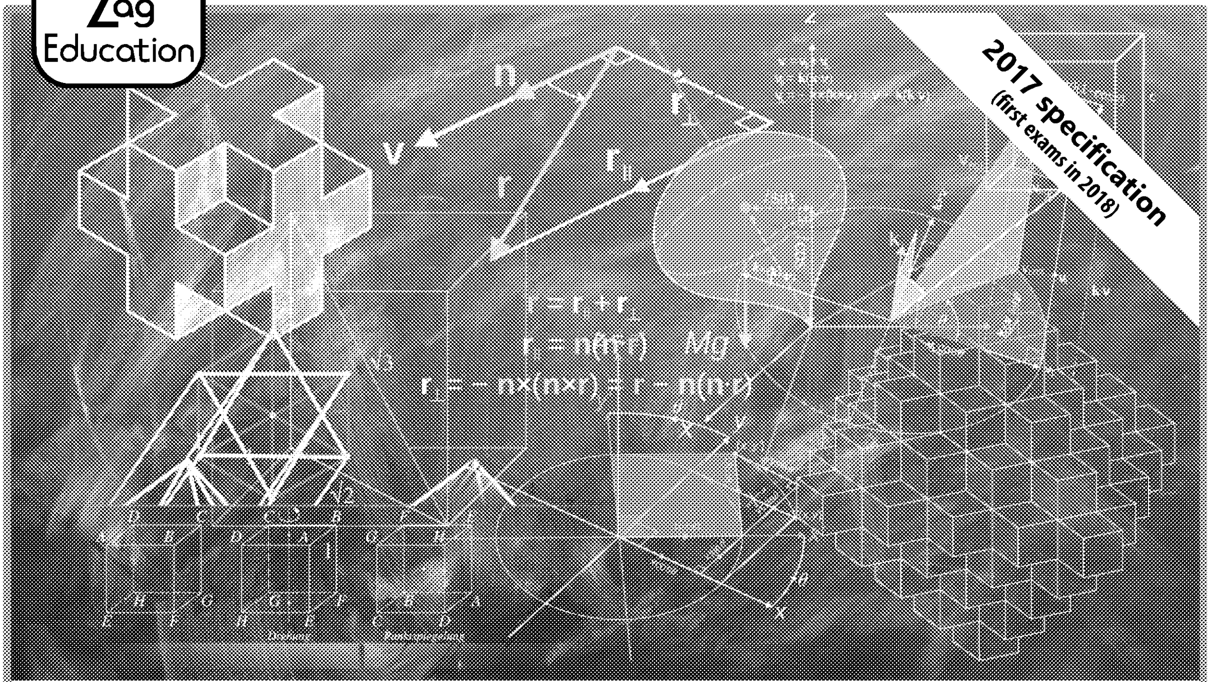
Maths A Level | Edexcel | 9MA0




# Topic Tests: Fundamentals Tests – Set A

For A Level Year 2 Edexcel
Pure Mathematics

zigzageducation.co.uk

POD 8924

# Contents

# Teacher's Introduction

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

## Content

This pack contains 14 fundamentals level topic tests and solutions for the Edexcel Pure Mathematics Year 2 A Level content.

## About the fundamentals tests

These **fundamentals** tests focus on isolating and testing the core skills of each topic. The questions are designed to use simple numbers and contexts **so that students can show what they can do,** and to allow them to easily identify any weaknesses.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

## Timings

The recommended times for students to complete each test are given at the top of individual tests.

## Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

## Also available from ZigZag Education

For students who have mastered the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and combines and extends the concepts covered.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

# Cross-referencing Grid

| Topic | Edexcel spec. points | Subtopics |
|---|---|---|
| Algebraic Methods | 1.1, 2.6, 2.10 | Proof by contradiction, algebraic fractions, partial fractions with repeated factors, algebraic division |
| Functions and Graphs | [illegible]–2.9 | The modulus function, functions and mappings, composite functions, inverse functions, $y = \|f(x)\|$ and $y = f(\|x\|)$, combining transformations, solving modulus problems |
| Sequences and Series | 4.2–4.6 | Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series |
| Binomial Expansion | 4.1 | Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions |
| Radians | 5.1–5.3 | Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations |
| Trigonometry Part I | 5.4–5.5 | Secant, cosecant and cotangent, graphs of sec $x$, cosec $x$ and cot $x$, using sec $x$, cosec $x$ and cot $x$, trigonometric identities, inverse trigonometric functions |
| Trigonometry Part II | 2.11, 5.6–5.9 | Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a\cos x \pm b\sin x$, proving trigonometric identities, modelling with trigonometric functions |
| Parametric Equations | 3.3–3.4 | Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations |
| Differentiation Part I | 7.1–7.5 | Differentiating sin $x$ and cos $x$, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives |
| Differentiation Part II | 7.2, 7.4–7.6 | The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change |
| Numerical Methods | 9.1–9.3, 9.5 | Locating roots, iteration, the Newton-Raphson method, applications to modelling |
| Integration Part I | [illegible].2, 5[illegible] | Integrating standard functions, integrating $f(ax + b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions |
| Integration Part II | 8.3–8.4, 8.7–8.8, 9.4 | Finding areas, the trapezium rule, solving differential equations, modelling with differential equations |
| Vectors | 10.1–10.5 | 3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics |

## **Vectors – Test A** (40 mins)

*Subtopics: 3D coordinates, vectors in 3D, solving geometric problems, app*

1. Find the **distances** between the following pairs of points:
   a) (0, 0, 0) and (2, 6, 9)
   b) (1, 0, –1) and (3, 10, 10)
   c) (–1, –1, 2) and (5, –7, –15)

2. Let $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{w} = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find the following vectors, giv
   a) $\mathbf{v} + \mathbf{w}$
   b) $\mathbf{v} - \mathbf{w}$
   c) $3\mathbf{v} - 2\mathbf{w}$

3. Let $\mathbf{a} = \begin{pmatrix} 14 \\ 10 \\ \text{[illegible]} \end{pmatrix}$ [illegible] $\begin{pmatrix} \text{[illegible]} \\ 0 \\ 7 \end{pmatrix}$. Show that the **resultant** of **a** and **b** is **pa**

4. Given that $\mathbf{v} = 2\mathbf{i} + p\mathbf{j} - 14\mathbf{k}$ and that $|\mathbf{v}| = 15$, find the **two** possible va

5. Find the **unit vector** that is in the direction of each of the following ve **exactly**:

   a) $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ b) $\begin{pmatrix} -12 \\ 16 \\ 15 \end{pmatrix}$ c) $\begin{pmatrix} \sqrt{2} \\ -3 \\ \sqrt{5} \end{pmatrix}$

6. Find, to **3 significant figures**, the angle in **degrees** made by the vector

   a) the positive *x*-axis
   b) the positive *y*-axis
   c) the positive *z*-axis

7. *O* is the origin, while the points *P* and *Q* have coordinates (3, 4, 0) and **vectors** to find the coordinates of the point *R* such that *OPQR* is a **par**

8. The points *P* and *Q* have coordinates [illegible] and (5, –2, –3) respecti
   a) Show that △*OPQ* is [illegible] triangle.
   b) Find the **exact area** of the triangle △*OPQ*.

9. A body of mass 3 kg is acted on by a force $(-6\mathbf{i} + 6\mathbf{j} - 12\mathbf{k})$ N. Find th giving your answer in **ijk** notation.

10. A body is acted on by forces $(a\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ N, $(2\mathbf{i} + b\mathbf{j} + \mathbf{k})$ N and $(-5\mathbf{i}$ – *a*, *b* and *c*. The body is in a state of **equilibrium**. Find the values of *a*

## Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

## Solutions to Integration Part II – Test A

1. a) $\int_0^{\pi} \sin x\,dx = [-\cos x]_0^{\pi}$ **M1**

$= -\cos\pi - (-\cos 0)$ **M1**

$= -(-1) - (-1)$

$= 2$ **A1**

Tech… curve… the $x$… $x = b$… $f(x) \geq$… Other… the ar… $x$-axis…

b) $\int_0^1 \frac{1}{x+1}\,dx = \left[\ln|x+1|\right]_0^1$ **M1**

$= \ln 2 - \ln 1$ **M1**

$= \ln 2$ **A1**

Tech… $\int$ …… a…

c) $\int_0^{\ln 10} e^x\,dx = \left[e^x\right]_0^{\ln 10}$ **M1**

$= e^{\ln 10} - e^0$ **M1**

$= 10 - 1$

Tech…

d) $\ldots x\,dx = [\tan x]_0^{\frac{\pi}{3}}$ **M1**

$= \tan\frac{\pi}{3} - \tan 0$ **M1**

$= \sqrt{3} - 0$

$= \sqrt{3}$ **A1** **[12 Marks]**

Tip: T… exam… of the…

2. Area of $R$ is $\int_0^3 \frac{x}{\sqrt{x+1}}\,dx$ **M1**

Let $u = x + 1$, so $\frac{du}{dx} = 1$

The lower limit of integration $x = 0$ becomes $u = 0 + 1 = 1$
The upper limit of integration $x = 3$ becomes $u = 3 + 1 = 4$

So $\int_0^3 \frac{x}{\sqrt{x+1}}\,dx = \int_1^4 \frac{u-1}{\sqrt{u}}\,du$ **M1**

$= \int_1^4 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$

$= \left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^4$ **M1**

$= \left(\frac{2}{3}\times 4^{\frac{3}{2}} - 2\times 4^{\frac{1}{2}}\right) - \left(\frac{2}{3}\times 1^{\frac{3}{2}} - 2\times 1^{\frac{1}{2}}\right)$ **M1**

$= \frac{4}{3} - \left(-\frac{4}{3}\right)$

$= \frac{8}{3}$ **A1** **[5 Marks]**

Tech… substi… • $x$ b… of $u$… • d… • the… corr…

3. a) **Prove that $\sec^2 x - \tan^2 x \equiv 1$.**

$\sin^2 x + \cos^2 x \equiv 1$

Provided $\cos x \neq 0$ we can divide through by $\cos^2 x$ to get:

$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} \equiv \frac{1}{\cos^2 x}$ **M1**

$\therefore \tan^2 x + 1 \equiv \sec^2 x$

$\therefore \sec^2 x - \tan^2 x \equiv 1$ **A1**

b) Area between the two curves is $\int_0^1 \left(\sec^2 x - \tan^2 x\right) \mathrm{d}x$ **M1**

From part a) this is equal to $\int_0^1 1\,\mathrm{d}x$

And so the area is $\int_0^1 1\,\mathrm{d}x = [x]_0^1$ **M1**

$= 1 - 0$

$= 1$ **A1** **[5 Marks]**

4. By the trapezium rule,

$$\int_0^{\pi} \cos(\sin x)\,\mathrm{d}x \approx \frac{1}{2}h\left\{(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right\}$$

where $h = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4}$ **M1**

and the $y$ values are given in the tab[illegible]

So $\int_0^{\pi} \cos(\sin x)\,\mathrm{d}x \approx \frac{1}{[illegible]}$ [illegible] $\frac{\pi}{4}$ [illegible] $+ 2(0.760 + 0.540 + 0.760)\}$ **M1**

$= \frac{\pi}{8} \times 6.12$

$= 2.40331\ldots$

$= 2.40$ (3 s.f.) **A1** **[3 Marks]**

5. The graph of $y = \sin x$ is concave **B1**
So the line forming the top of each trapezium will be beneath the curve, and thus it
[Allow use of sketch graph or other reasonable explanations; for example, referring to th
**[2 Marks]**

6. $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2$

$\therefore y = \int x^2\,\mathrm{d}x$ **M1**

$\therefore y = \frac{1}{3}x^3 + c$ **A1** **[2 Marks]**

7. $x^3 \frac{\mathrm{d}y}{\mathrm{d}x} = 2x^4 + x$

$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x^4 + x}{x^3} = 2x + \frac{1}{x^2}$ **M1**

$\therefore y = \int 2x + x^{-2}\,\mathrm{d}x$

$\therefore y = x^2 - x^{-1} + c$ **A1**

We are told that $y = 1$ when $x = 1$, so $1 = 1^2 - 1^{-1} + c = c$ [illegible] $= 1$ **M1**

So the particular solution is $y = x^2 - x^{-1} + 1$ [or $y =$ [illegible] $\frac{\,}{x} + 1$] **A1** **[4 Marks]**

8. a) $\dfrac{dP}{dt} = 0.1P$

$\therefore \dfrac{1}{P}\dfrac{dP}{dt} = 0.1$

$\therefore \displaystyle\int \frac{1}{P}\,dP = \int 0.1\,dt$ **M1**

$\therefore \ln P = 0.1t + c$ **A1**

$\therefore P = e^{0.1t+c}$ **A1**

We can rewrite this as $P = e^{0.1t}e^{c} = Ae^{0.1t}$, where $A = e^{c}$

We are told that when $t = 0$, $P = 150$, so $150 = Ae^{0.1\times0} = A$, so $A = 150$ **M1**

And so $P = 150e^{0.1t}$ **A1**

b) When $t = 25$, $P = 150e^{0.1\times25} = 150e^{2.5} = 1$[illegible]$7$ [illegible] $1800$ to the nearest hun

c) No, exponential growth eventu[illegible]larger than realistically possibl

[Can also reference th[illegible]gure when $t = 250$ of $P \approx 10^{13}$ penguins

## Preview of Answers Ends Here