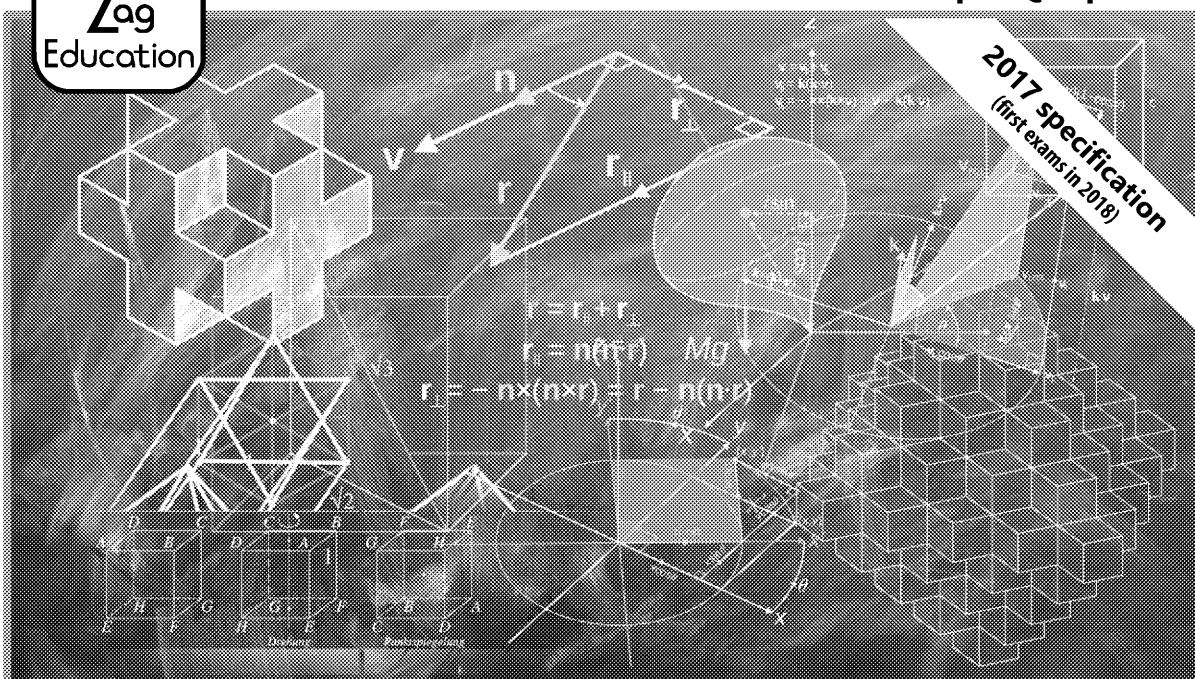




Topic Tests: Expert Tests – Set B

For A Level Year 2 AQA
Pure Mathematics

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Solutions

Teacher's Introduction

Content

This pack contains 14 expert level topic tests and solutions for the AQA Pure Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework:** Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test:** Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork:** Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test:** Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these expert tests, the relevant times are the last two listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

Free Updates!

Register your email address to receive any future free updates* made to this resource or other Maths resources your school has purchased, and details of any promotions for your subject.

* resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers

Go to [zzed.uk/freeupdates](https://www.zzed.uk/freeupdates)

Cross-referencing Grid

Topic	AQA spec. points	Subtopics
Algebraic Methods	A1, B6, B10	Proof by induction, algebraic fractions, partial fractions, repeated factors, algebraic division
Functions and Graphs	7–11	Graphs of the modulus function, functions and mappings, composite functions, inverse functions, $y = f(x) $, combining transformations, solving modulus problems
Sequences and Series	D2–D6	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	D1	Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions
Radians	E1–E3	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	E4–E5	Secant, cosecant and cotangent, graphs of $\sec x$, $\csc x$ and $\cot x$, using $\sec x$, $\csc x$ and $\cot x$, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	B11, E6–E9	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a \cos x \pm b \sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	C3–C4	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	G1–G5	Differentiating $\sin x$ and $\cos x$, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	G2, G4–G6	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	I1–I2, I4	Locating roots, iteration, the Newton-Raphson method, applications to modelling
Integration Part I	F1–F6	Integrating standard functions, integrating $f(ax + b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	H3–H4, H7–H8, I3	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	J1–J5	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

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Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.

For the **challenge** tests, refer to the tests marked X.2a and X.2b.

For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Algebraic Methods		Trigonometry Part I	
1.1.a	35	6.1a	30
1.1b	35	6.1b	30
1.2a	40	6.2a	40
1.2b	40	6.2b	40
1.3a	40	6.3a	50
1.3b	40	6.3b	50
Functions and Graphs		Trigonometry Part II	
2.1a	20	7.1a	55
2.1b	20	7.1b	55
2.2a	35	7.2a	65
2.2b	35	7.2b	65
2.3a	40	7.3a	65
2.3b	40	7.3b	65
Sequences and Series		Parametric Equations	
3.1a	30	8.1a	30
3.1b	30	8.1b	30
3.2a	35	8.2a	50
3.2b	35	8.2b	50
3.3a	50	8.3a	50
3.3b	50	8.3b	50
Binomial Expansion		Differentiation Part I	
4.1a	25	9.1a	25
4.1b	25	9.1b	25
4.2a	50	9.2a	30
4.2b	50	9.2b	30
4.3a	60	9.3a	45
4.3b	60	9.3b	45
Radians		Differentiation Part II	
5.1a	30	10.1a	30
5.1b	16	10.1b	30
5.2a	20	10.2a	40
5.2b	20	10.2b	40
5.3a	35	10.3a	45
5.3b	35	10.3b	45

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Functions and Graphs – Test B (40 mins)

Subtopics: The modulus function, functions and mappings, composite functions, inverse functions, transformations, solving modulus problems

1. Given that $f(x) = |2x - 8|$ and $g(x) = 7 - x$:
 - a) write down the value of: i) $f(3)$ ii) $f(4)$
 - b) **sketch** $y = f(x)$ and $y = g(x)$ on the same axes indicating **all** points where the graphs touch the axes
 - c) **solve** the equation $7 - x = |2x - 8|$
 - d) hence **solve** $7 - x > |2x - 8|$
2. Given that $f(x) = x^2 + 10$ and $g(x) = 2 - x$:
 - a) find the value of: i) $f(g(3))$ ii) $g(f(3))$
 - b) find an expression for $f(g(x))$
 - c) **solve** $f(g(x)) = g(x)$
3. The graph of $y = f(x)$ is shown to the right. Points $A(1, 2)$ and $B(-2, 1)$ are on the graph.
On **separate** diagrams, **sketch**, indicating the new coordinates of A and B for the following functions:
 - a) $y = -f(x + 2)$
 - b) $y = f(2x) + 1$
4. The function $f(x)$ has domain $-6 \leq x \leq 10$ and is **linear** from $(-6, 16)$ to $(10, 72)$.
 - a) **sketch** the graph of $y = f(x)$
 - b) state the **range** of $f(x)$
 - c) find the **two values** of a such that $f(a) = 2$
5. Given that $g(x) = x^2 - 10x + 19$ is a **one-to-one** function with domain $5 \leq x \leq 10$, find a possible value of k such that $g(x) = k$ has **two** solutions.
6. The function h is defined by $h: x \rightarrow \frac{x+6}{3x+1}, x \neq -\frac{1}{3}$
 - a) Find an expression for h^{-1} , stating the value **excluded** from its domain.
 - b) Find the **two values** of a for which $h(a) = h^{-1}(a)$, leaving your answers in surd form.
7. The function $f(x) = \begin{cases} 5x + 50 & -22 \leq x < 0 \\ 50 \cos x & 0 \leq x \leq \pi \end{cases}$

For each of the following, **sketch** the graph, indicating **all** points where the graph touches the axes:

 - a) $y = f(x)$
 - b) $y = |f(x)|$
8. The population of a small island can be **modelled** using the formula $N(t) = 1000(1.02)^t$, where N is the number of people living on the island and t is the number of years after 1st January 2000.
 - a) Find the population of the island on 1st January 2000.
 - b) Find the population of the island, **to the nearest person**, on 1st January 2010.
 - c) **Sketch** the graph of the island population against time for $0 \leq t \leq 10$.
 - d) Explain whether this is a **realistic** model for the population of a small island.

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Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Solutions to Trigonometry Part I – Test B

1. a) $\sec 120^\circ = \frac{1}{\cos 120^\circ}$ M1
 $= \frac{1}{-\frac{1}{2}} = -2$ A1

b) $\operatorname{cosec} 60^\circ = \frac{1}{\sin 60^\circ}$ M1
 $= \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$ A1

c) $\cot\left(-\frac{5\pi}{12}\right) = \frac{1}{\tan\left(-\frac{5\pi}{12}\right)}$ M1
 $= -\frac{1}{\frac{1}{\sqrt{3}}} = -\sqrt{3}$ A1

[6 Marks]

Tip: Check the correct radian question

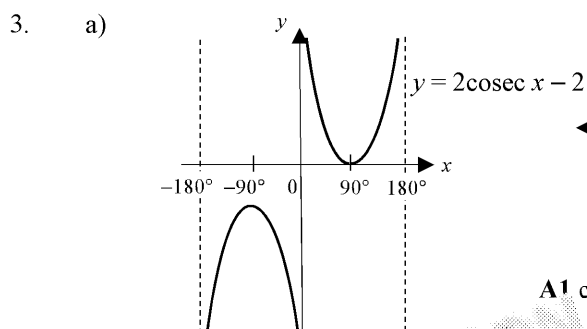
2. a) Prove that $\frac{1 + \cot \theta}{\sin \theta + \cos \theta} \equiv \operatorname{cosec} \theta$

$\frac{1 + \cot \theta}{\sin \theta + \cos \theta} \equiv \frac{1 + \frac{\cos \theta}{\sin \theta}}{\sin \theta + \cos \theta}$ M1
 $= \frac{\frac{\sin \theta + \cos \theta}{\sin \theta}}{\sin \theta + \cos \theta}$ M1
 $= \frac{1}{\sin \theta} \equiv \operatorname{cosec} \theta$ A1

Technique in terms of

b) $\frac{1 + \cot \theta}{\sin \theta + \cos \theta} = \frac{3}{4}$
 $\therefore \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{3}{4}$ and so $\sin \theta = \frac{4}{3}$

But $-1 \leq \sin \theta \leq 1$ so there are no real solutions A1 [4 Marks]



Technique stretch factor translate by 2

Asymptotes at $x = -180^\circ$, $x = 0^\circ$ and $x = 180^\circ$ A1
 Touches the x -axis at $x = 90^\circ$ A1

b) $\tan \theta = \frac{1}{2}$ A1A1

Hint: $y = \cot \theta$

c) Solutions are where the line $y = a$ intersects with $y = 2\operatorname{cosec} x - 2$
 There will be no solutions for $-4 < a < 0$ A1 [6 Marks]

4. The domain of $y = \cos^{-1} x$ is $-1 \leq x \leq 1$

So the domain of $y = \cos^{-1} \frac{x}{4}$ is $-4 \leq x \leq 4$ A1

[1 Mark]

Tip: $y = \cos^{-1} x$ is the inverse of $y = \cos x$

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5. a) Show that $CD = 6(\sec \theta - \cos \theta)$ cm

$$CD = AC - AD \quad \text{M1}$$

$$\text{Using triangle } ABC, \cos \theta = \frac{AB}{AC} \text{ so } AC = \frac{AB}{\cos \theta} = \frac{6}{\cos \theta} = 6 \sec \theta \quad \text{M1}$$

$$\text{Using triangle } ABD, \cos \theta = \frac{AD}{AB} \text{ so } AD = AB \cos \theta = 6 \cos \theta \quad \text{M1}$$

$$\therefore CD = AC - AD = 6 \sec \theta - 6 \cos \theta = 6(\sec \theta - \cos \theta) \quad \text{A1}$$

b) $CD = \frac{16\sqrt{17}}{17}$

$$\therefore 6(\sec \theta - \cos \theta) = \frac{16\sqrt{17}}{17}$$

$$51 \sec \theta - 51 \cos \theta = 8\sqrt{17}$$

$$51 - 51 \cos^2 \theta = 8\sqrt{17} \cos \theta$$

$$51 \cos^2 \theta + 8\sqrt{17} \cos \theta - 51 = 0 \quad \text{M1}$$

Let $\alpha = \cos \theta$ and solve using the quadratic formula with $a = 51$, $b = 8\sqrt{17}$, $c = -51$

$$\alpha = \frac{-8\sqrt{17} \pm \sqrt{(8\sqrt{17})^2 - 4 \times 51 \times (-51)}}{2 \times 51} \quad \text{M1}$$

$$= \frac{-8\sqrt{17} \pm 26\sqrt{17}}{102} = \frac{3\sqrt{17}}{17} \text{ or } -\frac{\sqrt{17}}{3}$$

But $-1 \leq \cos \theta \leq 1$ so $\alpha = -\frac{\sqrt{17}}{3}$ is not a solution, and $\alpha = \cos \theta = \frac{3\sqrt{17}}{17}$ or

$$\text{So } \theta = 43.3138... = 43.3^\circ \text{ (3 s.f.)} \quad \text{A1}$$

[8 Marks]

6. a) Show that $\frac{\sec x}{\operatorname{cosec} x \cot x} \equiv \tan^2 x$

$$\frac{\sec x}{\operatorname{cosec} x \cot x} \equiv \frac{1/\cos x}{(1/\sin x) \times (\cos x/\sin x)} \quad \text{M1}$$

$$\equiv \frac{1/\cos x}{\cos x/\sin^2 x} \equiv \frac{\sin^2 x}{\cos^2 x} \quad \text{M1}$$

$$\equiv \tan^2 x \quad \text{A1}$$

b) $\frac{\sec x}{\operatorname{cosec} x \cot x} = \frac{5}{4}$

$$\text{So using part a), } \tan^2 x = \frac{5}{4} \quad \text{M1}$$

$$1 + \tan^2 x \equiv \sec^2 x \therefore \sec^2 x = 1 + \frac{5}{4} = \frac{9}{4} \quad \text{M1}$$

$$\therefore \sec x = \pm \sqrt{\frac{9}{4}} = \pm \frac{3}{2}$$

For $\sec x = \frac{3}{2}$ only $\sec x = \frac{3}{2}$ is a solution A1 [7 Marks]

7. $\sin^{-1}(3\sqrt{3} + 2x) = \frac{2\pi}{3}$ so $\sin \frac{2\pi}{3} = 3\sqrt{3} + 2x \quad \text{M1}$

$$\therefore \frac{\sqrt{3}}{2} = 3\sqrt{3} + 2x \quad \text{M1}$$

$$\therefore x = \frac{\sqrt{3}}{4} - \frac{3\sqrt{3}}{2} = -\frac{5\sqrt{3}}{4} \quad \text{A1}$$

[3 Marks]

Hint:
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Hint:
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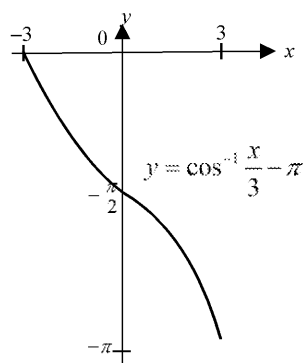
Hint:
 $\frac{\pi}{2}$
a value

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8.

a)



A1 domain is $-3 \leq x \leq 3$

A1 y-intercept is $-\pi/2$

A1 range is $-\pi \leq y \leq 0$

A1 correct shape

Technique

stretch
in the
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get y

b)

$$\cos^{-1} \frac{x}{3} - \pi = -1$$

$$\therefore \cos^{-1} \frac{x}{3} = \pi - 1 \quad \text{M1}$$

$$\therefore \frac{x}{3} = \cos(\pi - 1) = -0.4161 \quad \text{M1}$$

$$x = -1.2483 \approx -1.25 \quad \text{A1}$$

c)

the graph, $\cos^{-1} \frac{x}{3} - \pi = k$ has no solutions where $y > 0$ or $y < -\pi$ A

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