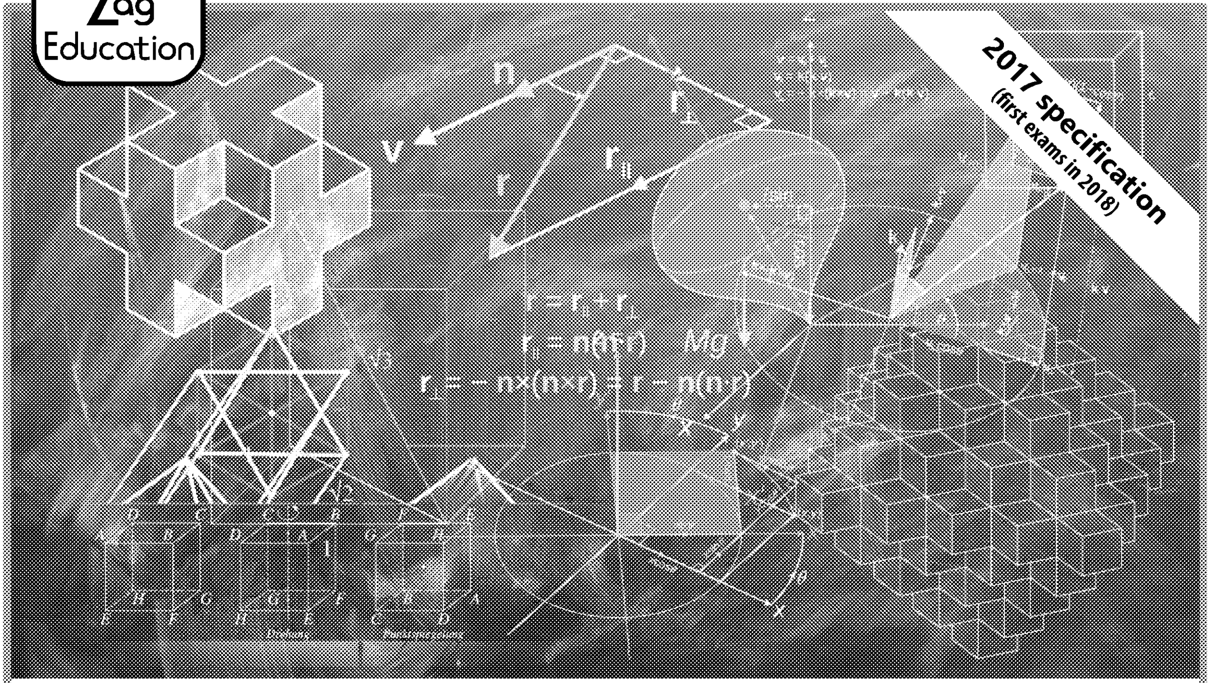
Maths

A Level | AQA | 7357



Topic Tests: Challenge Tests – Set A

For A Level Year 2 AQA Pure Mathematics

zigzageducation.co.uk

POD 8941

Contents

Teacher's Introduction

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

Content

This pack contains 14 challenge level topic tests and solutions for the AQA Pure Mathematics Year 2 A Level content.

About the challenge tests

These **challenge** tests have been designed to **stretch and challenge** your students. 50% of the marks come from questions similar in style to our fundamentals tests. These questions isolate and test the core skills in each topic. The other 50% of the marks come from questions of increased difficulty that progress and start to combine the concepts in the topic. Due to the increased challenge they pose, we recommend these tests for students who have already mastered the fundamentals by scoring 70% or more on our fundamentals tests.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Cross-referencing Grid

Topic	AQA spec. points	Subtopics
Algebraic Methods	A1, B6, B10	Proof by contradiction, algebraic fractions, partial fractions with repeated factors, algebraic division
Functions and Graphs	[illegible]	The modulus function, functions and mappings, composite functions, inverse functions, $y = \|f(x)\|$, combining transformations, solving modulus problems
Sequences and Series	D2–D6	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	D1	Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions
Radians	E1–E3	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	E4–E5	Secant, cosecant and cotangent, graphs of sec x, cosec x and cot x, using sec x, cosec x and cot x, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	B11, E6–E9	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a\cos x \pm b\sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	C3–C4	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	G1–G5	Differentiating sin x and cos x, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	G2, G4–G6	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	I1–I2, I4	Locating roots, iteration, the Newton-Raphson method, application to modelling
Integration Part I	H2, [illegible]	Integrating standard functions, integrating $f(ax + b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	H3–H4, H7–H8, I3	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	J1–J5	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

Subtopics: The product rule, the quotient rule, differentiating trigonometric functions, im

1. Use the **product rule** to differentiate each of the following with respec

   a) $x^2 \cos x$ b) $\cos 3x \ln x$ c) $e^{4x} \sin 2x$

2. Find $\frac{dy}{dx}$ given that:

   a) $y = \frac{x}{3x+1}$ b) $y = \frac{2x^2}{3x+1}$ c) $y = \frac{e^{3x}}{\sin x}$

3. a) If $y = 4\sec x$, use the q… …t …le to show that $\frac{dy}{dx} = 4\sec x \tan x$

   b) If … an …, …e the **quotient rule** to show that $\frac{dy}{dx} = 3\sec^2 3x$.

   c) If $y = \cot 2x$, use the **quotient rule** to show that $\frac{dy}{dx} = -2\operatorname{cosec}^2 2x$

   (For this question you may use the standard results for the derivatives o

4. If $x = \sin 2y$, find an expression for $\frac{dy}{dx}$ in terms of x.

5. Given that $x = y^2 + 4y$ and that $\frac{dy}{dt} = 2$, find $\frac{dx}{dt}$ when $y = 3$.

6. Find the **exact** gradient of the curve with equation $2^x = x^3 y - 4y$ at the

7. A stalagmite forms on a cave floor, growing in height with **constant ra**

   On day t the stalagmite is also eroded from its tip at a rate of $\frac{1}{5}h$ mm p

   of the stalagmite at the start of that day. Show that $-5\frac{dh}{dt} = h - 15$.

Preview of Questions Ends Here

1. a) $y = 3\sin x \therefore \frac{dy}{dx} = 3\cos x$ **A1**

   b) $y = \cos 4x \therefore \frac{dy}{dx} = -4\sin 4x$ **A1**

   c) $y = 2\sin 3x \therefore \frac{dy}{dx} = 2\times 3\cos 3x = 6\cos 3x$ **A1**

   d) $y = \cos\frac{1}{3}x - 5\sin 6x \therefore \frac{dy}{dx} = -\frac{1}{3}\sin\frac{1}{3}x - 5\times 6\cos 6x$

   $= -\frac{1}{3}\sin\frac{1}{3}x - 30\cos 6x$ **1A1** **[5 Marks]**

2. a) $f(x) = 4^x \ (a = 4, k = 1)$

   $\therefore f'(x) = 4^x \ln 4$

   b) [illegible]

   $f'(x) = \frac{1}{x}$ **A1**

   c) $f(x) = \ln\frac{1}{7}x = \ln\frac{1}{7} + \ln x$ **M1**

   $\therefore f'(x) = 0 + \frac{1}{x} = \frac{1}{x}$ **A1**

   d) $f(x) = e^{3x}$

   $\therefore f'(x) = 3e^{3x}$ **A1**

   e) $f(x) = e^{3x} - 2^{5x} \ (a = 2, k = 5)$ **M1**

   $\therefore f'(x) = 3e^{3x} - 2^{5x}5\ln 2$ **A1**

   f) $f(x) = 6e^{-x} + \ln\frac{x}{4} = -6e^{-x} + \ln\frac{1}{4} + \ln x$ **M1**

   $\therefore f'(x) = -6e^{-x} + \frac{1}{x}$ **A1A1** **[10 Marks]**

3. a) $y = \sin(x-1)$; let $u = x - 1$ so $y = \sin u$

   $\therefore \frac{du}{dx} = 1$

   $\therefore \frac{dy}{du} = \cos u$ **A1**

   Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \cos u \times 1 = \cos u$

   $= \cos(x-1)$ **A1**

   b) $y = \sin(x^2)$; let $u = x^2$ so $y = \sin u$

   $\therefore \frac{du}{dx} = 2x$

   $\therefore \frac{dy}{du} = \cos u$ **A1**

   Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \cos u \times 2x = 2x\cos u$

   $= 2x\cos(x^2)$ **A1**

c) $y = \sin\left(x^2 - 4x + 1\right)$; let $u = x^2 - 4x + 1$ so $y = \sin u$

$\therefore \frac{du}{dx} = 2x - 4$ **A1**

$\therefore \frac{dy}{du} = \cos u$ **A1**

Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times (2x - 4) = (2x - 4)\cos u$

Techn…
$u = x$…

$= (2x - 4)\cos\left(x^2 - 4x + \ldots\right.$

(also accept expanded form $2x\cos(x^2 - 4x + 1) - 4\cos(x^2 - 4x + 1)$) **A1**

d) $y = \left(-x^2 + 2x - 4\right)^3$; let $u = -x^2 + 2x - 4$ so $y = u^3$

$\therefore \frac{du}{dx} = -2x + 2$ **A1**

$\therefore \frac{dy}{du} = 3u^2$ **A1**

…the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 3u^2 \times (-2x + 2)$

Techn…
$u = -x$…

$= 3(-2x + 2)\left(-x^2 + 2x - 4\right)^2$

(also accept equivalent expressions, e.g. $6(-x^2 + 2x - 4)(-x + 1)$) **A1** **[11 …**

4. a) $x = t^2, y = 3t$

$\therefore \frac{dx}{dt} = 2t$ **M1**

$\therefore \frac{dy}{dt} = 3$ **M1**

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3}{2t}$ (also accept $1.5t^{-1}$) **A1**

b) $x = e^{2t}, y = 4t^2$

$\therefore \frac{dx}{dt} = 2e^{2t}$ **M1**

Techn…
$y = e$…

$\therefore \frac{dy}{dt} = 8t$ **M1**

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{8t}{2e^{2t}} = \frac{4t}{e^{2t}}$ (also accept $4te^{-2t}$) **A1**

c) $x = \frac{3}{t+5} = 3(t+5)^{-1}, y = 6t - 4$

$\therefore \frac{dx}{dt} = -3(t+5)^{-2} = \frac{-3}{(t+5\ldots}$ M…

$\therefore \frac{dy}{dt} = 6$ M…

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6}{-3/(t+5)^2} = -2(t+5)^2$ **A1** **[9 …**

5. $y = e^{\cos x}$; let $u = \cos x$ so $y = e^u$

$\therefore \dfrac{du}{dx} = -\sin x$ **M1**

$\therefore \dfrac{dy}{du} = e^u$ **M1**

Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = e^u \times (-\sin x) = -e^u \sin x$

$= -e^{\cos x} \sin x$ **M1**

At $x = \dfrac{\pi}{2}$, $-e^{\cos x} \sin x = -e^{\cos \pi/2} \sin \dfrac{\pi}{2}$ **M1**

$= -e^0 \times 1 = -1$ so the gradient there i[illegible]1 **A1** **[5 Marks]**

6. $y = \ln(5-3x)$; let $u = 5-3x$ so $y = \ln u$

$\therefore \dfrac{du}{dx} = -3$ **A1**

$\therefore \dfrac{dy}{du}$ [illegible]1

Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = \dfrac{1}{u} \times -3 = \dfrac{-3}{u}$

$= \dfrac{-3}{5-3x}$ **A1**

At $x = 2$, $\dfrac{-3}{5-3x} = \dfrac{-3}{5-3(2)}$ **M1**

$= \dfrac{-3}{-1} = 3$ so the gradient there is 3 **A1** **[5 Marks]**

7. **Show that $f(x) = x^3 + 4x^2 - x + 2$ is concave on the interval [−4, −2] and conv**

$f(x) = x^3 + 4x^2 - x + 2$

$\therefore f'(x) = 3x^2 + 8x - 1$ **M1**

$\therefore f''(x) = 6x + 8$ **M1**

$6x + 8 = -16$ when $x = -4$, and -4 when $x = -2$

$\therefore 6x + 8 \le 0$ for all $-4 \le x \le -2$ $\therefore f(x)$ is concave on this interval **A1**

$6x + 8 = 2$ when $x = -1$, and 14 when $x = 1$

$\therefore 6x + 8 \ge 0$ for all $-1 \le x \le 1$ $\therefore f(x)$ is convex on this interval **A1** **[4 Marks]**

8. $y = 2x^3 - 6x^2 + 6x + 4$

$\therefore \dfrac{dy}{dx} = 6x^2 - 12x + 6$ **M1**

$\therefore \dfrac{d^2y}{dx^2} = 12x - 12$ **M1**

If $12x - 12 = 0$ [illegible] 1

$\dfrac{d^2y}{dx^2} <$ [illegible] 1 and $\dfrac{d^2y}{dx^2} > 0$ if $x > 1$ so second derivative changes sign at $x = 1$ **A1**

At $x = 1$, $y = 2\times1^3 - 6\times1^2 + 6\times1 + 4$ **M1**

$= 2 - 6 + 6 + 4 = 6$

So the coordinates of the point of inflection are $(1, 6)$ **A1** **[5 Marks]**

9. **Prove from first principles that the derivative of cos x is −sin x.**

Let $f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} \quad \textbf{M1}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \quad \textbf{M1}$$

Techni
cos(A

$$= \lim_{h \to 0} \left(\cos x \left(\frac{\cos h - 1}{h} \right) - \sin x \left(\frac{\sin h}{h} \right) \right) \quad \textbf{M1}$$

Techni
$h \to$
from

This tends to $\cos x \times 0 - \sin x \times 1$ **M1**

So $\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} \to -\sin x$

Hence the derivative of $\cos x$ is $-\sin x$ **A1** **[5 Marks]**

10. $x = 4 + \cos 2t, y = \sin 2t$

$\therefore \frac{dx}{dt} = -2\sin 2t$ **M1**

$\therefore \frac{dy}{dt} = 2\cos 2t$ **M1**

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos 2t}{-2\sin 2t} = -\frac{\cos 2t}{\sin 2t} \quad \textbf{M1}$$

Techni
using

So at $t = \frac{\pi}{12}$, gradient of the tangent is $\frac{-\cos 2\pi/12}{\sin 2\pi/12} = -\sqrt{3}$ **M1**

So at $t = \frac{\pi}{12}$, gradient of the normal is $\frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$ **A1**

Also at $t = \frac{\pi}{12}$:

$x = 4 + \cos\frac{2\pi}{12} = 4 + \frac{\sqrt{3}}{2} = \frac{8+\sqrt{3}}{2}$ **M1**

$y = \sin\frac{2\pi}{12} = \frac{1}{2}$ **M1**

Alter
$y - y$
$m =$

So at $t = \frac{\pi}{12}$, equation of the normal to the curve is $y = mx + c$ with $y = \frac{1}{2}$, $m = \frac{\sqrt{3}}{3}$

$\therefore \frac{1}{2} = \frac{\sqrt{3}}{3} \times \frac{8+\sqrt{3}}{2} + c$

$\therefore c = \frac{1}{2} - \frac{8\sqrt{3}+3}{6} = -\frac{4\sqrt{3}}{3}$ **A1**

So $y = \frac{\sqrt{3}}{3}x - \frac{4\sqrt{3}}{3}$ (or equivalent, e.g. $3y = \sqrt{3}x - 4\sqrt{3}$) **A1** **[9 Marks]**

Preview of Answers Ends Here