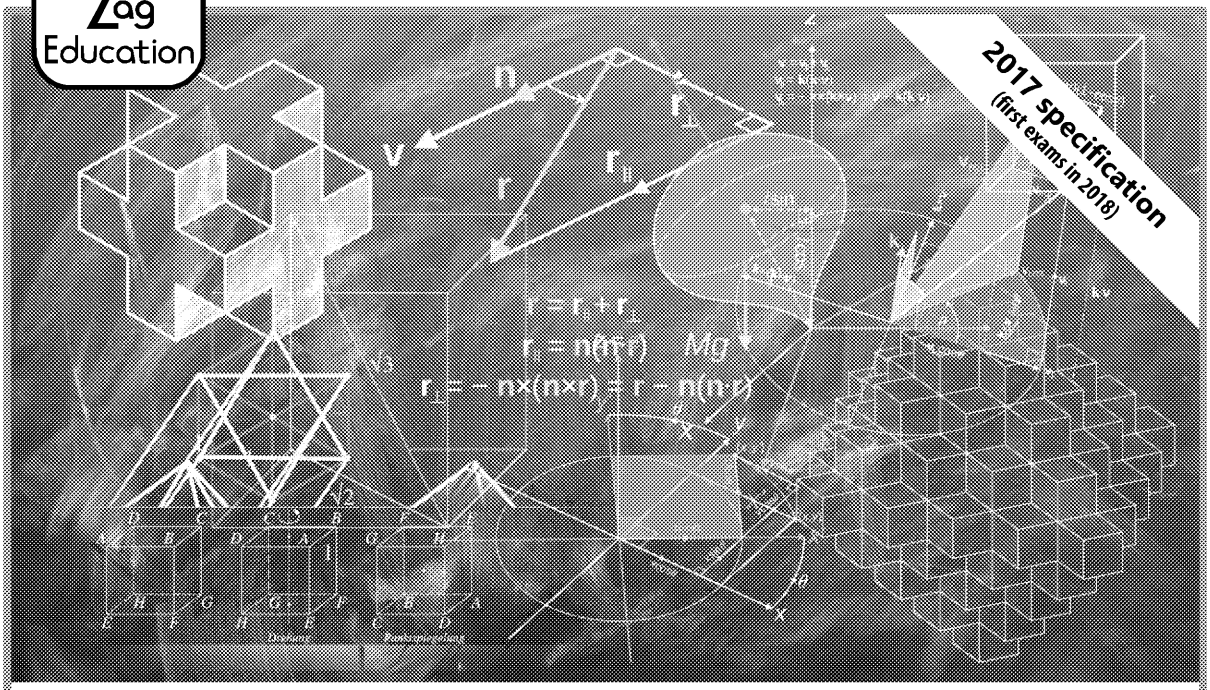
Zig Zag Education

Maths **A Level | OCR A | H240**



Topic Tests: Expert Tests – Set B

For A Level Year 2 OCR A Pure Mathematics

zigzageducation.co.uk

POD 8962

Contents

Teacher's Introduction

Content

This pack contains 14 expert level topic tests and solutions for the OCR A Pure Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these expert tests, the relevant times are the last two listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

Cross-referencing Grid

Topic	OCR A spec. points	Subtopics
Algebraic Methods	1.01a–d, 1.02j, 1.02[illegible]	Proof by [illegible]tion, algebraic fractions, partial [illegible]act[illegible] [illegible]peated factors, algebraic division
Functions and Graphs	1.02[illegible]–w	The modulus function, functions and mappings, composite functions, inverse functions, $y = \|f(x)\|$, combining transformations, solving modulus problems
Sequences and Series	1.04e–k	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	1.04a–d	Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions
Radians	1.05a–g	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	1.05h–k	Secant, cosecant and cotangent, graphs of sec x, cosec x and cot x, using sec x, cosec x and cot x, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	1.02z, 1.05l–q	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a\cos x \pm b\sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	1.03g–h	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	1.07a–p, 1.07r	Differentiating sin x and cos x, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	1.07k, 1.07q, 10.7s–t	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	1.09a–e, 1.09g	Locating roots by ra[illegible] the Newton-Raphson [illegible] pr[illegible]tions to modelling
Integration Part I	[illegible] 1.08h–j	[illegible]ating standard functions, integrating $f(ax + b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	1.08e–g, 1.08k–l, 1.09f	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	1.10a–h	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Algebraic Methods		**Trigonometry Part I**	
1.1a	35	6.1a	30
1.1b	35	6.1b	30
1.2a	40	6.2a	40
1.2b	40	6.2b	40
1.3a	40	6.3a	50
1.3b	40	6.3b	50
Functions and Graphs		**Trigonometry Part II**	
2.1a	20	7.1a	55
2.1b	20	7.1b	55
2.2a	35	7.2a	65
2.2b	35	7.2b	65
2.3a	40	7.3a	65
2.3b	40	7.3b	65
Sequences and Series		**Parametric Equations**	
3.1a	30	8.1a	30
3.1b	30	8.1b	30
3.2a	35	8.2a	50
3.2b	35	8.2b	50
3.3a	50	8.3a	50
3.3b	50	8.3b	50
Binomial Expansion		**Differentiation Part I**	
4.1a	25	9.1a	25
4.1b	25	9.1b	25
4.2a	50	9.2a	30
4.2b	50	9.2b	30
4.3a	60	9.3a	45
4.3b	60	9.3b	45
Radians		**Differentiation Part II**	
5.1a	15	10.1a	30
5.1b	16	10.1b	30
5.2a	20	10.2a	40
5.2b	20	10.2b	40
5.3a	35	10.3a	45
5.3b	35	10.3b	45

Functions and Graphs – Test B (40 mins)

Subtopics: The modulus function, functions and mappings, composite functions, inverse transformations, solving modulus problems

1. Given that $f(x) = |2x - 8|$ and $g(x) = 7 - x$:
   a) write down the value of: i) $f(3)$ ii) $f(4)$
   b) **sketch** $y = f(x)$ and $y = g(x)$ on the same axes indicating **all** points touch the **axes**
   c) **solve** the equation $7 - x = |2x - 8|$
   d) hence **solve** $7 - x > |2x - 8|$

2. Given that $f(x) = x^2 + 10$ and $g(x) = x - 5$:
   a) find the value of: i) $fg(5)$ ii) $gf($
   b) find an expression for $fg(x)$
   c) **so**$g($ $= g(x)$

3. The graph of $y = f(x)$ is shown to the right. Points A (1, 2) and B $(-2$ are on the graph.
   On **separate** diagrams, **sketch**, indicating the new coordinates of A and
   a) $y = -f(x+2)$
   b) $y = f(2x) + 1$

4. The function $f(x)$ has domain $-6 \le x \le 10$ and is **linear** from (–6, 16) (10, 72).
   a) **sketch** the graph of $y = f(x)$
   b) state the **range** of $f(x)$
   c) find the **two values** of a such that $f(a) = 2$

5. Given that $g(x) = x^2 - 10x + 19$ is a **one-to-one** function with **domain** possible value of k.

6. The function h is defined by $h : x \to \dfrac{x+6}{3x+1}$, $x \neq -\dfrac{1}{3}$
   a) Find an expression for h^{-1}, stating the value **excluded** from its do
   b) Find the **two values** of a for which $h(a) = h^{-1}(a)$, leaving your ans

7. The function $f(x) = \begin{cases} 5x + 50 & -22 \le x < 0 \\ 50\cos x & 0° \le \end{cases}$
   For each of the following equa, **sketch** the graph, indicating **all** po touch the **axes**:
   a) y b) $y = |f(x)|$

8. The population of a small island can be **modelled** using the formula N number of people living on the island and t is the number of years afte
   a) Find the population of the island on 1st January 2000.
   b) Find the population of the island, **to the nearest person**, on 1st Ja
   c) **Sketch** the graph of the island population against time for $0 \le t \le$
   d) Explain whether this is a **realistic** model for the population of a s

Preview of Questions Ends Here

Solutions to Trigonometry Part I – Test B

1. a) $\sec 120° = \dfrac{1}{\cos 120°}$ **M1**

$= \dfrac{1}{-1/2} = -2$ **A1**

b) $\operatorname{cosec} 60° = \dfrac{1}{\sin 60°}$ **M1**

$= \dfrac{1}{\sqrt{3}/2} = \dfrac{2\sqrt{3}}{3}$ **A1**

c) $\cot\left(-\dfrac{5\pi}{12}\right) = \dfrac{1}{\tan\left(-5\pi/12\right)}$ **M1**

$= -\dfrac{1}{\sqrt{3}} = -2+\sqrt{3}$ **A1** **[6 Marks]**

Tip: C… the co… radia… quest…

2. a) **Prove that** $\dfrac{1+\cot\theta}{\sin\theta+\cos\theta} \equiv \operatorname{cosec}\theta$

$\dfrac{1+\cot\theta}{\sin\theta+\cos\theta} \equiv \dfrac{1+\cos\theta/\sin\theta}{\sin\theta+\cos\theta}$ **M1**

Techn… in ter…

$\equiv \dfrac{\sin\theta+\cos\theta}{\sin\theta(\sin\theta+\cos\theta)}$ **M1**

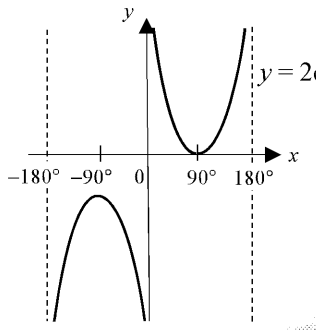
$\equiv \dfrac{1}{\sin\theta} \equiv \operatorname{cosec}\theta$ **A1**

b) $\dfrac{1+\cot\theta}{\sin\theta+\cos\theta} = \dfrac{3}{4}$

$\therefore \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{3}{4}$ and so $\sin\theta = \dfrac{4}{3}$

But $-1 \le \sin\theta \le 1$ so there are no real solutions **A1** **[4 Marks]**

3. a)



$y = 2\operatorname{cosec} x - 2$

Techn… stretc… facto… transl… by 2 …

A1 correct shape

Asymptotes at $x = -18$…, $x =$ … and $x = 180°$ **A1**

Touches the … at $-90°$ **A1**

b) … **A1A1**

Hint: … $y = c$…

c) …ns are where the line $y = a$ intersects with $y = 2\operatorname{cosec} x - 2$

…ere will be no solutions for $-4 < a < 0$ **A1** **[6 Marks]**

4. The domain of $y = \cos^{-1} x$ is $-1 \le x \le 1$

So the domain of $y = \cos^{-1}\dfrac{x}{4}$ is $-4 \le x \le 4$ **A1** **[1 Mark]**

Tip: y… $y = c$… x-dir…

5. a) **Show that $CD = 6(\sec\theta - \cos\theta)$ cm**

$CD = AC - AD$ **M1**

Using triangle ABC, $\cos\theta = \dfrac{AB}{AC}$ so $AC = \dfrac{AB}{\cos\theta} = \dfrac{6}{\cos\theta} = 6\sec\theta$ **M1**

Using triangle ABD, $\cos\theta = \dfrac{AD}{AB}$ so $AD = AB\cos\theta = 6\cos\theta$ **M1**

$\therefore CD = AC - AD = 6\sec\theta - 6\cos\theta = 6(\sec\theta - \cos\theta)$ **A1**

b) $CD = \dfrac{16\sqrt{17}}{17}$

$\therefore 6(\sec\theta - \cos\theta) = \dfrac{16\sqrt{17}}{17}$

$51\sec\theta - 51\cos\theta = 8\sqrt{17}$

$51 - 51\cos^2\theta = 8\sqrt{17}\cos\theta$

$51\cos^2\theta + 8\sqrt{17}\cos\theta - 51 = 0$ **M1**

Let $\cos\theta = \alpha$ and solve using the quadratic formula with $a = 51$, $b = 8\sqrt{17}$, $c = -51$

$\alpha = \dfrac{-8\sqrt{17} \pm \sqrt{(8\sqrt{17})^2 - 4\times 51\times(-51)}}{2\times 51}$ **M1**

$= \dfrac{-8\sqrt{17} \pm 26\sqrt{17}}{102} = \dfrac{3\sqrt{17}}{17}$ or $-\dfrac{\sqrt{17}}{3}$

But $-1 \le \cos\theta \le 1$ so $\alpha = -\dfrac{\sqrt{17}}{3}$ is not a solution, and $\alpha = \cos\theta = \dfrac{3\sqrt{17}}{17}$ o

So $\theta = 43.3138\ldots = 43.3°$ (3 s.f.) **A1** **[8 Marks]**

6. a) **Show that $\dfrac{\sec x}{\operatorname{cosec} x \cot x} \equiv \tan^2 x$**

$\dfrac{\sec x}{\operatorname{cosec} x \cot x} \equiv \dfrac{1/\cos x}{(1/\sin x)\times(\cos x/\sin x)}$ **M1**

$\equiv \dfrac{1/\cos x}{\cos x/\sin^2 x} \equiv \dfrac{\sin^2 x}{\cos^2 x}$ **M1**

$\equiv \tan^2 x$ **A1**

b) $\dfrac{\sec x}{\operatorname{cosec} x \cot x} = \dfrac{5}{4}$

So using part a), $\tan^2 x = \dfrac{5}{4}$ **M1**

$1 + \tan^2 x \equiv \sec^2 x \therefore \sec^2 x = 1 + \dfrac{5}{4} = \dfrac{9}{4}$ **M1**

$\therefore \sec x = \pm\sqrt{\dfrac{9}{4}} = \pm\dfrac{3}{2}$ **M1**

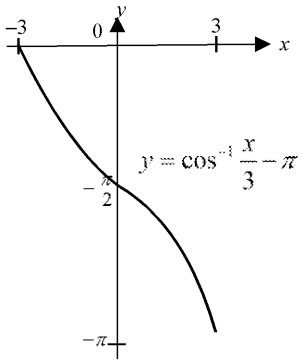
For $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$ only $\sec x = \dfrac{3}{2}$ is a solution **A1** **[7 Marks]**

7. $\sin^{-1}(3\sqrt{3} + 2x) = \dfrac{2\pi}{3}$ so $\sin\dfrac{2\pi}{3} = 3\sqrt{3} + 2x$ **M1**

$\therefore \dfrac{\sqrt{3}}{2} = 3\sqrt{3} + 2x$ **M1**

$\therefore x = \dfrac{\sqrt{3}}{4} - \dfrac{3\sqrt{3}}{2} = -\dfrac{5\sqrt{3}}{4}$ **A1** **[3 Marks]**

8. a)



A1 domain is $-3 \le x \le 3$

A1 y-intercept is $-\pi/2$

A1 range is $-\pi \le y \le 0$
A1 correct shape

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b) $\cos^{-1}\frac{x}{3} - \pi = -1$

$\therefore \cos^{-1}\frac{x}{3} = \pi - 1$ **M1**

$\therefore \frac{x}{3} = \cos(\pi - 1) = -0.\ldots$ **M1**

$x = \ldots = -1.62$ (3 s.f.) **A1**

c) [illegible] the graph, $\cos^{-1}\frac{x}{3} - \pi = k$ has no solutions where $y > 0$ or $y < -\pi$ **A**

Preview of Answers Ends Here