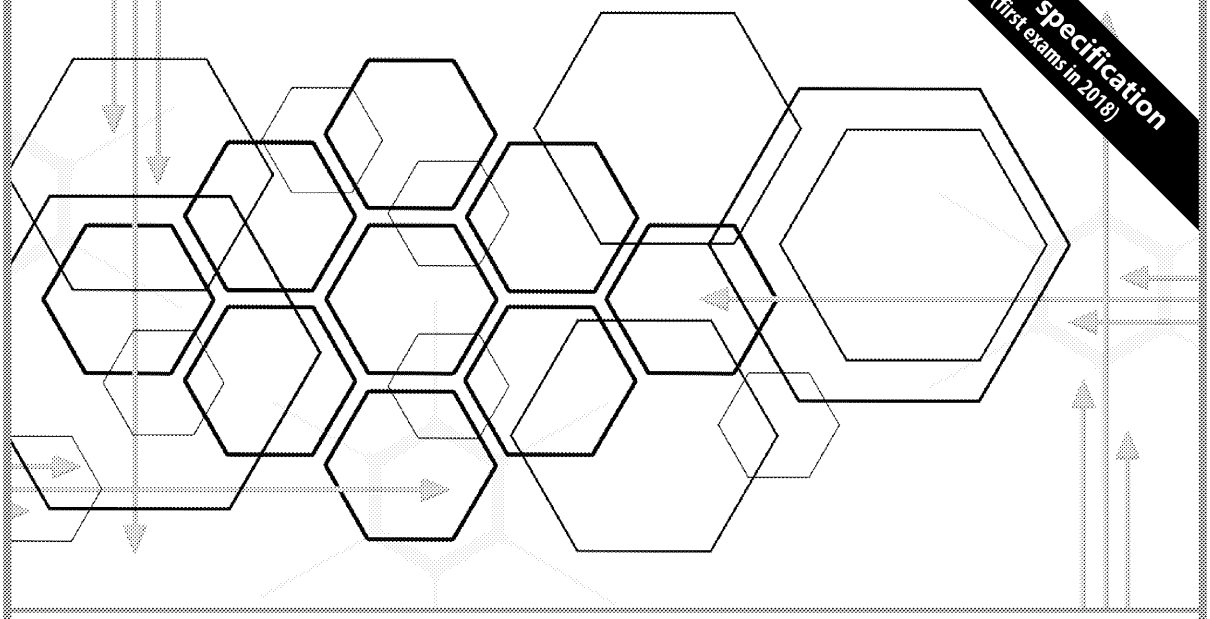
Zig Zag Education

Maths

A Level | OCR A | H240

2017 specification (first exams in 2018)



Topic Tests: Challenge Tests – Set B

For A Level Year 2 OCR A

Statistics and Mechanics

zigzageducation.co.uk

POD 8969

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Content

This pack contains 6 challenge level topic tests and solutions for the OCR A Applied Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the challenge tests

These **challenge** tests have been designed to **stretch and challenge** your students. 50% of the marks come from questions similar in style to our fundamentals tests. These questions isolate and test the core skills in each topic. The other 50% of the marks come from questions of increased difficulty that progress and start to combine the concepts in the topic. Due to the increased challenge they pose, we recommend these tests for students who have already mastered the fundamentals by scoring 70% or more on our fundamentals tests.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these challenge tests, the relevant times are the third and fourth listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests. In particular, students should be comfortable using the statistical functions on their calculator.

The large data set

As part of their assessment, students will be tested on data from the **large data set** provided by OCR. This data set contains data on workers' commutes and the age structure of the England & Wales population from two years in various locations. Familiarity with the large data set is assumed in these topic tests, but a copy of it is not needed to take the tests themselves.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

Cross-referencing Grid

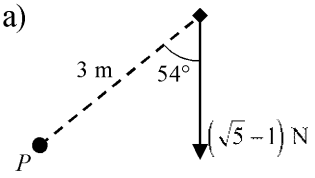
Topic	OCR A spec. points	Sub-topics
Regression and Correlation	2.02c–e, 2.05a, [illegible]	Exponential models, measuring correlation, hypothesis testing for zero correlation
Conditional Probability	2.03a–e	Set notation, conditional probability, conditional probabilities in Venn diagrams, probability formulae, tree diagrams
The Normal Distribution	2.04e–h, 2.05b	The normal distribution, finding probabilities, the inverse normal distribution, the standard normal distribution, finding μ and σ, approximating a binomial distribution, hypothesis testing with the normal distribution
Moments	3.01a–c, 3.04a–c	Moments, resultant moments, equilibrium, centres of mass, tilting
Forces and Friction	3.03e, 3.03l–v	Resolving forces, inclined planes, friction, modelling with statics, friction and static particles, dynamics and inclined planes, connected particles
Kinematics and Projectiles	3.02a, 3.02e, 3.02g–i	Horizontal projection, projection at any angle, projection motion formulae, vectors in kinematics, variable acceleration in one dimension, differentiating vectors, integrating vectors

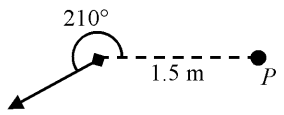
Timings Sheet

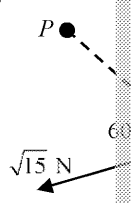
For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Regression and Correlation		**The Normal Distribution**	
1.1.a	25	3.1a	35
1.1b	25	3.1b	35
1.2a	25	3.2a	50
1.2b	25	3.2b	50
1.3a	30	3.3a	65
1.3b	30	3.3b	65
Conditional Probability		**Moments**	
2.1a	35	4.1a	35
2.1b	35	4.1b	35
2.2a	35	4.2a	55
2.2b	35	4.2b	55
2.3a	65	4.3a	70
2.3b	65	4.3b	70

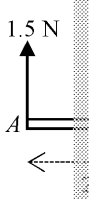
1. Calculate the **moment about the point *P*** of the forces acting on a lam

a) 

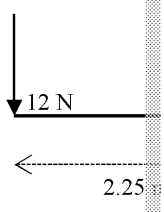
b) 

c) 

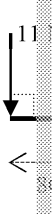
2. The diagram to the right shows vertical forces acting on a **light horizontal rod** *AB*. Calculate the **resultant moment** about:
   a) the point *A* **[2]**
   b) the point *B* **[2]**
   c) the point *C* **[2]**



3. The diagram to the right shows vertical forces acting on a light rod that is hanging horizontally from a string at *P*.
   a) Show that the resultant moment about *P* is zero. **[2]**
   b) Does this prove that the rod is in a state of equilibrium? **[1]**



4. The light rod in the diagram to the right is resting on supports at points *P* and *Q*. The rod is in a state of **equilibrium**. By taking moments about *P* and about *Q*, find the magnitudes of the reaction forces marked R_P and R_Q. **[6]**



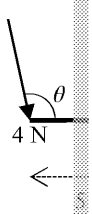
5. A uniform rod *AB* of length 4 m and mass 3.5 kg is resting horizontally on a support at *A*. At *B* the rod is attached to a light, inextensible string that is at an angle of 40° to the rod, as shown in the diagram to the right. For this question you should assume $g = 9.8$ m s^{-2}
   a) Draw a force diagram showing the forces acting on the rod. **[3**

The rod is in **equilibrium**.
   b) By taking moments about a suitable point, find the tension in the string. Give your answer to **3 significant figures**. **[3**

6. A **non-uniform** rod *AB* of mass 10 kg and length 20 cm is resting hori endpoints *A* and *B*. The centre of mass of the rod is *d* cm from *A*. For t
   a) Draw a force diagram showing the forces acting on the rod.

The reaction force from the support at *B* has magnitude 83.3 N.
   b) By taking moments about *A*, find the value of *d*.

7. A light rod is supported horizontally at the two points *C* and *D*. Forces of magnitude 10 N and 1 N are applied vertically to the rod, while a force of magnitude 4 N is applied at an obtuse angle of θ to the rod, as shown in the diagram to the right. The rod is on the point of tilting about *C*.
   a) Find the angle θ to **3 significant figures**. **[4]**
   b) A mass of 1 kg is added to the right-hand end of the rod and, at th changed, but remains obtuse. The rod is now on the point of tiltin angle θ correct to **the nearest degree**. You should take $g = 9.8$ m



Preview of Questions Ends Here

Solutions to The Normal Distribution – Test B

1. 22, 7 **A1A1** **[2 Marks]**

2. a) $P(H \leq 162) = 0.5$ **A1**
   b) $P(H = 162) = 0$ **A1**
   c) $P(H > 162) = 0.5$ **A1** **[3 Marks]**

3. a) $n = 1200$ is large **A1**
   and
   $p = 0.55$ is close to 0.5 **A1**
   b) $\mu = np$
   $= 1200 \times 0.55$ **M1**
   $= 660$ **A1**
   $\sigma^2 = np(1-p)$
   $= 1200 \times 0.5$ [illegible] **M1**
   [illegible] **[6 Marks]**

4. $H_0 : \mu = 30$, $H_1 : \mu > 30$, so test is one-tailed
   Sample size = 75
   Level of significance = 0.1
   Assume H_0 is true, so $X \sim N(30, 147)$ **M1**
   Sample mean $\overline{X}$ is normally distributed with $\overline{X} \sim N\left(30, \frac{147}{75}\right)$ i.e. $\overline{X} \sim N(30, 1.4$
   $P(\overline{X} > 31.6) = 0.126548...$ **M1**
   0.126548… > 0.1 so there is insufficient evidence at the 10% level of significance to conclude that the mean of the whole population is greater than 30 **A1** **[4 Marks]**

5. 95% of the data in a normal distribution lies within 2 standard deviations of the me
   So $5.3 = 8.1 - 2\sigma$ and $10.9 = 8.1 + 2\sigma$
   Hence $2\sigma = 2.8$, so $\sigma = 1.4$ m s^{-1} **A1** **[2 Marks]**

6. $Y \sim N(\mu, \sigma^2)$ approximates B(375, 0.44)
   a) mean $= \mu = np$
   $= 375 \times 0.44$ **M1**
   $= 165$ **A1**
   b) standard deviation $= \sigma = \sqrt{np(1-p)}$
   $= \sqrt{375 \times 0.44 \times (1-0.44)}$ **M1**
   $= \sqrt{92.4}$
   $= 9.61249... = 9.61$ (3 s.f.) **A1**
   c) $P(152 < X < 162) \approx P(152.5 < Y < 161.5)$ **M1**
   $= 0.261154...$ $= 0.261$ (3 s.f.) **A1** **[6 Marks]**

7. Points of inflection at $\mu - \sigma$ and $\mu + \sigma$ **M1**
   $\mu - \sigma$ [illegible] **(1)**
   $\mu + \sigma$ [illegible] **(2)** **M1**
   (1) + **(2)**:
   $2\mu = 26.8$
   $\therefore \mu = 13.4$ **A1**
   Substitute this value into **(2)**:
   $13.4 + \sigma = 17.3$
   $\therefore \sigma = 17.3 - 13.4 = 3.9$ **A1** **[4 Marks]**

8. $X \sim N(19, 4^2)$

a) $P(X < 23) = P\left(Z < \frac{23-19}{4}\right)$ **M1**

$= P(Z < 1)$

$= \Phi(1)$ **A1**

b) $P(X \geq 21) = P\left(Z \geq \frac{21-19}{4}\right)$ **M1**

$= P(Z \geq 0.5)$

$= 1 - P(Z < 0.5)$

$= 1 - \Phi(0.5)$ **A1**

c) $P(16 < X < 18) = P(X < 18) - P(X < 16)$

$= P\left(Z < \frac{18-19}{4}\right) - P\left(Z < \frac{16-19}{4}\right)$ **M1**

$= P(Z < -0.25) - P(Z < -0.75)$

$= \Phi(-0.25) - \Phi(-0.75)$ **A1**

9. $X \sim N(90, \sigma^2)$ and $P(X > 99) = 0.025$

So $P\left(Z > \frac{99-90}{\sigma}\right) = 0.025$ **M1**

$\therefore \frac{9}{\sigma} = 1.95996...$ **M1**

Rearranging this gives $\sigma = \frac{9}{1.95996...} = 4.59192... = 4.59$ (3 s.f.) **A1**

10. $H_0 : \mu = 0.5$, $H_1 : \mu > 0.5$, so test is one-tailed **M1**

Sample size = 36

Level of significance = 0.1

Assume H_0 is true, so $X \sim N(0.5, 0.126^2)$ **M1**

Sample mean $\overline{X}$ is normally distributed with $\overline{X} \sim N\left(0.5, \frac{0.126^2}{36}\right)$ i.e. $\overline{X} \sim N(0.5$

$P(\overline{X} > 0.535) = 0.0477903...$ **M1**

0.0477903… < 0.1 so there is sufficient evidence at the 10% level of significance t
conclude that the mean proportion of people who travelled to work by car in each l
in 2011 was greater than 0.5 **A1**

Preview of Answers Ends Here