, [illegible]	Exponential models, measuring correlation, hypothesis testing for zero correlation
Conditional Probability	2.03a–e	Set notation, conditional probability, conditional probabilities in Venn diagrams, probability formulae, tree diagrams
The Normal Distribution	2.04e–h, 2.05b	The normal distribution, finding probabilities, the inverse normal distribution, the standard normal distribution, finding μ and σ, approximating a binomial distribution, hypothesis testing with the normal distribution
Moments	3.01a–c, 3.04a–c	Moments, resultant moments, equilibrium, centres of mass, tilting
Forces and Friction	3.03e, 3.03l–v	Resolving forces, inclined planes, friction, modelling with statics, friction and static particles, dynamics and inclined planes, connected particles
Kinematics and Projectiles	3.02a, 3.02e, 3.02g–i	Horizontal projection, projection at any angle, projection motion formulae, vectors in kinematics, variable acceleration in one dimension, differentiating vectors, integrating vectors

Subtopics: resolving forces, inclined planes, friction, modelling with statics, fri dynamics and inclined planes, connected particles

1. A block of mass m kg is placed on a slope that is inclined at an angle o frictional force opposing the block's motion down the slope is F.
   a) By modelling the block as a particle, draw a force diagram showin block in directions **perpendicular** and **parallel** to the slope.
   The block lies in a state of limiting equilibrium.
   b) Show that the **coefficient of friction** between the block and the slo

2. Two particles A and B are connected by a light, inextensible string that smooth pulley. Particle A has mass 5 kg and is held on a smooth slope angle θ to the horizontal, where tan θ = $^{24}/_{7}$. The string lies over a line slope. The system is illustrated to the right.
   When particle A is released from rest, it moves up the slope with accel
   a) Find the **exact** tension in the string. You should assume g **= 10 m**
   After A has travelled up the slope for 0.4 seconds, the string snaps.
   b) Find the magnitude of particle A's **acceleration** down the slope aft
   c) Hence determine how long it takes from the time the string snaps f point.

3. A particle of mass m kg is acted on by the forces 5**i** + 2**j** N, x**i** + 2**j** N a forces produce an acceleration of 3**i** + 4**j** m s^{-2} on the particle.
   a) Find expressions for x and y in terms of m.
   A fourth force of $(-4x + y + z)$**i** + $(y - 2z)$**j** N is applied to the particle. changes to 4**i** + 10**j** m s^{-2}.
   b) Show that the total force on the particle is now $(32 - 5m + z)$**i** + (– find m.
   c) What force must be applied to the particle for it to reach a state of

4. Two particles P and Q are held on different sides of a block that is in th of an isosceles triangle (see right). The side of the triangle that particle is **rough**, with coefficient of friction μ. The side of the triangle that pa on is **smooth**. The two particles are joined by a light, inextensible strin passes over a smooth pulley. Particle P has mass m_1 kg and particle Q m_2 kg. Assume for this question that g **= 10 m s^{-2}**.
   When the system is released, particle Q moves up its side of the block
   a) By considering the motion of Q, show that the tension in the strin
   b) Hence, by considering the motion of P, show that $\dfrac{m_1}{m_2} = \dfrac{5}{1-4\mu}$

5. A scientist finds a metallic block of mass 2.9 kg, and wants to know what metal it is made of. She releases the block from rest from the top of a 2.5 m long steel slope that is inclined at an angle of θ to the horizontal, where tan θ = $^{20}/_{21}$. The block reaches the bottom of the slope with velocity 4.1 m s^{-1}. Using the table to the right, which shows the coefficient of friction μ between various metals and steel, determine what metal the block is made of. **State any modelling assumptions** you make. In this question you should take g **= 9.8 m s^{-2}**.

Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Solutions to Kinematics and Projectiles – Test A

1. a) Consider the horizontal and vertical components of velocity separately
Treating downwards as positive, the vertical acceleration of the cork is g = 9
Acceleration is constant, so we can use $v = u + at$ with $u = 0$ m s^{-1}, $a = 9.8$ m

$v = 0 + 9.8 \times \frac{5}{7}$ **M1**

$= 7$ m s^{-1} **A1**

Treating the direction of projection as positive, the horizontal acceleration of
So the horizontal velocity of the cork does not change from its initial value,
So the cork's velocity after $^5/_7$ seconds has horizontal component U m s^{-1}
and vertical component 7 m s^{-1}, and its speed is 25 m s^{-1} (see right)
By Pythagoras' theorem:

$U^2 = 25^2 - 7^2 = 576$ **M1**

$\therefore U = \sqrt{576} = 24$ m s^{-1}

b) Cork takes $^6/_7$ s to drop from the bottle to the water with downward a
Acceleration is constant, so can use $s = ut + \frac{1}{2}at^2$ with $u = 0$ m s^{-1}, $t = {}^6/_7$ s a

$s = 0 \times \frac{6}{7} + \frac{1}{2} \times 9.8 \times \left(\frac{6}{7}\right)^2$ **M1**

$= 3.6$ m **A1**

c) We need the vertical component of the velocity at the instant when the cork
happens at $t = {}^6/_7$ s.
In the air:
Acceleration is constant, so use $v = u + at$ with $u = 0$ m s^{-1}, $a = 9.8$ m s^{-2} an

$v = 0 + 9.8 \times \frac{6}{7}$ **M1**

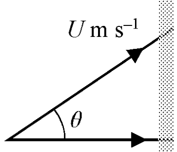
$= 8.4$ m s^{-1} **A1**

In the water:
Cork now accelerates upwards at 2.4 m s^{-2}. Set the time when the cork hits
Acceleration is constant, so we can use $s = ut + \frac{1}{2}at^2$ with $s = 0$ m, $u = 8.4$ m

$0 = 8.4t + \frac{1}{2} \times (-2.4)t^2 = 8.4t - 1.2t^2 = 1.2t(7 - t)$ **M1**

So the cork is at the water's surface at $t = 0$ s and $t = 7$ s **M1**
So the time when the cork returns to the surface is $t = 7$ s **A1** **[11 Marks]**

2. a) A sketch diagram makes this easier (see right)
The horizontal component of the initial velocity is
$U \cos \theta$ m s^{-1} **A1**
The vertical component of the initial velocity is
$U \sin \theta$ m s^{-1} **A1**



b) **Show that $R = \dfrac{U^2 \sin 2\theta}{g}$**

Consider vertical component of motion first

Taking upwards as positive, the vertical acceleration of the particle is $-g$

Acceleration is constant, so we can use $s = ut + \frac{1}{2}at^2$ with $s = 0$ m, $u = U\sin\theta$

$$0 = (U\sin\theta)t + \frac{1}{2}\times(-g)t^2 = t\left(U\sin\theta - \frac{gt}{2}\right) \quad \textbf{M1}$$

So particle is at the ground when $t = 0$ s and when $t = \dfrac{2U\sin\theta}{g}$ **A1**

Now consider the horizontal component of motion

Velocity is constant at $U\cos\theta$ m s^{-1}

$$\text{Distance travelled} = U\cos\theta \times \frac{2U\sin\theta}{g} \quad \textbf{M1}$$

$$= \frac{U^2 \times (2\sin\theta\cos\theta)}{g}$$

$$= \frac{U^2 \sin 2\theta}{g} \text{ m} \quad \textbf{A1}$$

[6 Marks]

Tech… then …

Tech… angl…

3. a) Work with column vectors for easier notation

Acceleration is constant, so we can use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $\mathbf{u} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ m s^{-1}, $\mathbf{a} =$

$$\mathbf{v} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \begin{pmatrix} 0.15 \\ -0.12 \end{pmatrix} \times 60 \quad \textbf{M1}$$

$$= \begin{pmatrix} -5 + 0.15\times 60 \\ 6 - 0.12\times 60 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -1.2 \end{pmatrix} \text{ m s}^{-1} \quad \textbf{A1}$$

Tip: … You … 1 mi…

b) The ship is moving north-east when its velocity is of the form $\begin{pmatrix} v \\ v \end{pmatrix}$ m s^{-1} for

Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for an unknown $\mathbf{v} = \begin{pmatrix} v \\ v \end{pmatrix}$ m s^{-1} and with **u** and **a** as in part a)

$$\begin{pmatrix} v \\ v \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \begin{pmatrix} 0.15 \\ -0.12 \end{pmatrix} t = \begin{pmatrix} -5 + 0.15t \\ 6 - 0.12t \end{pmatrix} \quad \textbf{M1}$$

So we get simultaneous equations:

$v - 0.15t = -5$ **(1)**

$v + 0.12t = 6$ **(2)** **M1**

(2) – (1):

$0.27t = 11$

$\therefore t = \dfrac{11}{0.27}$ **A1**

Alte… comp… equa… −5 + … for t…

So the ship is moving north-east at this time

Substituting this into **(1)**:

$$v - 0.15\times\frac{11}{0.27} = -5$$

$$v = -5 + 0.15\times\frac{11}{0.27} = \frac{10}{9} \quad \textbf{M1}$$

So velocity at this time is $\mathbf{v} = \begin{pmatrix} 10/9 \\ 10/9 \end{pmatrix}$ m s^{-1}

Speed is the magnitude of this vector, i.e.:

$$\left|\begin{pmatrix} 10/9 \\ 10/9 \end{pmatrix}\right| = \sqrt{\left(\frac{10}{9}\right)^2 + \left(\frac{10}{9}\right)^2} \quad \textbf{M1}$$

$$= \frac{10\sqrt{2}}{9} \text{ m s}^{-1} \quad \textbf{A1}$$

c) Island's displacement is $\begin{pmatrix}880\\-390\end{pmatrix}$ m

We need to find ship's position when it has displacement $\begin{pmatrix}880\\y\end{pmatrix}$ m

Tech… east … harbo… or du… is 88… north…

If $y > -390$ then ship passes north of the island; if $y < -390$ then ship passes s…

Use $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **u** and **a** as in part a) and $\mathbf{r} = \begin{pmatrix}880\\y\end{pmatrix}$ m

$$\begin{pmatrix}880\\y\end{pmatrix} = \begin{pmatrix}-5\\6\end{pmatrix}t + \frac{1}{2}\begin{pmatrix}0.15\\-0.12\end{pmatrix}t^2 = \begin{pmatrix}-5t+0.075t^2\\6t-0.06t^2\end{pmatrix}$$

First component gives the quadratic equation $0.075t^2 - 5t - 880 = 0$ **M1**

$$\therefore t = \frac{5 \pm \sqrt{(-5)^2 - 4\times 0.075\times(-880)}}{2\times 0.075} = \frac{5\pm\sqrt{289}}{0.15} = \frac{5\pm 17}{0.15}$$ **M1**

So $t = -80$ s or $t = \frac{440}{3}$ s

Ship leaves harbour at $t = 0$ s, so need $t \geq 0$, so $t = \frac{440}{3}$ s **A1**

We know from the second component of the vector equation above that $y = 6$…

$$y = 6\times\frac{440}{3} - 0.06\times\left(\frac{440}{3}\right)^2 = -\frac{1232}{3} = -410.666...$$ **M1**

So $y < -390$, and hence the ship passes due south of the island **A1** [13 …

4. a) **Show that $\dot{\mathbf{r}} = \left((4\pi - t)\cos t - \sin t\right)\mathbf{i} - \left((4\pi - t)\sin t + \cos t\right)\mathbf{j}$ cm s^{-1}**

$\mathbf{r} = (4\pi - t)\sin t\,\mathbf{i} + (4\pi - t)\cos t\,\mathbf{j}$ cm

Tip: …

$$\therefore \dot{\mathbf{r}} = \frac{\mathrm{d}}{\mathrm{d}t}\left((4\pi-t)\sin t\,\mathbf{i} + (4\pi-t)\cos t\,\mathbf{j}\right)$$ **M1**

Tech… $\frac{\mathrm{d}}{\mathrm{d}x}$(…

$$= \left((4\pi-t)\cos t + (-1)\sin t\right)\mathbf{i} + \left((4\pi-t)(-\sin t) + (-1)\cos t\right)\mathbf{j}$$

$$= \left((4\pi-t)\cos t - \sin t\right)\mathbf{i} - \left((4\pi-t)\sin t + \cos t\right)\mathbf{j}$$ **A1**

b) **Show that when the moth hits the edge of the lamp its speed is $\sqrt{\pi^2+1}$ c…**

Displacement, **r**, is measured from centre of lamp, which is a circle with dia…

So the moth hits the lamp when $|\mathbf{r}| = \frac{2\pi}{2} = \pi$ cm **M1**

$\mathbf{r} = (4\pi - t)\sin t\,\mathbf{i} + (4\pi - t)\cos t\,\mathbf{j}$ cm

$$\therefore |\mathbf{r}| = \sqrt{\left((4\pi-t)\sin t\right)^2 + \left((4\pi-t)\cos t\right)^2}$$

$$= \sqrt{(4\pi-t)^2(\sin^2 t + \cos^2 t)}$$

Hint: …

$$= 4\pi - t$$ **M1**

So moth hits the lamp when $4\pi - t = \pi$, i.e. when $t = 4\pi - \pi = 3\pi$ s **M1**

Substituting this value of t into the equation in part a) gives:

$$\dot{\mathbf{r}} = \left((4\pi-3\pi)\cos 3\pi - \sin 3\pi\right)\mathbf{i} - \left((4\pi-3\pi)\sin 3\pi + \cos 3\pi\right)\mathbf{j}$$

Tip: …

$$= -\pi\,\mathbf{i} + \mathbf{j} \text{ cm s}^{-1}$$

So that the moth's speed at this time is:

$$|-\pi\,\mathbf{i} + \mathbf{j}|$$

$$= \sqrt{\pi^2+1} \text{ cm s}^{-1}$$ **A1** [7 …

5. a) **Show that $g_C = \dfrac{U^2}{2800}$ m s^{-2}**

Model the dust as a particle. Can ignore air resistance since there is no air.

Dust is thrown up at a 45° angle, so the vertical component of initial velocit

Treating upwards as positive, the acceleration of the dust is $-g_C$
Dust reaches its maximum height when vertical velocity is zero

Acceleration is constant, so use $v^2 = u^2 + 2as$ with $v = 0$ m s^{-1}, $u = \dfrac{U\sqrt{2}}{2}$ m s

$$0^2 = \left(\frac{U\sqrt{2}}{2}\right)^2 + 2\times(-g_C)\times 700 \quad \textbf{M1}$$

$$\therefore g_C = \frac{2U^2}{4\times2\times700} = \frac{U^2}{2800} \text{ m s}^{-2} \quad \textbf{A1}$$

b) On Earth:

Dust is thrown up at 60° angle, so the vertical component of its initial veloci

Treating upwards as positive, the acceleration of the dust is $-g_E$
Dust reaches its maximum height when vertical velocity is zero

Acceleration is constant, so use $v^2 = u^2 + 2as$ with $v = 0$ m s^{-1}, $u = \dfrac{U\sqrt{3}}{2}$ m s

$$0^2 = \left(\frac{U\sqrt{3}}{2}\right)^2 + 2\times(-g_E)\times 0.105 \quad \textbf{M1}$$

Tip: P
heig
10.5

$$\therefore g_E = \frac{3U^2}{4\times2\times0.105} = \frac{25U^2}{7} \text{ m s}^{-2} \quad \textbf{A1}$$

And so

$$\frac{g_C}{g_E} = \frac{U^2/2800}{25U^2/7} = \frac{7}{25\times2800} = \frac{1}{10\ 000} \quad \textbf{A1}$$

[7 Marks]

6. $\mathbf{a} = (20t^3 + 16)\mathbf{i} + (30t^4 - 18t)\mathbf{j}$ m s^{-2}

$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$

$= \int\left[\left(20t^3+16\right)\mathbf{i}+\left(30t^4-18t\right)\mathbf{j}\right]\mathrm{d}t$ **M1**

$= \left(5t^4+16t+c_1\right)\mathbf{i}+\left(6t^5-9t^2+c_2\right)\mathbf{j}$ **A1**

We are told the particle is moving due east when $t = 0$ s, so $\mathbf{v}$ is of the form $a\mathbf{i}$ m s^{-1}

At $t = 0$:

$\mathbf{v} = \left(5\times0^4+16\times0+c_1\right)\mathbf{i}+\left(6\times0^5-9\times0^2+c_2\right)\mathbf{j}$

$= c_1\,\mathbf{i}+c_2\,\mathbf{j}$ m s^{-1}

So $c_1 > 0$ and $c_2 = 0$ **M1**

We are also told that the particle has speed 205 m s^{-1} when $t = 2$ s

Velocity when $t = 2$ s is:

$\mathbf{v} = \left(5\times2^4+16\times2+c_1\right)\mathbf{i}+\left(6\times2^5-9\times2^2\right)\mathbf{j}$

$= \left(112+c_1\right)\mathbf{i}+156\mathbf{j}$ m s^{-1}

Speed at $t = 2$ s is the magnitude of this vector, which the question states is 205 m s^{-1}, so

$205 = \left|\left(112+c_1\right)\mathbf{i}+156\mathbf{j}\right|$

$= \sqrt{\left(112+c_1\right)^2+156^2}$

$\therefore\ 205^2 = \left(112+c_1\right)^2+156^2$

$\therefore\ 112+c_1 = \pm\sqrt{205^2-156^2} = \pm133$

So $c_1 = 133 - 112 = 21$ or $c_1 = -133 - 112 = -245$

But we know $c_1 > 0$ so $c_1 = 21$ **M1**

$\therefore\ \mathbf{v} = \left(5t^4+16t+21\right)\mathbf{i}+\left(6t^5-9t^2\right)\mathbf{j}$ m s^{-1} **M1**

$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$

$= \int\left[\left(5t^4+16t+21\right)\mathbf{i}+\left(6t^5-9t^2\right)\mathbf{j}\right]\mathrm{d}t$ **M1**

$= \left(t^5+8t^2+21t+c_3\right)\mathbf{i}+\left(t^6-3t^3+c_4\right)\mathbf{j}$ m **M1**

We are told that $\mathbf{r} = 0\mathbf{i} + 0\mathbf{j}$ when $t = 0$, so:

$\left(0^5+8\times0^2+21\times0+c_3\right)\mathbf{i}+\left(0^6-3\times0^3+c_4\right)\mathbf{j} = 0\mathbf{i}+0\mathbf{j}$

and hence $c_3\mathbf{i} + c_4\mathbf{j} = 0\mathbf{i} + 0\mathbf{j}$, so $c_3 = c_4 = 0$ **M1**

And so $\mathbf{r} = \left(t^5+8t^2+21t\right)\mathbf{i}+\left(t^6-3t^3\right)\mathbf{j}$ m **A1** **[9 Marks]**

Preview of Answers Ends Here

This is a limited inspection copy. Sample of answers ends here to stop students looking up answers to their assessments. See contents page for details of the rest of the resource.