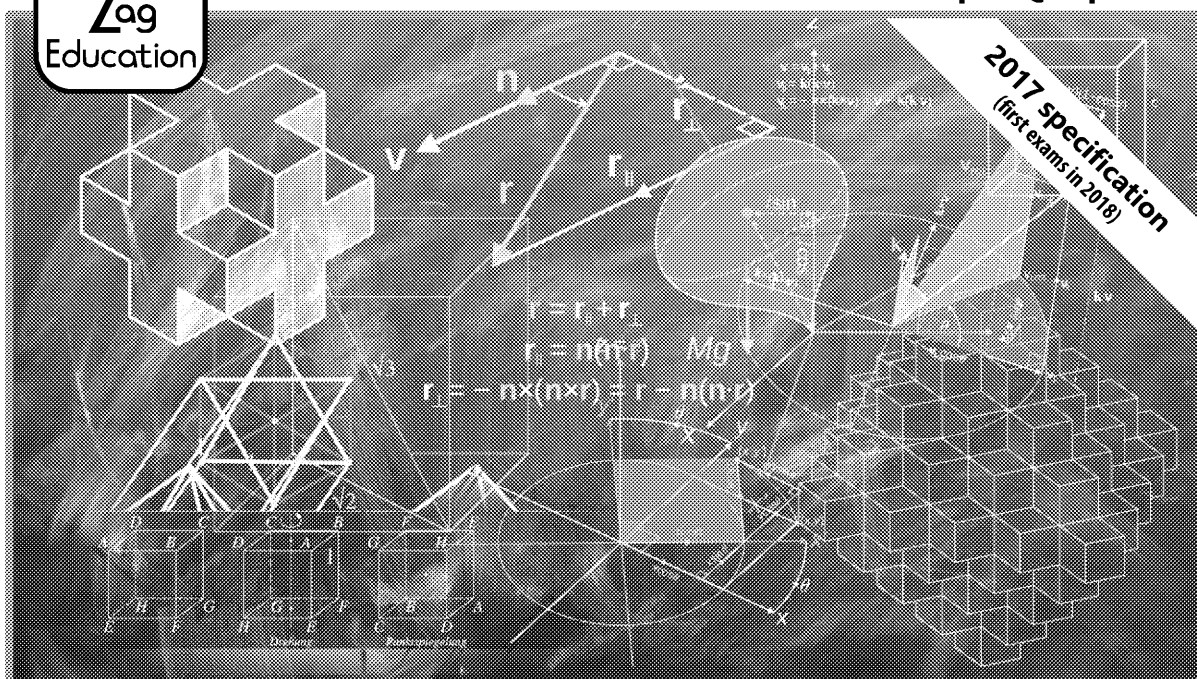




Topic Tests:

Fundamentals Tests – Set A

For A Level Year 2 AQA
Pure Mathematics

zigzageducation.co.uk

**POD
8940**

Publish your own work... Write to a brief...
Register at publishmenow.co.uk

Contents

Thank You for Choosing ZigZag Education	ii
Teacher Feedback Opportunity	iii
Terms and Conditions of Use	iv
Teacher's Introduction	1
Cross-referencing Grid	2

Tests

- Test 1 – Algebraic Methods
- Test 2 – Functions and Graphs
- Test 3 – Sequences and Series
- Test 4 – Binomial Expansion
- Test 5 – Radians
- Test 6 – Trigonometry Part I
- Test 7 – Trigonometry Part II
- Test 8 – Parametric Equations
- Test 9 – Differentiation Part I
- Test 10 – Differentiation Part II
- Test 11 – Numerical Methods
- Test 12 – Integration Part I
- Test 13 – Integration Part II
- Test 14 – Vectors

Solutions

Teacher's Introduction

Content

This pack contains 14 fundamentals level topic tests and solutions for the AQA Pure Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the fundamentals tests

These **fundamentals** tests focus on isolating and testing the core skills of each topic. The questions are designed to use simple numbers and contexts **so that students can show what they can do**, and to allow them to easily identify any weaknesses.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework:** Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test:** Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork:** Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test:** Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Free Updates!

Register your email address to receive any future free updates* made to this resource or other Maths resources your school has purchased, and details of any promotions for your subject.

* resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers

Go to **zzed.uk/freeupdates**

Cross-referencing Grid

Topic	AQA spec. points	Subtopics
Algebraic Methods	A1, B6, B10	Proof by contradiction, algebraic fractions, partial fractions, connected factors, algebraic division
Functions and Graphs	7–8	Modulus function, functions and mappings, composite functions, inverse functions, $y = f(x) $, combining transformations, solving modulus problems
Sequences and Series	D2–D6	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	D1	Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions
Radians	E1–E3	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	E4–E5	Secant, cosecant and cotangent, graphs of $\sec x$, $\csc x$ and $\cot x$, using $\sec x$, $\csc x$ and $\cot x$, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	B11, E6–E9	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a \cos x \pm b \sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	C3–C4	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	G1–G5	Differentiating $\sin x$ and $\cos x$, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	G2, G4–G6	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	I1–I2, I4	Locating roots, iteration, Newton-Raphson method, application to modelling
Integration Part I	H2, H7–H8	Integrating standard functions, integrating $f(ax + b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	H3–H4, H7–H8, I3	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	J1–J5	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

INSPECTION COPY

**COPYRIGHT
PROTECTED**



Vectors – Test A (40 mins)

Subtopics: 3D coordinates, vectors in 3D, solving geometric problems, applications

1. Find the **distances** between the following pairs of points:
 - a) $(0, 0, 0)$ and $(2, 6, 9)$
 - b) $(1, 0, -1)$ and $(3, 10, 10)$
 - c) $(-1, -1, 2)$ and $(5, -7, -15)$
2. Let $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{w} = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find the following vectors, giving your answers in $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation.
 - a) $\mathbf{v} + \mathbf{w}$
 - b) $\mathbf{v} - \mathbf{w}$
 - c) $3\mathbf{v} - 2\mathbf{w}$
3. Let $\mathbf{a} = \begin{pmatrix} 14 \\ 10 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$. Show that the **resultant** of $\mathbf{a}$ and $\mathbf{b}$ is parallel to the yz -plane.
4. Given that $\mathbf{v} = 2\mathbf{i} + p\mathbf{j} - 14\mathbf{k}$ and that $|\mathbf{v}| = 15$, find the **two possible values** of p .
5. Find the **unit vector** that is in the direction of each of the following vectors, giving your answers in $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation **exactly**:
 - a) $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$
 - b) $\begin{pmatrix} -12 \\ 16 \\ 15 \end{pmatrix}$
 - c) $\begin{pmatrix} \sqrt{2} \\ -3 \\ \sqrt{5} \end{pmatrix}$
6. Find, to **3 significant figures**, the angle in **degrees** made by the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$ with
 - a) the positive x -axis
 - b) the positive y -axis
 - c) the positive z -axis
7. O is the origin, while the points P and Q have coordinates $(3, 4, 0)$ and $(5, 2, 3)$ respectively. Find the coordinates of the point R such that $OPQR$ is a **parallelepiped** by using **vectors** to find the coordinates of the point R such that $OPQR$ is a **parallelepiped**.
8. The points P and Q have coordinates $(3, 3, 3)$ and $(5, -2, -3)$ respectively.
 - a) Show that $\triangle OPQ$ is a **right-angled** triangle.
 - b) Find the **exact area** of the triangle $\triangle OPQ$.
9. A body of mass 3 kg is acted on by a force $(-6\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}) \text{ N}$. Find the **acceleration** of the body, giving your answer in **ijk** notation.
10. A body is acted on by forces $(a\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \text{ N}$, $(2\mathbf{i} + b\mathbf{j} + \mathbf{k}) \text{ N}$ and $(-5\mathbf{i} - 3\mathbf{j} + c\mathbf{k}) \text{ N}$. The body is in a state of **equilibrium**. Find the values of a , b and c .

INSPECTION COPY

**COPYRIGHT
PROTECTED**



Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Solutions to Integration Part II – Test A

1. a) $\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi}$ M1
 $= -\cos \pi - (-\cos 0)$ M1
 $= -(-1) - (-1)$
 $= 2$ A1

b) $\int_0^1 \frac{1}{x+1} \, dx = [\ln|x+1|]_0^1$ M1
 $= \ln 2 - \ln 1$ M1
 $= \ln 2$ A1

c) $\int_0^{\ln 10} e^x \, dx = [e^x]_0^{\ln 10}$ M1
 $= e^{\ln 10} - e^0$ M1
 $= 10 - 1$

d) $\int_0^{\frac{\pi}{3}} x \, dx = \left[\tan x \right]_0^{\frac{\pi}{3}}$ M1
 $= \tan \frac{\pi}{3} - \tan 0$ M1
 $= \sqrt{3} - 0$
 $= \sqrt{3}$ A1

[12 Marks]

2. Area of R is $\int_0^3 \frac{x}{\sqrt{x+1}} \, dx$ M1

Let $u = x + 1$, so $\frac{du}{dx} = 1$

The lower limit of integration $x = 0$ becomes $u = 0 + 1 = 1$

The upper limit of integration $x = 3$ becomes $u = 3 + 1 = 4$

So $\int_0^3 \frac{x}{\sqrt{x+1}} \, dx = \int_1^4 \frac{u-1}{\sqrt{u}} \, du$ M1
 $= \int_1^4 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}} \right) du$
 $= \left[\frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right]_1^4$ M1
 $= \left(\frac{2}{3} \times 4^{\frac{3}{2}} - 2 \times 4^{\frac{1}{2}} \right) - \left(\frac{2}{3} \times 1^{\frac{3}{2}} - 2 \times 1^{\frac{1}{2}} \right)$ M1
 $= \frac{4}{3} - \left(-\frac{4}{3} \right)$
 $= \frac{8}{3}$ A1

[5 Marks]

3. a) Prove that $\sec^2 x - \tan^2 x \equiv 1$.

$\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} \equiv 1$

Provided $\cos x \neq 0$ we can divide through by $\cos^2 x$ to get:

$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} \equiv \frac{1}{\cos^2 x}$ M1

$\therefore \tan^2 x + 1 \equiv \sec^2 x$

$\therefore \sec^2 x - \tan^2 x \equiv 1$ A1

Technique
curve
the x-axis
 $x = b$

$f(x) \geq 0$
Other
the curve
x-axis

Technique
 $\int_a^b \dots$

Technique

Tip: The
example
of the

Technique
substitution
• $x = b$
of u
• dx
• the
correct

INSPECTION COPY

COPYRIGHT
PROTECTED



b) Area between the two curves is $\int_0^1 (\sec^2 x - \tan^2 x) dx$ **M1**

From part a) this is equal to $\int_0^1 1 dx$

And so the area is $\int_0^1 1 dx = [x]_0^1$ **M1**

$= 1 - 0$

$= 1$ **A1**

[5 Marks]

4. By the trapezium rule,

$$\int_0^\pi \cos(\sin x) dx \approx \frac{1}{2} h \{ (y_0 + y_4) + 2(y_1 + y_2 + y_3) \}$$

where $h = \frac{b-a}{n} = \frac{\pi-0}{4} = \frac{\pi}{4}$ **M1**

and the y values are given in the table:

So $\int_0^\pi \cos(\sin x) dx \approx \frac{1}{2} \times \frac{\pi}{4} \{ 1 + 2(0.760 + 0.540 + 0.760) \}$ **M1**

$= \frac{\pi}{8} \times 6.12$

$= 2.40331...$

$= 2.40$ (3 s.f.) **A1**

[3 Marks]

5. The graph of $y = \sin x$ is concave **B1**

So the line forming the top of each trapezium will be beneath the curve, and thus it is an underestimate.
[Allow use of sketch graph or other reasonable explanations; for example, referring to the graph of $y = \sin x$]

[2 Marks]

6. $\frac{dy}{dx} = x^2$

$\therefore y = \int x^2 dx$ **M1**

$\therefore y = \frac{1}{3}x^3 + c$ **A1**

[2 Marks]

7. $x^3 \frac{dy}{dx} = 2x^4 + x$

$\therefore \frac{dy}{dx} = \frac{2x^4 + x}{x^3} = 2x + \frac{1}{x^2}$ **M1**

$\therefore y = \int 2x + x^{-2} dx$

$\therefore y = x^2 - x^{-1} + c$ **A1**

We are told that $y = 1$ when $x = 1$, so $1 = 1^2 - 1^{-1} + c = c - 1 = 1$ **M1**

So the particular solution is $y = x^2 - x^{-1} + 1$ [or $y = x^2 - \frac{1}{x} + 1$] **A1** [4 Marks]

8. a) $\frac{dP}{dt} = 0.1P$

$$\therefore \frac{1}{P} \frac{dP}{dt} = 0.1$$

$$\therefore \int \frac{1}{P} dP = \int 0.1 dt \quad \mathbf{M1}$$

$$\therefore \ln P = 0.1t + c \quad \mathbf{A1}$$

$$\therefore P = e^{0.1t+c} \quad \mathbf{A1}$$

We can rewrite this as $P = e^{0.1t} e^c = A e^{0.1t}$, where $A = e^c$

We are told that when $t = 0$, $P = 150$, so $150 = A e^{0.1 \times 0} = A$, so $A = 150 \quad \mathbf{M1}$

And so $P = 150 e^{0.1t} \quad \mathbf{A1}$

b) When $t = 25$, $P = 150 e^{0.1 \times 25} = 150 e^{2.5} = 1'277.7 \approx 1800$ to the nearest hundred

c) No, exponential growth eventually gives values larger than realistically possible

[Can also reference the predicted figure when $t = 250$ of $P \approx 10^{13}$ penguins as



**COPYRIGHT
PROTECTED**



Preview of Answers Ends Here

This is a limited inspection copy. Sample of answers ends here to stop students looking up answers to their assessments. See contents page for details of the rest of the resource.