Zig Zag Education

Maths

A Level | OCR A | H240

2017 specification
(first exams in 2018)

Topic Tests: Expert Tests – Set B

For A Level Year 2 OCR A

Statistics and Mechanics

zigzageducation.co.uk

POD 8970

Contents

Teacher's Introduction

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

Content

This pack contains 6 expert level topic tests and solutions for the OCR A Applied Mathematics Year 2 A Level content.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these expert tests, the relevant times are the last two listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests. In particular, students should be comfortable using the statistical functions on their calculator.

The large data set

As part of their assessment, students will be tested on data from the **large data set** provided by OCR. This data set contains data on workers' commutes and the age structure of the England & Wales population from two years in various locations. Familiarity with the large data set is assumed in these topic tests, but a copy of it is not needed to take the tests themselves.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

Cross-referencing Grid

Topic	OCR A spec. points	Sub-topics
Regression and Correlation	2.02c–e, 2.05a, 2.[illegible]f	Exponential models, measuring correlation, hypothesis testing for zero correlation
Conditional Probability	2.03a–e	Set notation, conditional probability, conditional probabilities in Venn diagrams, probability formulae, tree diagrams
The Normal Distribution	2.04e–h, 2.05b	The normal distribution, finding probabilities, the inverse normal distribution, the standard normal distribution, finding μ and σ, approximating a binomial distribution, hypothesis testing with the normal distribution
Moments	3.01a–c, 3.04a–c	Moments, resultant moments, equilibrium, centres of mass, tilting
Forces and Friction	3.03e, 3.03l–v	Resolving forces, inclined planes, friction, modelling with statics, friction and static particles, dynamics and inclined planes, connected particles
Kinematics and Projectiles	3.02a, 3.02e, 3.02g–i	Horizontal projection, projection at any angle, projection motion formulae, vectors in kinematics, variable acceleration in one dimension, differentiating vectors, integrating vectors

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Regression and Correlation		**The Normal Distribution**	
1.1.a	25	3.1a	35
1.1b	25	3.1b	35
1.2a	25	3.2a	50
1.2b	25	3.2b	50
1.3a	3[illegible]	3.3a	65
1.3b	30	3.3b	65
Conditional Probability		**Moments**	
2.1a	35	4.1a	35
2.1b	35	4.1b	35
2.2a	35	4.2a	55
2.2b	35	4.2b	55
2.3a	65	4.3a	70
2.3b	65	4.3b	70

Subtopics: resolving forces, inclined planes, friction, modelling with statics, fr dynamics and inclined plane, connected particles

1. A block of mass m kg rests on a rough surface. A force of magnitude applied to the block at an angle θ above the horizontal, as shown to the
   a) By modelling the block as a particle, draw a force diagram showin forces acting on the block.
   The block lies in a state of limiting equilibrium.
   b) Show that the **coefficient of friction** between the block and the su

2. Two particles A and B are connected b a li nextensible string that a smooth pulley. Particle A h s ma kg and is held on a smooth slo at an angle θ to the h , where $\tan \theta = ¾$. The string lies over a greatest pe. ystem is illustrated to the right.
   When e A is released from rest, it moves up the slope with accel
   a) Fi the tension in the string. You should assume $\boldsymbol{g} = \mathbf{10\ m\ s^{-2}}$.
   After A has travelled up the slope for 0.5 seconds, the string snaps.
   b) Find the magnitude of particle A's **acceleration** down the slope af
   c) Hence determine the time between the string snapping and A reach

3. A particle of mass m kg is acted on by the forces $x\mathbf{i} + \mathbf{j}$ N and $-3\mathbf{i} + y\mathbf{j}$ produce an acceleration of $\mathbf{i} + 3\mathbf{j}$ m s^{-2} on the particle.
   a) Find expressions for x and y in terms of m.
   A third force of $(3x - 2y - z)\mathbf{i} + (x - y - 4z)\mathbf{j}$ N is applied to the particle changes to $-\mathbf{j}$ m s^{-2}.
   b) Show that the total force on the particle is now $(-2m - z + 11)\mathbf{i}$ + (find m.
   c) What force must be applied to the particle for it to reach a state of

4. Two particles P and Q are held on different sides of a block that is in t of an isosceles triangle (see right). The side of the triangle that particle is **rough**, with coefficient of friction 0.5. The side of the triangle that is on is **smooth**. The two particles are joined by a light, inextensible st passes over a smooth pulley. Particle P has mass m_1 kg and particle Q When the system is released, particle Q accelerates **down** its side of th
   a) By considering the motion of Q, show th t t nsion in the string
   b) Hence, by consideri the t on of P, show that $\dfrac{m_1}{m_2} = \dfrac{7g - 20}{19g + 20}$

5. A scie trying to identify a lump of metal. She uses the metal to create a ck of mass 6.25 kg together with a long slope that she inclines at an angle of θ to the horizontal, where $\tan \theta = 3/4$. She then projects the block up the slope at 6 m s^{-1}. The block travels 1.5 m up the slope before coming to instantaneous rest. Using the table to the right, which shows the coefficient of friction μ for various metals, determine what metal the lump was made of. **State any modelling assumptions** you make. In this question you should take $\boldsymbol{g} = \mathbf{9.8\ m\ s^{-2}}$

Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

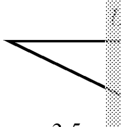
Solutions to Kinematics and Projectiles – Test B

1. a) Consider the horizontal and vertical components of velocity separately
Treating downwards as positive, the vertical acceleration of the helicopter i
Acceleration is constant, so we can use $v = u + at$ with $u = 0$ m s^{-1}, $a = 9.8$ m

$v = 0 + 9.8 \times 0.1$ **M1**

$= 0.98$ m s^{-1} **A1**

Treating the direction of projection as positive, the horizontal acceleration o
So the horizontal velocity of the helicopter does not change from its initial v
So the helicopter's velocity after 0.1 seconds has horizontal component U m s^{-1} and vertical component 0.98 m s^{-1}, and its speed is 3.5 m s^{-1} (see right)



3.5 m s

By Pythagoras' theorem:

$U^2 = 3.5^2 - 0.98^2 = 11.2896$ **M1**

$\therefore U = \sqrt{11.2896} =$ [illegible] m [illegible]

b) Rotors ar[illegible] at [illegible] 2 seconds. Consider the helicopter's vertical and ho
[illegible]ert[illegible] direction:
[illegible]eration is constant, so we can use $s = ut + \frac{1}{2}at^2$ with $u = 0$ m s^{-1}, $t = 2$ s

$s = 0 \times 2 + \frac{1}{2} \times 9.8 \times 2^2$ **M1**

$= 19.6$ m **A1**

In a horizontal direction, velocity is a constant 3.36 m s^{-1}, so horizontal disp

$s = 3.36 \times 2$ **M1**

$= 6.72$ m **A1**

Tech
then
should

And so the distance from the controller is:

$d = \sqrt{19.6^2 + 6.72^2}$

$= \sqrt{429.3184}$

$= 20.72$ m **A1**

[Accept 20.7 m (3 s.f.) or 21 m (2 s.f.)]

c) We need the vertical component of the velocity at the instant when the rotor
We know this happens at $t = 2$ s
Acceleration is constant, so use $v = u + at$ with $u = 0$ m s^{-1}, $a = 9.8$ m s^{-2} an

$v = 0 + 9.8 \times 2$ **M1**

$= 19.6$ m s^{-1} **A1**

The helicopter now accelerates upwards at 6.4 m s^{-2}. Set the time when the
of the helicopter when they start as a vertical displacement of $s = 0$ m. From
below the height at which the helicopter was thrown.

Acceleration is constant, so we can use $s = ut + \frac{1}{2}at^2$ [illegible]th $s = 19.6$ m, $u = -$

$19.6 = -19.6t + \frac{1}{2} \times 6.4t^2 = -19.6t$ [illegible] t^2 [illegible]

$\therefore 3.2t^2 - 19.6t - 19$[illegible] 0

[illegible]$\frac{19\text{[illegible]}\ \text{[illegible]}.6)^2 - 4 \times 3.2 \times (-19.6)}{2 \times 3.2}$

$= \frac{19.6 \pm \sqrt{635.04}}{6.4}$

$= \frac{19.6 \pm 25.2}{6.4}$

So the solutions are $t = -0.875$ s and $t = 7$ s **M1**

Want $t > 0$, so $t = 7$ s **A1** **[14 Marks]**

2. a) A sketch diagram makes this easier (see right)
The horizontal component of the initial velocity is
$U\cos\alpha$ m s^{-1} **A1**
The vertical component of the initial velocity is
$U\sin\alpha$ m s^{-1} **A1**

b) **Show that $V^2 = U^2 + 2gh$**

Consider vertical component of motion first
Taking downwards as positive, the vertical acceleration of the stone is g
Acceleration is constant, so we can use $v^2 = u^2 + 2as$ with $u = U\sin\alpha$ m s^{-1}, ...

$v^2 = (U\sin\alpha)^2 + 2\times g\times h = U^2\sin^2\alpha + 2gh$ **M1**

Here, v is the vertical component of the stone's velocity when it hits the wat...
In a horizontal direction, velocity is constant at $U\cos\alpha$ m s^{-1} **M1**
The stone's speed when it hits the water is V, so by Pythagoras' theorem:

$V^2 = U^2\sin^2\alpha + 2gh + (U\cos\alpha)^2$

$= U^2(\sin^2\alpha + \cos^2\alpha) + 2gh$

$= U^2 + 2gh$ **A1** **[6 Marks]**

Tech...

3. a) Work with column vectors for easier notation

Acceleration is constant, so we can use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with $\mathbf{u} = \begin{pmatrix}1\\6\end{pmatrix}$ m s^{-1}, $\mathbf{a} =$ ...

$\mathbf{v} = \begin{pmatrix}1\\6\end{pmatrix} + \begin{pmatrix}-0.02\\-0.06\end{pmatrix}\times 300$ **M1**

$= \begin{pmatrix}1-0.02\times300\\6-0.06\times300\end{pmatrix}$

$= \begin{pmatrix}-5\\-12\end{pmatrix}$ m s^{-1} **A1**

Tip: ...
want ...
minute...

b) The ship is moving south-west when its velocity is of the form $\begin{pmatrix}-v\\-v\end{pmatrix}$ m s^{-1} f...

Use $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for an unknown $\mathbf{v} = \begin{pmatrix}-v\\-v\end{pmatrix}$ m s^{-1} and with $\mathbf{u}$ and $\mathbf{a}$ as in part a)

$\begin{pmatrix}-v\\-v\end{pmatrix} = \begin{pmatrix}1\\6\end{pmatrix} + \begin{pmatrix}-0.02\\-0.06\end{pmatrix}t = \begin{pmatrix}1-0.02t\\6-0.06t\end{pmatrix}$ **M1**

So we get simultaneous equations:
$v - 0.02t = -1$ **(1)**
$v - 0.06t = -6$ **(2)** **M1**

Alte...
comp...
equa...
1 − 0...
for t

(1) − (2):
$0.04t = 5$

$\therefore t = \frac{5}{0.04} = 125$ s **A1**

So the ship is moving south-west at this time
Substituting this into (1):
$v - 0.02\times125 = -1$
$v = -1 + 0.02\times125 = 1.5$ **M1**

So velocity at this time is $\mathbf{v} = \begin{pmatrix}-1.5\\-1.5\end{pmatrix}$ m s^{-1}

Speed is the magnitude of this vector, i.e.:

$\left|\begin{pmatrix}-1.5\\-1.5\end{pmatrix}\right| = \sqrt{(-1.5)^2 + (-1.5)^2}$ **M1**

$= \frac{3\sqrt{2}}{2}$ m s^{-1} **A1**

c) The iceberg's displacement is $\begin{pmatrix} -875 \\ -1500 \end{pmatrix}$ m

We need to find ship's position when it has displacement $\begin{pmatrix} -875 \\ y \end{pmatrix}$ m

If $y > -1500$ then the ship passes north of the iceberg; if $y < -1500$ then the s

Use $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **u** and **a** as in part a) and $\mathbf{r} = \begin{pmatrix} -875 \\ y \end{pmatrix}$ m

$$\begin{pmatrix} -875 \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} -0.02 \\ -0.06 \end{pmatrix} t^2 = \begin{pmatrix} t - 0.01t^2 \\ 6t - 0.03t^2 \end{pmatrix}$$

First component gives the quadratic equation $0.01t^2 - t - 875 = 0$ **M1**

$$\therefore t = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 0.01 \times (-875)}}{2 \times 0.01} = \frac{1 \pm \sqrt{36}}{0.02} = \frac{1 \pm 6}{0.02} \quad \textbf{M1}$$

So $t = -250$ s or $t = 350$ s

Ship leaves port at $t = 0$ s, so need $t \geq 0$, so $t = 350$ s **A1**

We know from the second component of the vector equation above that $y = 6$

$\times 350 - 0.03 \times 350^2 = -1575$ **M1**

$-1575 < -1500$, and hence the ship passes due south of the iceberg **A1** [13]

4. a) **Show that $\dot{\mathbf{r}} = 2te^{10}\cos(t^2)\mathbf{i} + 2te^{10}\sin(t^2)\mathbf{j}$ km s^{-1}**

$\mathbf{r} = e^{10}\sin(t^2)\mathbf{i} - e^{10}\cos(t^2)\mathbf{j}$ km

$$\therefore \dot{\mathbf{r}} = \frac{d}{dt}\left(e^{10}\sin(t^2)\mathbf{i} - e^{10}\cos(t^2)\mathbf{j}\right) \quad \textbf{M1}$$

$$= 2te^{10}\cos(t^2)\mathbf{i} - 2te^{10}\left(-\sin(t^2)\right)\mathbf{j}$$

$$= 2te^{10}\cos(t^2)\mathbf{i} + 2te^{10}\sin(t^2)\mathbf{j} \text{ km s}^{-1} \quad \textbf{A1}$$

b) Speed is the magnitude of the velocity vector, $\dot{\mathbf{r}}$

So the astronaut's speed at time t is:

$$|\dot{\mathbf{r}}| = \left|2te^{10}\cos(t^2)\mathbf{i} + 2te^{10}\sin(t^2)\mathbf{j}\right|$$

$$= \sqrt{\left(2te^{10}\cos(t^2)\right)^2 + \left(2te^{10}\sin(t^2)\right)^2} \quad \textbf{M1}$$

$$= \sqrt{4t^2e^{20}\left(\cos^2(t^2) + \sin^2(t^2)\right)}$$

$$= 2te^{10} \text{ km s}^{-1} \quad \textbf{A1}$$

Set this equal to $2e^{12}$:

$2te^{10} = 2e^{12}$

$$\therefore t = \frac{2e^{12}}{2e^{10}} = e^2 \text{ s} \quad \textbf{A1}$$

Astronaut's displacement at this time is:

$$\mathbf{r} = e^{10}\sin(e^4)\mathbf{i} - e^{10}\cos(e^4)\mathbf{j} \quad \textbf{M1}$$

$$= 17953.9...\mathbf{i} - 12760.0...\mathbf{j}$$

$$= 18000\mathbf{i} - 12800\mathbf{j} \text{ km (3 s.f.)} \quad \textbf{A1}$$

[7]

Tech west port due 875 north

Tip:

Tech $\frac{d}{dx}$

Hint:

5. a) **Show that $g_M = \dfrac{8\sin 2\theta}{5}$ m s^{-2}**

Model the ball as a particle. Can ignore air resistance since there is no air o
Ball is projected at an angle of θ with speed 40 m s^{-1}, so vertical component
and horizontal component is $40\cos\theta$ m s^{-1} **M1**
Treating upwards as positive, the acceleration of the ball is $-g_M$
The ball lands back on the Moon's surface when the vertical displacement i

Acceleration is constant, so use $s = ut + \frac{1}{2}at^2$ with $s = 0$ m,
$u = 40\sin\theta$ m s^{-1} and $a = -g_M$ m s^{-2}

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time
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θ, th

$$0 = (40\sin\theta)t + \frac{1}{2}\times(-g_M)\times t^2 = t\left(40\sin\theta - \frac{g_M}{2}t\right) \quad \textbf{M1}$$

So ball lands when $t = \dfrac{2\times 40\sin\theta}{g_M} = \dfrac{80\sin\theta}{g_M}$ **A1**

In this time it travels $40\cos\theta \times \dfrac{80\sin\theta}{g_M} = \dfrac{1600\times(2\cos\theta\sin\theta)}{g_M} = \dfrac{1600\sin 2\theta}{g_M}$ m

We are told the ball travelled 1 km = 1000 m, so

$$1000 = \frac{1600\sin 2\theta}{g_M}$$

So $g_M = \dfrac{1600\sin 2\theta}{1000} = \dfrac{8\sin 2\theta}{5}$ m s^{-2} **A1**

b) On Earth:
Ball is projected with angle θ and with speed 60 m s^{-1}, so vertical compone
and horizontal component is $60\cos\theta$ m s^{-1} **M1**
Treating upwards as positive, the acceleration of the ball is $-g_E$
The ball lands back on the ground when the vertical displacement is zero

Acceleration is constant, so use $s = ut + \frac{1}{2}at^2$ with $s = 0$ m, $u = 60\sin\theta$ m s^{-1}

$$0 = (60\sin\theta)t + \frac{1}{2}\times(-g_E)\times t^2 = t\left(60\sin\theta - \frac{g_E}{2}t\right) \quad \textbf{M1}$$

So ball lands when $t = \dfrac{2\times 60\sin\theta}{g_E} = \dfrac{120\sin\theta}{g_E}$ **A1**

In this time it travels $60\cos\theta \times \dfrac{120\sin\theta}{g_E} = \dfrac{3600\times(2\cos\theta\sin\theta)}{g_E} = \dfrac{3600\sin 2\theta}{g_E}$

We are told the ball would have travelled 375 m, so

$$375 = \frac{3600\sin 2\theta}{g_E}$$

So $g_E = \dfrac{3600\sin 2\theta}{375} = \dfrac{48\sin 2\theta}{5}$ m s^{-2} **A1**

And so

$$\frac{g_M}{g_E} = \frac{8\sin 2\theta/5}{48\sin 2\theta/5} = \frac{8}{48} = \frac{1}{6}$$

[11 Marks]

6. $\mathbf{a} = (6t - 12)\mathbf{i} + (36t^2 - 96t + 36)\mathbf{j}$ m s^{-2}

$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$

$= \int \left[(6t-12)\mathbf{i} + (36t^2 - 96t + 36)\mathbf{j} \right] \mathrm{d}t$ **M1**

$= (3t^2 - 12t + c_1)\mathbf{i} + (12t^3 - 48t^2 + 36t + c_2)\mathbf{j}$ **A1**

We are told that the particle is instantaneously at rest at $t = 1$. So:

$0\mathbf{i} + 0\mathbf{j} = (3\times1^2 - 12\times1 + c_1)\mathbf{i} + (12\times1^3 - 48\times1^2 + 36\times1 + c_2)\mathbf{j} = (-9 + c_1)\mathbf{i} + c_2\mathbf{j}$

So that $c_1 = 9$ and $c_2 = 0$ **M1**

So $\mathbf{v} = (3t^2 - 12t + 9)\mathbf{i} + (12t^3 - 48t^2 + 36t)\mathbf{j}$ m s^{-1} **A1**

Can factorise this as $\mathbf{v} = 3(t-1)(t-3)\mathbf{i} + 12t(t-1)(t-3)\mathbf{j}$ m s^{-1}, so the second time is $t = 3$ s **A1**

$\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$

$= \int \left[(3t^2 - 12t + 9)\mathbf{i} + (12t^3 - 48t^2 + 36t)\mathbf{j} \right] \mathrm{d}t$ **M1**

$= (t^3 - 6t^2 + 9t + c_3)\mathbf{i} + (3t^4 - 16t^3 + 18t^2 + c_4)\mathbf{j}$ **A1**

We are told that $\mathbf{r} = 2\mathbf{i} - 30\mathbf{j}$ m the second time the particle is at rest, i.e. at $t = 3$ s. So:

$2\mathbf{i} - 30\mathbf{j} = (3^3 - 6\times3^2 + 9\times3 + c_3)\mathbf{i} + (3\times3^4 - 16\times3^3 + 18\times3^2 + c_4)\mathbf{j} = c_3\mathbf{i} + (-27 + c_4)\mathbf{j}$

So $c_3 = 2$ and $c_4 = -30 + 27 = -3$ **M1**

So $\mathbf{r} = (t^3 - 6t^2 + 9t + 2)\mathbf{i} + (3t^4 - 16t^3 + 18t^2 - 3)\mathbf{j}$ m **A1** **[9 Marks]**

Preview of Answers Ends Here

This is a limited inspection copy. Sample of answers ends here to stop students looking up answers to their assessments. See contents page for details of the rest of the resource.