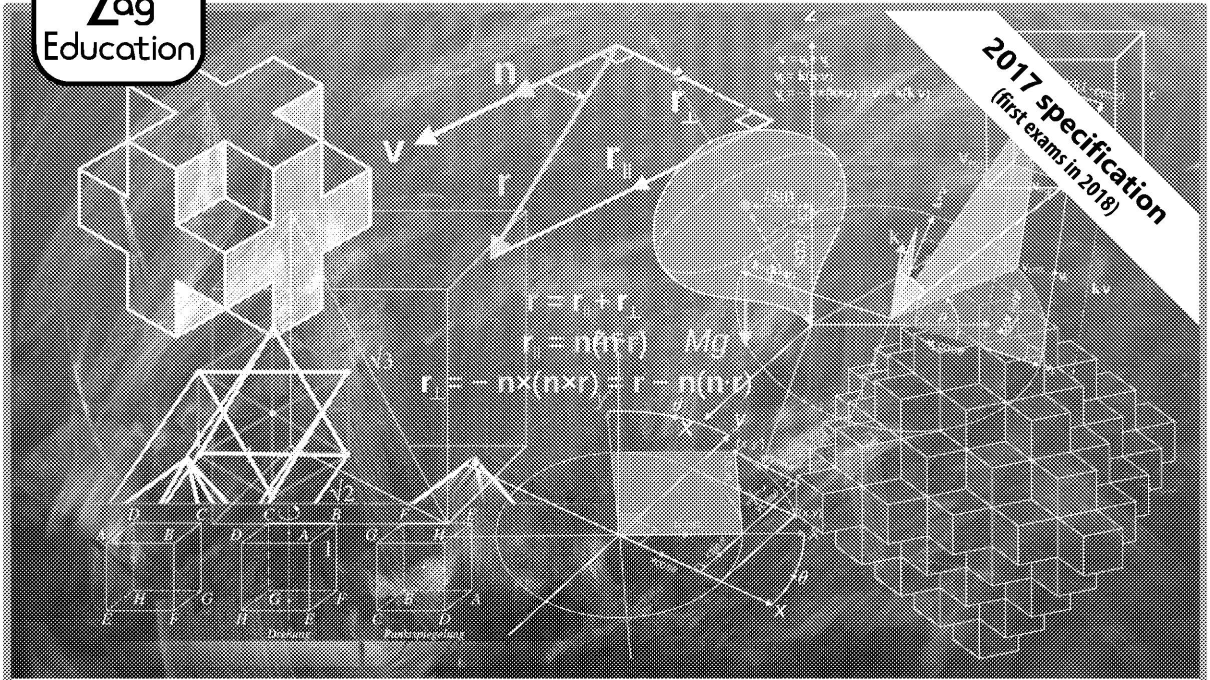
Maths

A Level | Edexcel | 9MA0



Topic Tests: Expert Tests – Set A

For A Level Year 2 Edexcel
Pure Mathematics

zigzageducation.co.uk

POD 8926

Contents

Teacher’s Introduction

Content

This pack contains 14 expert level topic tests and solutions for the Edexcel Pure Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student’s weaknesses lie.

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students’ confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Cross-referencing Grid

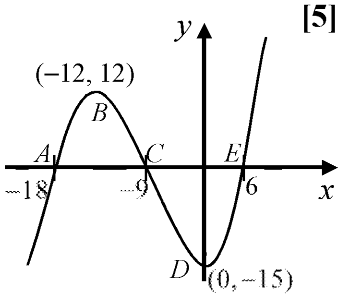
Topic	Edexcel spec. points	Subtopics
Algebraic Methods	1.1, 2.6, 2.10	Proof by contradiction, algebraic fractions, partial fractions with repeated factors, algebraic division
Functions and Graphs	[illegible]–2.9	The modulus function, functions and mappings, composite functions, inverse functions, $y = \|f(x)\|$ and $y = f(\|x\|)$, combining transformations, solving modulus problems
Sequences and Series	4.2–4.6	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	4.1	Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions
Radians	5.1–5.3	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	5.4–5.5	Secant, cosecant and cotangent, graphs of sec x, cosec x and cot x, using sec x, cosec x and cot x, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	2.11, 5.6–5.9	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a\cos x \pm b\sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	3.3–3.4	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	7.1–7.5	Differentiating sin x and cos x, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	7.2, 7.4–7.6	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	9.1–9.3, 9.5	Locating roots, iteration, the Newton-Raphson method, applications to modelling
Integration Part I	[illegible]	Integrating standard functions, integrating $f(ax + b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	8.3–8.4, 8.7–8.8, 9.4	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	10.1–10.5	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

Subtopics: The modulus function, functions and mappings, composite functions, inverse functions, y = |f(x)|, combining transformations, solving modulus problems

1. Given that $f(x)=|3x-9|$ and $g(x)=7-x$:
   a) write down the value of: i) $f(2)$ ii) $f(3)$ iii) $f(4)$ **[3]**
   b) **sketch** $y=f(x)$ and $y=g(x)$ on the same axes, indicating **all** points where the graphs cross or touch the **axes** **[3]**
   c) **solve** the equation $7-x=|3x-9|$ **[3]**
   d) hence **solve** $7-x>|3x-9|$ **[2]**

2. Given that $f(x)=x^2+2$ and $g(x)=x-3$:
   a) find the value of: i) $fg(4)$ ii) $gf(2)$ **[4]**
   b) find an expression for $gf(x)$ **[2]**
   c) **solve** $fg(x)=gf(x)$ **[5]**

3. The graph of $y=f(x)$ is shown to the right. Points *A*, *C* and *E* are the *x*-intercepts, and points *B* and *D* are the stationary points of the graph. On **separate** diagrams, **sketch**, indicating the new coordinates of *B* and *D*:
   a) $y=-f(x+6)$ **[3]**
   b) $y=f(3x)-1$ **[3]**



4. The function $f(x)$ has domain $-12\le x\le 12$ and is **linear** from (–12, 28) to (4, –4) and from (4, –4) to (12, 60).
   a) **Sketch** the graph of $y=f(x)$ **[2]**
   b) State the **range** of $f(x)$ **[1]**
   c) Find the **two values** of *a* such that $f(a)=12$ **[4]**

5. Given that $g(x)=x^2-8x+28$ is a **one-to-one** function with **domain** $x\ge k$, find the **smallest** possible value of *k*. **[3]**

6. The function *h* is defined by $h: x\to\dfrac{x+4}{4x+1},\ x\ne-\dfrac{1}{4}$
   a) Find an expression for h^{-1}, stating the value **excluded** from its domain. **[3]**
   b) Find the **two values** of *a* for which $h(a)=h^{-1}(a)$ **[4]**

7. The function $f(x)=\begin{cases}4x+1 & -4\le x<0\\ 2^x & 0\le x\le 3\end{cases}$

   For each of the following equations, **sketch** the graph, indicating **all** points where the graphs cross or touch the **axes**:
   a) $y=f(x)$ b) $y=|f(x)|$ **[6]**

8. The price of a car can be **modelled** using the formula $P=15000\mathrm{e}^{-0.1t}$, where *P* is the price of the car in pounds and *t* is the age of the car in years.
   a) Find the price of the car when it is **new**. **[1]**
   b) Find the price of the car, **to the nearest pound**, when it is 6 years old. **[2]**
   c) **Sketch** the graph of the price of the car against time for $0\le t\le 10$. **[3]**
   d) Explain whether this is a **realistic** model for the price of a car over time. **[1]**

TOTAL 58 MARKS

Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

1. a) $\sec 45° = \dfrac{1}{\cos 45°}$ **M1**

$= \dfrac{1}{\sqrt{2}/2} = \sqrt{2}$ **A1**

b) $\operatorname{cosec}(-270°) = \dfrac{1}{\sin(-270°)}$ **M1**

$= \dfrac{1}{1} = 1$ **A1**

c) $\cot \dfrac{\pi}{12} = \dfrac{1}{\tan \pi/12}$ **M1**

$= \dfrac{1}{2 \ldots}$ [illegible] 3 [illegible] 1 **[6 Marks]**

Tip: C[illegible]
the co[illegible]
radian[illegible]
questi[illegible]

2. a) [illegible]e that $\dfrac{\tan\theta - \sec\theta}{\sin\theta - 1} \equiv \sec\theta$

$\dfrac{\tan\theta - \sec\theta}{\sin\theta - 1} \equiv \dfrac{\sin\theta/\cos\theta - 1/\cos\theta}{\sin\theta - 1}$ **M1**

$\equiv \dfrac{(\sin\theta - 1)/\cos\theta}{\sin\theta - 1}$ **M1**

$\equiv \dfrac{1}{\cos\theta} \equiv \sec\theta$ **A1**

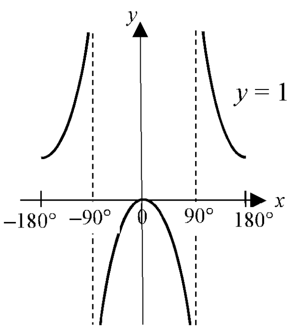
Techn[illegible]
in ter[illegible]

b) $\dfrac{\tan\theta - \sec\theta}{\sin\theta - 1} = \dfrac{1}{2}$

$\therefore \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{2}$ and so $\cos\theta = 2$

But $-1 \le \cos\theta \le 1$ so there are no real solutions **A1** **[4 Marks]**

3. a)



Techn[illegible]
reflec[illegible]
x-axis[illegible]
reflec[illegible]
y = 1[illegible]

A1 correct [illegible]

A1 c[illegible]ve [illegible]s the origin

Asymptotes at $x = -9$[illegible] **A1**

b) $y \ge 2$ and y [illegible]

c) [illegible]on[illegible]ere the line $y = a$ intersects with $y = 1 - \sec x$

[illegible] will be two solutions when $a \ge 2$ or $a < 0$ **A1A1** **[7 Marks]**

Hint: [illegible]
y = s[illegible]

4. The domain of arcsin x is $-1 \le x \le 1$, so the domain of $y = \arcsin 2x$ is $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$ **A1** **[1 Mark]**

Tip: y[illegible]
y = ar[illegible]
x-dire[illegible]

5. a) **Show that $CD = 8(\sec\theta - \cos\theta)$ cm**

$CD = AC - AD$ **M1**

Using triangle ABC, $\cos\theta = \dfrac{AB}{AC}$ so $AC = \dfrac{AB}{\cos\theta} = \dfrac{8}{\cos\theta} = 8\sec\theta$ **M1**

Using triangle ABD, $\cos\theta = \dfrac{AD}{AB}$ so $AD = AB\cos\theta = 8\cos\theta$ **M1**

$\therefore CD = AC - AD = 8\sec\theta - 8\cos\theta = 8(\sec\theta - \cos\theta)$ **A1**

Hint: … triangl… betw… tange…

Hint: … triangl… subte… the di…

b) $CD = \dfrac{16\sqrt{3}}{3}$

$\therefore 8(\sec\theta - \cos\theta) = \dfrac{16\sqrt{3}}{3}$

$3\sec\theta - 3\cos\theta = 2\sqrt{3}$

$3 - 3\cos^2\theta = 2\sqrt{3}\cos\theta$

$3\cos^2\theta + 2\sqrt{3}\cos\theta - 3 = 0$ **M1**

Techni… coeffic… to get… terms…

Let $\cos\theta = \alpha$ and solve using the quadratic formula with $a = 3$, $b = 2\sqrt{3}$, $c = \dots$

$$\alpha = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4\times 3\times(-3)}}{2\times 3} \quad \textbf{M1}$$

$$= \frac{-2\sqrt{3} \pm 4\sqrt{3}}{6} = \frac{\sqrt{3}}{3} \text{ or } -\sqrt{3}$$

But $-1 \le \cos\theta \le 1$ so $\alpha = -\sqrt{3}$ is not a solution, and $\alpha = \cos\theta = \dfrac{\sqrt{3}}{3}$ only …

So $\theta = 54.7356\ldots = 54.7°$ (3 s.f.) **A1** **[8 Marks]**

6. a) **Show that $\dfrac{\operatorname{cosec}^2 x}{\sec^2 x} \equiv \cot^2 x$**

$$\frac{\operatorname{cosec}^2 x}{\sec^2 x} \equiv \frac{1/\sin^2 x}{1/\cos^2 x} \quad \textbf{M1}$$

$$\equiv \frac{\cos^2 x}{\sin^2 x} \quad \textbf{M1}$$

$$\equiv \frac{1}{\tan^2 x} \equiv \cot^2 x \quad \textbf{A1}$$

b) $\dfrac{\operatorname{cosec}^2 x}{\sec^2 x} = 3$

So using part a), $\cot^2 x = 3$ **M1**

$1 + \cot^2 x \equiv \operatorname{cosec}^2 x \therefore \operatorname{cosec}^2 x = 1 + 3 = 4$ **M1**

$\therefore \operatorname{cosec} x = \pm\sqrt{4} = \pm 2$ **M1**

But for $0 \le x \le \pi$ only $\operatorname{cosec} x = 2$ is a solution **A1** **[7 Marks]**

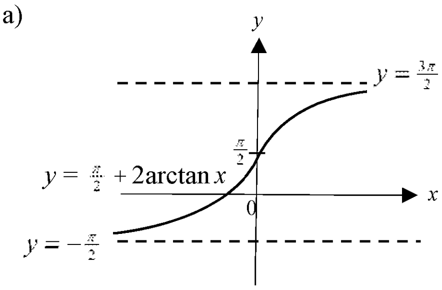
Hint: … $0 \le x$ … valid …

7. $\arccos(2\sqrt{2} - x) = \dfrac{\pi}{4}$ so $\cos\left(\dfrac{\pi}{4}\right) = 2\sqrt{2} - x$ **M1**

$\therefore \dfrac{\sqrt{2}}{2} = 2\sqrt{2} - x$ **M1**

$\therefore x = 2\sqrt{2} - \dfrac{\sqrt{2}}{2} = \dfrac{3\sqrt{2}}{2}$ **A1** **[3 Marks]**

8. a)



A1

A1 y-intercept is $\pi/2$

A1

A1 correct shape

Tech
stretch
in the
the s
get y

b) The x-intercept is where $\frac{\pi}{2} + 2\arctan x = 0$ **M1**

$2\arctan x = -\frac{\pi}{2}$

$\arctan x = -\frac{\pi}{4}$ **M1**

$\therefore x = \tan\left(-\frac{\pi}{4}\right) = -1$ so the x-intercept is at $(-1, 0)$ **A1**

c) From the graph, $\frac{\pi}{2} + 2\arctan x = k$ has no solutions where $y \geq \frac{3\pi}{2}$ or $y \leq -$

Preview of Answers Ends Here

This is a limited inspection copy. Sample of answers ends here to stop students looking up answers to their assessments. See contents page for details of the rest of the resource.