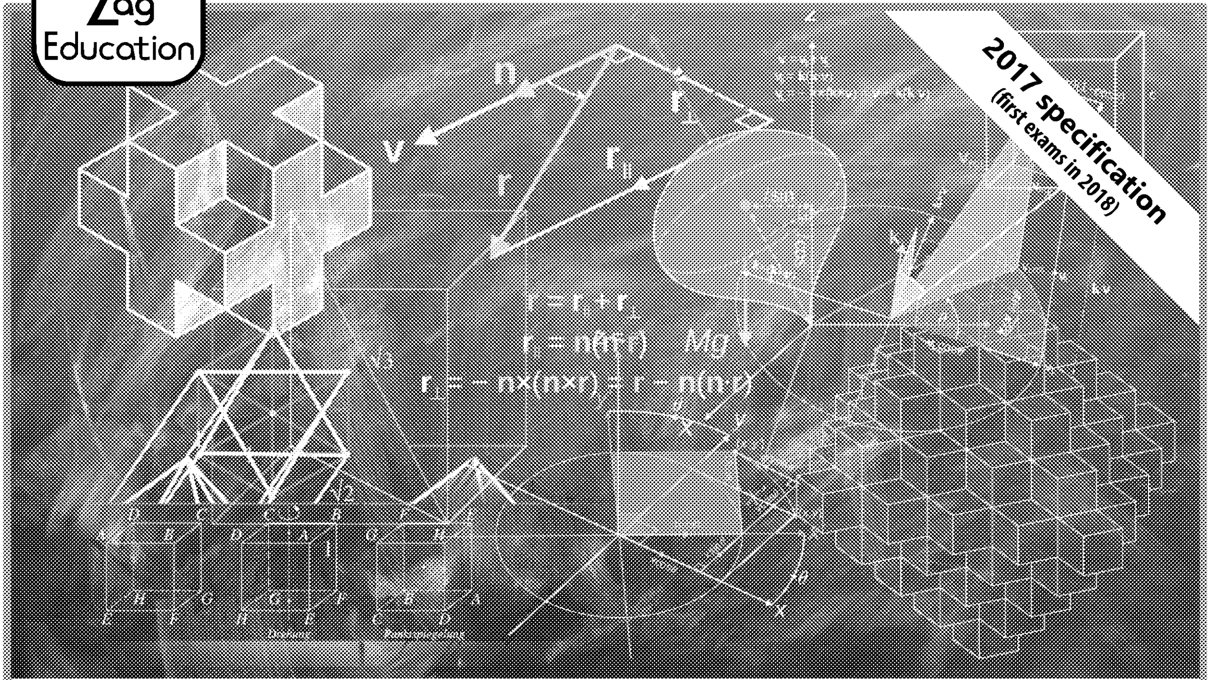
Zig Zag Education

Maths **A Level | Edexcel | 9MA0**



2017 specification
(first exams in 2018)

Topic Tests: Challenge Tests – Set B

For A Level Year 2 Edexcel Pure Mathematics

zigzageducation.co.uk **POD 8929**

Contents

Teacher's Introduction

Content

This pack contains 14 challenge level topic tests and solutions for the Edexcel Pure Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the challenge tests

These **challenge** tests have been designed to **stretch and challenge** your students. 50% of the marks come from questions similar in style to our fundamentals tests. These questions isolate and test the core skills in each topic. The other 50% of the marks come from questions of increased difficulty that progress and start to combine the concepts in the topic. Due to the increased challenge they pose, we recommend these tests for students who have already mastered the fundamentals by scoring 70% or more on our fundamentals tests.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these challenge tests, the relevant times are the third and fourth listed under each topic.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

Cross-referencing Grid

Topic	Edexcel spec. points	Subtopics
Algebraic Methods	1.1, 2.6, 2.10	Proof by contradiction, algebraic fractions, partial fractions with repeated factors, algebraic division
Functions and Graphs	[illegible]–2.9	The modulus function, functions and mappings, composite functions, inverse functions, $y = \|f(x)\|$ and $y = f(\|x\|)$, combining transformations, solving modulus problems
Sequences and Series	4.2–4.6	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	4.1	Expanding $(1 + x)^n$, expanding $(a + bx)^n$, using partial fractions
Radians	5.1–5.3	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	5.4–5.5	Secant, cosecant and cotangent, graphs of sec x, cosec x and cot x, using sec x, cosec x and cot x, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	2.11, 5.6–5.9	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a\cos x \pm b\sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	3.3–3.4	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	7.1–7.5	Differentiating sin x and cos x, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	7.2, 7.4–7.6	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	9.1–9.3, 9.5	Locating roots, iteration, the Newton-Raphson method, applications to modelling
Integration Part I	[illegible]	Integrating standard functions, integrating $f(ax + b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	8.3–8.4, 8.7–8.8, 9.4	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	10.1–10.5	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Algebraic Methods		**Trigonometry Part I**	
1.1.a	35	6.1a	30
1.1b	35	6.1b	30
1.2a	40	[illegible]	40
1.2b	40	6.2b	40
1.3a	[illegible]	6.3a	50
[illegible]	40	6.3b	50
[illegible] and Graphs		**Trigonometry Part II**	
2.1a	20	7.1a	55
2.1b	20	7.1b	55
2.2a	35	7.2a	65
2.2b	35	7.2b	65
2.3a	40	7.3a	65
2.3b	40	7.3b	65
Sequences and Series		**Parametric Equations**	
3.1a	30	8.1a	30
3.1b	30	8.1b	30
3.2a	35	8.2a	50
3.2b	35	8.2b	50
3.3a	50	8.3a	50
3.3b	50	8.3b	50
Binomial Expansion		**Differentiation Part I**	
4.1a	25	9.1a	25
4.1b	25	9.1b	25
4.2a	50	9.2a	30
4.2b	50	9.2b	30
4.3a	60	9.3a	45
4.3b	60	[illegible]	45
Radians		**Differentiation Part II**	
5.1a	[illegible]	10.1a	30
[illegible]	16	10.1b	30
[illegible]	20	10.2a	40
5.2b	20	10.2b	40
5.3a	35	10.3a	45
5.3b	35	10.3b	45

Differentiation Part II – Test B (40 mins)

Subtopics: The product rule, the quotient rule, differentiating trigonometric functions, im

1. Use the **product rule** to differentiate each of the following with respec

   a) xe^x b) $\sin 2x \ln x$ c) $e^{3x}\cos 2x$

2. Find $\frac{dy}{dx}$ given that:

   a) $y = \frac{x}{2x-5}$ b) $y = \frac{3x^2}{2x-5}$ c) $y = \frac{e^{-x}}{\cos x}$

3. a) If $y = 2\cot x$, use the quotient rule to show that $\frac{dy}{dx} = -2\operatorname{cosec}^2 x$.

   b) If $y = \tan 4x$, use the **quotient rule** to show that $\frac{dy}{dx} = 4\sec^2 4x$.

   c) If $y = \sec 3x$, use the **quotient rule** to show that $\frac{dy}{dx} = 3\sec 3x \tan 3$

   (For this question you may use the standard results for the derivatives o

4. If $x = \cos 2y$, find an expression for $\frac{dy}{dx}$ in terms of x.

5. Given that $x = y^3 - y$ and that $\frac{dy}{dt} = 5$, find $\frac{dx}{dt}$ when $y = 2$.

6. Find the **exact** gradient of the curve with equation $3^x = x^2y - y$ at the p

7. A stalactite forms on a cave ceiling, growing in length with **constant r** day. On day t the stalactite is also eroded from its tip at a rate of $\frac{1}{10}l$ n length of the stalactite at the start of that day. Show that $-10\frac{dl}{dt} = l - 4$

Preview of Questions Ends Here

Solutions to Differentiation Part I – Test B

1. a) $y = 5\sin x \therefore \dfrac{dy}{dx} = 5\cos x$ **A1**

   b) $y = \cos 7x \therefore \dfrac{dy}{dx} = -7\sin 7x$ **A1**

   c) $y = 3\cos 2x \therefore \dfrac{dy}{dx} = 3 \times -2\sin 2x = -6\sin 2x$ **A1**

   d) $y = \cos\frac{1}{5}x + 2\sin 4x \therefore \dfrac{dy}{dx}$

   $= -\frac{1}{5}\sin\frac{1}{5}x + 2\times 4\cos 4x = -\frac{1}{5}\sin\frac{1}{5}x + 8\cos 4x$ **A1** **[5 Marks]**

2. a) $f(x) = 10^x \ (a = 10, k = 1)$

   $\therefore f'(x) = 10^x$ [illegible]

   b) [illegible] $=$ [illegible]

   [illegible]$(x) = \dfrac{1}{x}$ **A1**

   c) $f(x) = \ln\frac{1}{2}x = \ln\frac{1}{2} + \ln x$ **M1**

   $\therefore f'(x) = 0 + \dfrac{1}{x} = \dfrac{1}{x}$ **A1**

   d) $f(x) = e^{5x}$

   $\therefore f'(x) = 5e^{5x}$ **A1**

   e) $f(x) = e^{5x} - 1^{3x} \ (a = 1, k = 3)$ **M1**

   $\therefore f'(x) = 5e^{5x} - 1^{3x}3\ln 1 = 5e^{5x} - 0 = 5e^{5x}$ **A1**

   f) $f(x) = 8e^{4x} + \ln\frac{x}{3} = 8e^{4x} + \ln\frac{1}{3} + \ln x$ **M1**

   $\therefore f'(x) = 8\times 4e^{4x} + 0 + \dfrac{1}{x} = 32e^{4x} + \dfrac{1}{x}$ **A1A1** **[10 Marks]**

3. a) $y = \cos(x+1)$; let $u = x+1$ so $y = \cos u$

   $\therefore \dfrac{du}{dx} = 1$

   $\therefore \dfrac{dy}{du} = -\sin u$ **A1**

   Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du}\times\dfrac{du}{dx} = -\sin u \times 1 =$ [illegible]

   $=$ [illegible] **A1**

   b) $y = \cos\left(x^2\right)$; let $u = x^2$ so [illegible] u

   $\dfrac{du}{}$ $=$ [illegible]

   [illegible] $= -\sin u$ **A1**

   Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du}\times\dfrac{du}{dx} = -\sin u \times 2x = -2x\sin u$

   $= -2x\sin\left(x^2\right)$ **A1**

c) $y = \cos\left(x^2 + 6x - 1\right)$; let $u = x^2 + 6x - 1$ so $y = \cos u$

$\therefore \frac{du}{dx} = 2x + 6$ **A1**

$\therefore \frac{dy}{du} = -\sin u$ **A1**

Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\sin u \times (2x + 6) = -(2x + 6)\sin u$

$= -(2x + 6)\sin\left(x^2 + 6\right.$

(also accept expanded form $-2x\sin(x^2 + 6x - 1) - 6\sin(x^2 + 6x - 1)$) **A1**

Techn
$u = x$

d) $y = \left(x^2 + 5x - 3\right)^4$; let $u = x^2 + 5x - 3$ so $y = u^4$

$\therefore \frac{du}{dx} = 2x + 5$ **A1**

$\therefore \frac{dy}{du} = 4u^3$ **A1**

the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 4u^3 \times (2x + 5)$

$= 4(2x + 5)\left(x^2 + 5x - 3\right)^3$ **A1** **[11 Mark**

Techn
$u = x$

4. a) $x = t^3, y = 4t$

$\therefore \frac{dx}{dt} = 3t^2$ **M1**

$\therefore \frac{dy}{dt} = 4$ **M1**

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4}{3t^2}$ $\left(\text{also accept } \frac{4}{3}t^{-2}\right)$ **A1**

b) $x = e^{6t}, y = 3t^2$

$\therefore \frac{dx}{dt} = 6e^{6t}$ **M1**

Techn
$y = e$

$\therefore \frac{dy}{dt} = 6t$ **M1**

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t}{6e^{6t}} = \frac{t}{e^{6t}}$ (also accept te^{-6t}) **A1**

c) $x = \frac{4}{t - 3} = 4(t - 3)^{-1}, y = 2t + 1$

$\therefore \frac{dx}{dt} = -4(t - 3)^{-2} = \frac{-4}{(t - 3)^2}$ **M1**

$\therefore \frac{dy}{dt} = 2$ **M1**

$\frac{dy/dt}{dx/dt} = \frac{2}{-4/(t - 3)^2} = -\frac{(t - 3)^2}{2}$ **A1** **[9 Mark**

5. $y = e^{\sin x}$; let $u = \sin x$ so $y = e^u$

$\therefore \dfrac{du}{dx} = \cos x$ **M1**

$\therefore \dfrac{dy}{du} = e^u$ **M1**

Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = e^u \times \cos x = e^u \cos x$

$= e^{\sin x} \cos x$ **M1**

At $x = \pi$, $e^{\sin x} \cos x = e^{\sin \pi} \cos \pi$ **M1**

$= e^0 \times (-1) = -1$ so the gradient there is -1 **A1** **[5 Marks]**

6. $y = \ln(8 - 2x)$; Let $u = 8 - 2x$ so $y = \ln u$

$\therefore \dfrac{du}{dx} = -2$ **A1**

$\therefore \dfrac{dy}{du}$ [illegible] **1**

Using the chain rule, $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} = \dfrac{1}{u} \times (-2) = \dfrac{-2}{u}$

$= \dfrac{-2}{8 - 2x}$ **A1** ← Tip: In $\dfrac{dy}{dx}$ =

At $x = 3$, $\dfrac{-2}{8 - 2x} = \dfrac{-2}{8 - 2(3)}$ **M1**

$= \dfrac{-2}{2} = -1$ so the gradient there is -1 **A1** **[5 Marks]**

7. **Show that $f(x) = x^3 - x^2 + 2x - 6$ is concave on the interval [–3, 0] and conve**

$f(x) = x^3 - x^2 + 2x - 6$

$\therefore f'(x) = 3x^2 - 2x + 2$ **M1**

$\therefore f''(x) = 6x - 2$ **M1**

$6x - 2 = -20$ when $x = -3$, and -2 when $x = 0$ ← Techn concav if f''(x and co for all

$\therefore 6x - 2 \le 0$ for all $-3 \le x \le 0$ $\therefore f(x)$ is concave on this interval **A1**

$6x - 2 = 4$ when $x = 1$, and 22 when $x = 4$

$\therefore 6x - 2 \ge 0$ for all $1 \le x \le 4$ $\therefore f(x)$ is convex on this interval **A1** **[4 Marks]**

8. $y = x^3 - 9x^2 + 3x - 5$

$\therefore \dfrac{dy}{dx} = 3x^2 - 18x + 3$ **M1**

$\therefore \dfrac{d^2y}{dx^2} = 6x - 18$ **M1**

If $6x - 18 = 0$, $x = \dfrac{18}{6}$ [illegible]

$\dfrac{d^2y}{dx^2}$ [illegible] 3 and $\dfrac{d^2y}{dx^2} > 0$ if $x > 3$ so second derivative changes sign at $x = 3$ **A** ← Techn point $\dfrac{d^2y}{dx^2}$

At $x = 3, y = 3^3 - 9 \times 3^2 + 3 \times 3 - 5$ **M1** ← Techn x-coo y to fi point

$= 27 - 81 + 9 - 5 = -50$

So the coordinates of the point of inflection are $(3, \ -50)$ **A1** **[5 Marks]**

9. **Prove from first principles that the derivative of sin *x* is cos *x*.**

Let $f(x) = \sin x$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} \quad \textbf{M1}$$

$$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x\sin h - \sin x}{h} \quad \textbf{M1}$$

Techni… sin(A …

$$= \lim_{h\to 0}\left(\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right) \quad \textbf{M1}$$

Techni… $h \to 0$, … from th…

This tends to $\sin x \times 0 + \cos x \times 1$ **M1**

So $\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} \to \cos x$

Hence the derivative of $\sin x$ is $\cos x$ **A1** **[5 Marks]**

10. $x = 3 - \cos 2t, y = 2\sin 2t$

$\therefore \frac{dx}{dt} = 2\sin 2t$ **M1**

$\therefore \frac{dy}{dt} = 2\times 2\cos 2t = 4\cos 2t$ **M1**

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos 2t}{2\sin 2t} = \frac{2\cos 2t}{\sin 2t} \quad \textbf{M1}$$

Techni… using …

So at $t = \frac{\pi}{6}$, gradient of the tangent is $\frac{2\cos \frac{2\pi}{6}}{\sin \frac{2\pi}{6}} = \frac{2\sqrt{3}}{3}$ **M1**

So at $t = \frac{\pi}{6}$, gradient of the normal is $-\frac{1}{\frac{2\sqrt{3}}{3}} = -\frac{\sqrt{3}}{2}$ **A1**

Also at $t = \frac{\pi}{6}$:

$x = 3 - \cos\frac{2\pi}{6} = 3 - \frac{1}{2} = \frac{5}{2}$ **M1**

$y = 2\sin\frac{2\pi}{6} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$ **M1**

Altern… $y - y$… $m = $…

So at $t = \frac{\pi}{6}$, equation of the normal to the curve is $y = mx + c$ with $y = \sqrt{3}$, $m = -$…

$\therefore \sqrt{3} = -\frac{\sqrt{3}}{2}\times\frac{5}{2} + c$

$\therefore c = \sqrt{3} + \frac{5\sqrt{3}}{4} = \frac{9\sqrt{3}}{4}$ **A1**

So $y = -\frac{\sqrt{3}}{2}x + \frac{9\sqrt{3}}{4}$ (or equivalent, e.g. $4y = -2\sqrt{3}x + 9\sqrt{3}$) **A1** **[9 Marks]**

Preview of Answers Ends Here