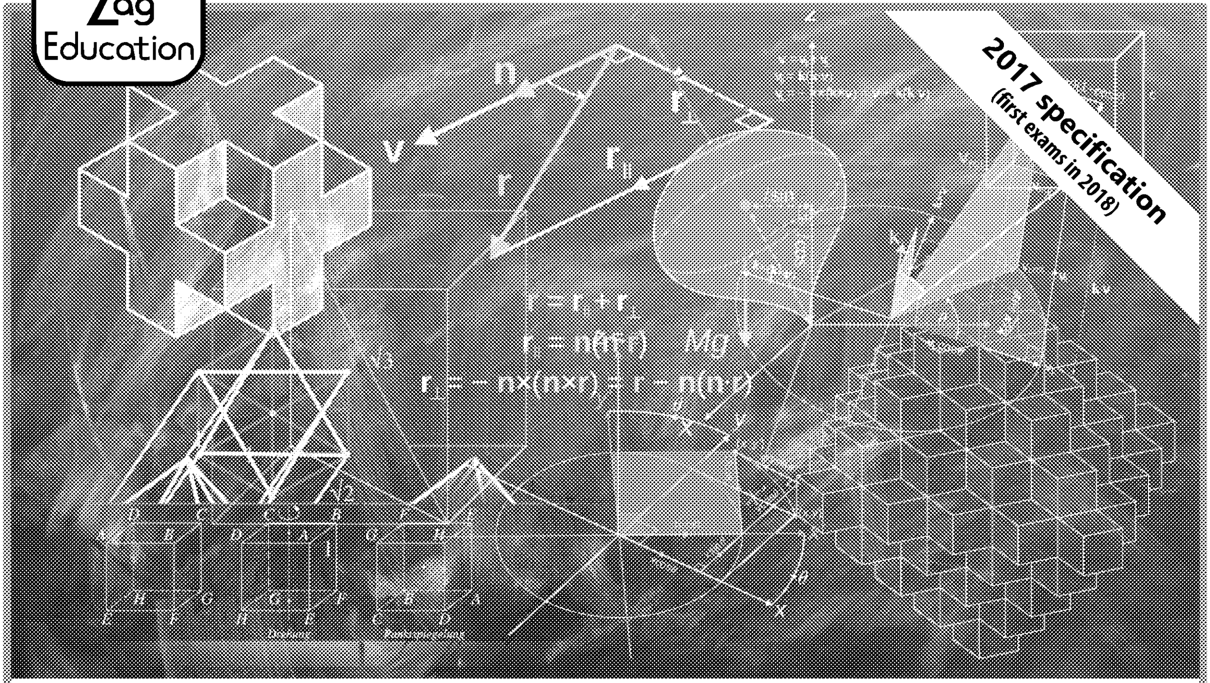
Maths

A Level | OCR A | H240



Topic Tests: Fundamentals Tests – Set B

For A Level Year 2 OCR A Pure Mathematics

zigzageducation.co.uk

POD 8960

Contents

Teacher's Introduction

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

Content

This pack contains 14 fundamentals level topic tests and solutions for the OCR A Pure Mathematics Year 2 A Level content.

About the fundamentals tests

These **fundamentals** tests focus on isolating and testing the core skills of each topic. The questions are designed to use simple numbers and contexts **so that students can show what they can do,** and to allow them to easily identify any weaknesses.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these fundamentals tests, the relevant times are the first two listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

Cross-referencing Grid

Topic	OCR A spec. points	Subtopics
Algebraic Methods	1.01a–d, 1.02j, 1.02y	Proof by contradiction, algebraic fractions, partial fractions with repeated factors, algebraic division
Functions and Graphs	1.02r–w	The modulus function, functions and mappings, composite functions, inverse functions, $y = \|f(x)\|$, combining transformations, solving modulus problems
Sequences and Series	1.04e–k	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	1.04a–d	Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions
Radians	1.05a–g	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	1.05h–k	Secant, cosecant and cotangent, graphs of sec x, cosec x and cot x, using sec x, cosec x and cot x, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	1.02z, 1.05l–q	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a\cos x \pm b\sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	1.03g–h	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	1.07a–p, 1.07r	Differentiating sin x and cos x, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	1.07k, 1.07q, 10.7s–t	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	1.09a–e, 1.09g	Locating roots, iteration, the Newton-Raphson method, application to modelling
Integration Part I	1.08a [illegible]	Integrating standard functions, integrating f($ax + b$), using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	1.08e–g, 1.08k–l, 1.09f	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	1.10a–h	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Algebraic Methods		**Trigonometry Part I**	
1.1.a	35	6.1a	30
1.1b	35	6.1b	30
1.2a	40	6.2a	40
1.2b	40	6.2b	40
1.3a	[illegible]	6.3a	50
1.3b	40	6.3b	50
Functions and Graphs		**Trigonometry Part II**	
2.1a	20	7.1a	55
2.1b	20	7.1b	55
2.2a	35	7.2a	65
2.2b	35	7.2b	65
2.3a	40	7.3a	65
2.3b	40	7.3b	65
Sequences and Series		**Parametric Equations**	
3.1a	30	8.1a	30
3.1b	30	8.1b	30
3.2a	35	8.2a	50
3.2b	35	8.2b	50
3.3a	50	8.3a	50
3.3b	50	8.3b	50
Binomial Expansion		**Differentiation Part I**	
4.1a	25	9.1a	25
4.1b	25	9.1b	25
4.2a	50	9.2a	30
4.2b	50	9.2b	30
4.3a	60	9.3a	45
4.3b	60	9.3b	45
Radians		**Differentiation Part II**	
5.1a	[illegible]	10.1a	30
5.1b	16	10.1b	30
5.2a	20	10.2a	40
5.2b	20	10.2b	40
5.3a	35	10.3a	45
5.3b	35	10.3b	45

Vectors – Test B (40 mins)

Subtopics: 3D coordinates, vectors in 3D, solving geometric problems, app

1. Find the **distances** between the following pairs of points:
   a) (0, 0, 0) and (2, 5, 14)
   b) (1, 0, –2) and (2, 12, 10)
   c) (–1, –10, 2) and (4, 10, –2)

2. Let $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = -\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$. Find the following vectors, g notation:
   a) $\mathbf{v} + \mathbf{w}$
   b) $\mathbf{v} - \mathbf{w}$
   c) $2\mathbf{v} - 3\mathbf{w}$

3. Let $\mathbf{a} = \begin{pmatrix} 24 \\ [illegible] \\ -5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \\ -10 \end{pmatrix}$. Show that the **resultant** of **a** and **b** is **p**

4. Given that $\mathbf{v} = 8\mathbf{i} - 12\mathbf{j} + p\mathbf{k}$ and that $|\mathbf{v}| = 17$, find the **two** possible va

5. Find the **unit vector** that is in the direction of each of the following vecto
   a) $\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$ b) $\begin{pmatrix} -2 \\ -11 \\ 10 \end{pmatrix}$ c) $\begin{pmatrix} 4 \\ -2 \\ -\sqrt{5} \end{pmatrix}$

6. Find, to **1 decimal place**, the angle in **degrees** made by the vector $\begin{pmatrix} 12 \\ -8 \\ 9 \end{pmatrix}$
   a) the positive x-axis
   b) the positive y-axis
   c) the positive z-axis

7. O is the origin, while the points P and Q have coordinates (1, 0, 5) and **vectors** to find the coordinates of the point R such that $OPQR$ is a **par**

8. The points P and Q have coordinates (4, –6, [illegible]) and (–2, –4, 6) respecti
   a) Show that $\triangle OPQ$ is an **equilateral** triangle.
   b) Find the **exact area** of the triangle $\triangle OPQ$.

9. A body [illegible] is acted on by a force $(12\mathbf{i} - 8\mathbf{j} + 20\mathbf{k})$ N. Find th giving [illegible] answer in **ijk** notation.

10. A body is acted on by forces $(a\mathbf{i} + 2\mathbf{j} - 7\mathbf{k})$ N, $(7\mathbf{i} + b\mathbf{j} + \mathbf{k})$ N and $(-9\mathbf{i}$ a, b and c. The body is in a state of **equilibrium**. Find the values of a

Preview of Questions Ends Here

1. a) $\int_0^{\frac{\pi}{2}} \cos x \, dx = [\sin x]_0^{\frac{\pi}{2}}$ **M1**

$= \sin\frac{\pi}{2} - \sin 0$ **M1**

$= 1 - 0$

$= 1$ **A1**

b) $\int_0^1 \frac{2}{2x+1} \, dx = \left[2 \times \frac{1}{2}\ln|2x+1|\right]_0^1$ **M1**

$= \ln|2\times1+1| - \ln|2\times0+1|$ **M1**

$= \ln 3$ **A1**

c) $\int_0^{\ln 3} 2e^x \, dx = \left[2e^x\right]_0^{\ln 3}$ **M1**

$= 2e^{\ln 3} - 2e^0$ **M1**

$= 2\times3 - 2\times1$

$= 4$ **A1**

d) $\int_0^{\frac{\pi}{3}} \tan x \, dx = \left[\ln|\sec x|\right]_0^{\frac{\pi}{3}}$ **M1**

$= \ln\left(\frac{1}{\cos\frac{\pi}{3}}\right) - \ln\left(\frac{1}{\cos 0}\right)$ **M1**

$= \ln 2 - \ln 1$

$= \ln 2$ **A1** **[12 Marks]**

2. Area of R is $\int_3^4 x\sqrt{x-3} \, dx$ **M1**

Let $u = x - 3$, so $\frac{du}{dx} = 1$

The lower limit of integration $x = 3$ becomes $u = 3 - 3 = 0$

The upper limit of integration $x = 4$ becomes $u = 4 - 3 = 1$

So $\int_3^4 x\sqrt{x-3} \, dx = \int_0^1 (u+3)\sqrt{u} \, du$ **M1**

$= \int_0^1 \left(u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\right) du$

$= \left[\frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}}\right]_0^1$ **M1**

$= \left(\frac{2}{5}\times1^{\frac{5}{2}} + 2\times1^{\frac{3}{2}}\right) - \left(\frac{2}{5}\times0^{\frac{5}{2}} + 2\times0^{\frac{3}{2}}\right)$ **M1**

$= \frac{12}{5} - 0$

$= \frac{12}{5}$ **A1** **[5 Marks]**

Tech… curve… the x… x = b… f(x) ≥… Other… the c… x-axis…

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3. a) **Prove that $2\cos^2 x - \cos 2x \equiv 1$.**

From the question we have $\cos 2x \equiv \cos^2 x - \sin^2 x$

Using the identity $\sin^2 x + \cos^2 x = 1$, we can, therefore, write:

$$\cos 2x \equiv \cos^2 x - \sin^2 x$$
$$\equiv \cos^2 x - \left(1 - \cos^2 x\right) \quad \textbf{M1}$$
$$\equiv 2\cos^2 x - 1$$
$$\therefore 2\cos^2 x - \cos 2x \equiv 1 \quad \textbf{A1}$$

b) Area between the two curves is $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2\cos^2 x - \cos 2x\right) \mathrm{d}x$ **M1**

From part a) this is equal to $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \, \mathrm{d}x$

And so the area is $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \, \mathrm{d}x = [x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ **M**

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$
$$= \pi \quad \textbf{A1}$$

[5 Marks]

4. By the trapezium rule:

$$\int_0^{2\pi} \mathrm{e}^{\sin x} \, \mathrm{d}x \approx \frac{1}{2} h \left\{ \left(y_0 + y_4\right) + 2\left(y_1 + y_2 + y_3\right) \right\}$$

where $h = \frac{b-a}{n} = \frac{2\pi - 0}{4} = \frac{\pi}{2}$ **M1**

and the y-values are given in the table

So $\int_0^{2\pi} \mathrm{e}^{\sin x} \, \mathrm{d}x \approx \frac{1}{2} \times \frac{\pi}{2} \times \left\{ (1+1) + 2(2.718 + 1 + 0.368) \right\}$ **M1**

$$= \frac{\pi}{4} \times 10.172$$
$$= 7.98907...$$
$$= 7.99 \text{ (3 s.f.)} \quad \textbf{A1}$$

[3 Marks]

5. The graph of $y = 2^x$ is convex **B1**

So the line forming the top of each trapezium will be above the curve, and thus it o

[Allow use of sketch graph or other reasonable explanations, for example referring

gradient] **[2 Marks]**

6. $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 1$

$$\therefore y = \int \left(x^2 + 1\right) \mathrm{d}x \quad \textbf{M1}$$
$$\therefore y = \frac{1}{3} x^3 + x + c \quad \textbf{A1}$$

[. Marks]

7. $x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = -3 - x^2$

$$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-3 - x^2}{x^2} = -\frac{3}{x^2} - 1$$
$$\therefore y = \int \left(-\frac{3}{x^2} - 1\right) \mathrm{d}x \quad \textbf{M1}$$
$$\therefore y = \frac{3}{x} - x + c \quad \textbf{A1}$$

We are told that $y = -3$ when $x = 3$, so $-3 = \frac{3}{3} - 3 + c$, so $c = -1$ **M1**

So the particular solution is $y = \frac{3}{x} - x - 1$ **A1** **[4 Marks]**

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8. a) $\frac{dB}{dt} = -0.25B$

$\therefore \frac{1}{B}\frac{dB}{dt} = -0.25$

$\therefore \int \frac{1}{B}\,dB = \int(-0.25)\,dt$ **M1**

$\therefore \ln B = -0.25t + c$ **A1**

$\therefore B = e^{-0.25t+c}$ **A1**

We can rewrite this as $B = e^{-0.25t}e^{c} = Ae^{-0.25t}$, where $A = e^{c}$

We are told that when $t = 0$, $B = 5000$, so $5000 = Ae^{-0.25\times 0} = A$, so $A = 5000$

And so $B = 5000e^{-0.25t}$ **A1**

b) When $t = 12$, $B = 5000e^{-0.25\times 12} = 5000e^{-3}$ = 24[illegible]... = 249 to the nearest w

c) As t increases B tends to zero. [illegible]

[Allow any acceptable response, e.g. mention of exponential decay or B alw

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