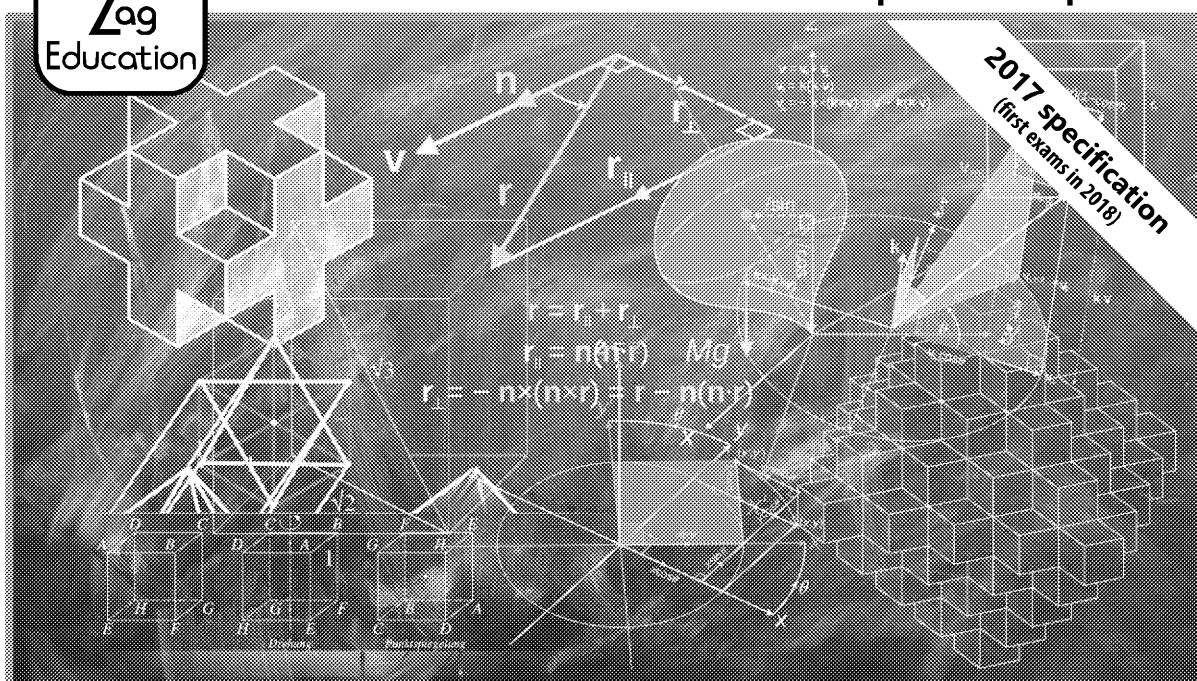




Topic Tests: Challenge Tests – Set A

For A Level Year 2 Edexcel
Pure Mathematics

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Teacher's Introduction

Content

This pack contains 14 challenge level topic tests and solutions for the Edexcel Pure Mathematics Year 2 A Level content.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the challenge tests

These **challenge** tests have been designed to **stretch and challenge** your students. 50% of the marks come from questions similar in style to our fundamentals tests. These questions isolate and test the core skills in each topic. The other 50% of the marks come from questions of increased difficulty that progress and start to combine the concepts in the topic. Due to the increased challenge they pose, we recommend these tests for students who have already mastered the fundamentals by scoring 70% or more on our fundamentals tests.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes to help students who may have made a mistake or who are struggling on a particular question.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework:** Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test:** Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork:** Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test:** Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Free Updates!

Register your email address to receive any future free updates* made to this resource or other Maths resources your school has purchased, and details of any promotions for your subject.

* resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers

Go to zzed.uk/freeupdates

Cross-referencing Grid

Topic	Edexcel spec. points	Subtopics
Algebraic Methods	1.1, 2.6, 2.10	Proof by contradiction, algebraic fractions, partial fractions, connected factors, algebraic division
Functions and Graphs	2.1–2.9	Graphs of the modulus function, functions and mappings, composite functions, inverse functions, $y = f(x) $ and $y = f(x)$, combining transformations, solving modulus problems
Sequences and Series	4.2–4.6	Arithmetic sequences, arithmetic series, geometric sequences, geometric series, sum to infinity, sigma notation, recurrence relations, modelling with series
Binomial Expansion	4.1	Expanding $(1+x)^n$, expanding $(a+bx)^n$, using partial fractions
Radians	5.1–5.3	Radian measure, arc length, areas of sectors and segments, solving trigonometric equations, small angle approximations
Trigonometry Part I	5.4–5.5	Secant, cosecant and cotangent, graphs of $\sec x$, $\csc x$ and $\cot x$, using $\sec x$, $\csc x$ and $\cot x$, trigonometric identities, inverse trigonometric functions
Trigonometry Part II	2.11, 5.6–5.9	Addition formulae, using the angle addition formulae, double-angle formulae, solving trigonometric equations, simplifying $a\cos x \pm b\sin x$, proving trigonometric identities, modelling with trigonometric functions
Parametric Equations	3.3–3.4	Parametric equations, using trigonometric identities, curve sketching, points of intersection, modelling with parametric equations
Differentiation Part I	7.1–7.5	Differentiating $\sin x$ and $\cos x$, differentiating exponentials and logarithms, the chain rule, parametric differentiation, using second derivatives
Differentiation Part II	7.2, 7.4–7.6	The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation, rates of change
Numerical Methods	9.1–9.3, 9.5	Locating roots, iteration, the Newton-Raphson method, applications to modelling
Integration Part I	6.2, 6.5–6.9	Integrating standard functions, integrating $f(ax+b)$, using trigonometric identities, reverse chain rule, integration by substitution, integration by parts, partial fractions
Integration Part II	8.3–8.4, 8.7–8.8, 9.4	Finding areas, the trapezium rule, solving differential equations, modelling with differential equations
Vectors	10.1–10.5	3D coordinates, vectors in 3D, solving geometric problems, applications to mechanics

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Differentiation Part II – Test A (40 mins)

Subtopics: The product rule, the quotient rule, differentiating trigonometric functions, implicit differentiation

1. Use the **product rule** to differentiate each of the following with respect to x .

- a) $x^2 \cos x$ b) $\cos 3x \ln x$ c) $e^{4x} \sin 2x$

2. Find $\frac{dy}{dx}$ given that:

- a) $y = \frac{x}{3x+1}$ b) $y = \frac{2x^2}{3x+1}$ c) $y = \frac{e^{3x}}{\sin x}$

3. a) If $y = 4 \sec x$, use the **product rule** to show that $\frac{dy}{dx} = 4 \sec x \tan x$.

b) If $y = \tan 3x$, use the **quotient rule** to show that $\frac{dy}{dx} = 3 \sec^2 3x$.

c) If $y = \cot 2x$, use the **quotient rule** to show that $\frac{dy}{dx} = -2 \operatorname{cosec}^2 2x$.

(For this question you may use the standard results for the derivatives of $\sec x$ and $\tan x$.)

4. If $x = \sin 2y$, find an expression for $\frac{dy}{dx}$ in terms of x .

5. Given that $x = y^2 + 4y$ and that $\frac{dy}{dt} = 2$, find $\frac{dx}{dt}$ when $y = 3$.

6. Find the **exact** gradient of the curve with equation $2^x = x^3 y - 4y$ at the point $(2, 1)$.

7. A stalagmite forms on a cave floor, growing in height with **constant** rate h mm per day.

On day t the stalagmite is also eroded from its tip at a rate of $\frac{1}{5}h$ mm per day.

Find the rate of change of the height of the stalagmite at the start of that day. Show that $-5 \frac{dh}{dt} = h - 15$.

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Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Solutions to Differentiation Part I – Test A

1. a) $y = 3 \sin x \therefore \frac{dy}{dx} = 3 \cos x$ A1 Technique: $y = \sin$
- b) $y = \cos 4x \therefore \frac{dy}{dx} = -4 \sin 4x$ A1 Technique: $y = \cos$
- c) $y = 2 \sin 3x \therefore \frac{dy}{dx} = 2 \times 3 \cos 3x = 6 \cos 3x$ A1
- d) $y = \cos \frac{1}{3}x - 5 \sin 6x \therefore \frac{dy}{dx} = -\frac{1}{3} \sin \frac{1}{3}x - 5 \times 6 \cos 6x$
 $= -\frac{1}{3} \sin \frac{1}{3}x - 30 \cos 6x$ A1 [5 Marks]

2. a) $f(x) = 4^x$ ($a = 4, k = 1$) Technique: $f(x) = a^x$
 $\therefore f'(x) = 4^x \ln 4$ A1
- b) $f(x) = \frac{1}{x}$ A1 Technique: law of log differentiation
- c) $f(x) = \ln \frac{1}{7}x = \ln \frac{1}{7} + \ln x$ M1 Tip: $\ln a^b = b \ln a$
 $\therefore f'(x) = 0 + \frac{1}{x} = \frac{1}{x}$ A1 Technique: $f(x) = \ln$
- d) $f(x) = e^{3x}$
 $\therefore f'(x) = 3e^{3x}$ A1 Technique: $f(x) = e^x$
- e) $f(x) = e^{3x} - 2^{5x}$ ($a = 2, k = 5$) M1
 $\therefore f'(x) = 3e^{3x} - 2^{5x} 5 \ln 2$ A1
- f) $f(x) = 6e^{-x} + \ln \frac{x}{4} = -6e^{-x} + \ln \frac{1}{4} + \ln x$ M1
 $\therefore f'(x) = -6e^{-x} + \frac{1}{x}$ A1A1 [10 Marks]

3. a) $y = \sin(x-1)$; let $u = x-1$ so $y = \sin u$
 $\therefore \frac{du}{dx} = 1$
 $\therefore \frac{dy}{du} = \cos u$ A1
 Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times 1 = \cos(x-1)$ A1 Technique: $u = x$
- b) $y = \sin(x^2)$; let $u = x^2$ so $y = \sin u$
 $\frac{du}{dx} = 2x$
 $\therefore \frac{dy}{du} = \cos u$ A1 Technique: $u = x^2$
 Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times 2x = 2x \cos u$
 $= 2x \cos(x^2)$ A1

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c) $y = \sin(x^2 - 4x + 1)$; let $u = x^2 - 4x + 1$ so $y = \sin u$

$$\therefore \frac{du}{dx} = 2x - 4 \quad \text{A1}$$

$$\therefore \frac{dy}{du} = \cos u \quad \text{A1}$$

Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos u \times (2x - 4) = (2x - 4) \cos u$ Technique
 $u = x^2 - 4x + 1$

$$= (2x - 4) \cos(x^2 - 4x + 1)$$

(also accept expanded form $2x \cos(x^2 - 4x + 1) - 4 \cos(x^2 - 4x + 1)$) **A1**

d) $y = (-x^2 + 2x - 4)^3$; let $u = -x^2 + 2x - 4$ so $y = u^3$

$$\therefore \frac{du}{dx} = -2x + 2 \quad \text{A1}$$

$$\therefore \frac{dy}{du} = 3u^2 \quad \text{A1}$$

 the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 3u^2 \times (-2x + 2)$ Technique
 $u = -x^2 + 2x - 4$

$$= 3(-2x + 2)(-x^2 + 2x - 4)^2$$

(also accept equivalent expressions, e.g. $6(-x^2 + 2x - 4)(-x + 1)$) **A1** **[11 M]**

4. a) $x = t^2, y = 3t$

$$\therefore \frac{dx}{dt} = 2t \quad \text{M1}$$

$$\therefore \frac{dy}{dt} = 3 \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3}{2t} \quad (\text{also accept } 1.5t^{-1}) \quad \text{A1}$$

b) $x = e^{2t}, y = 4t^2$

$$\therefore \frac{dx}{dt} = 2e^{2t} \quad \text{M1}$$

$$\therefore \frac{dy}{dt} = 8t \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{8t}{2e^{2t}} = \frac{4t}{e^{2t}} \quad (\text{also accept } 4te^{-2t}) \quad \text{A1}$$

c) $x = \frac{3}{t+5} = 3(t+5)^{-1}, y = 6t - 4$

$$\therefore \frac{dx}{dt} = -3(t+5)^{-2} = \frac{-3}{(t+5)^2} \quad \text{M1}$$

$$\therefore \frac{dy}{dt} = 6 \quad \text{M1}$$

 $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6}{-3/(t+5)^2} = -2(t+5)^2 \quad \text{A1}$

[9 M]

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5. $y = e^{\cos x}$; let $u = \cos x$ so $y = e^u$

$$\therefore \frac{du}{dx} = -\sin x \quad \text{M1}$$

$$\therefore \frac{dy}{du} = e^u \quad \text{M1}$$

Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u \times (-\sin x) = -e^u \sin x$
 $= -e^{\cos x} \sin x \quad \text{M1}$

At $x = \frac{\pi}{2}$, $-e^{\cos x} \sin x = -e^{\cos \pi/2} \sin \frac{\pi}{2} \quad \text{M1}$
 $= -e^0 \times 1 = -1$ so the gradient there is $-1 \quad \text{A1 [5 Marks]}$

6. $y = \ln(5-3x)$; let $u = 5-3x$ so $y = \ln u$

$$\therefore \frac{du}{dx} = -3 \quad \text{A1}$$

$$\therefore \frac{dy}{du} = \frac{1}{u} \quad \text{M1}$$

Using the chain rule, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times -3 = \frac{-3}{u}$
 $= \frac{-3}{5-3x} \quad \text{A1}$

At $x = 2$, $\frac{-3}{5-3x} = \frac{-3}{5-3(2)} \quad \text{M1}$
 $= \frac{-3}{-1} = 3$ so the gradient there is $3 \quad \text{A1 [5 Marks]}$

7. Show that $f(x) = x^3 + 4x^2 - x + 2$ is concave on the interval $[-4, -2]$ and convex on the interval $[-1, 1]$

$$f(x) = x^3 + 4x^2 - x + 2$$

$$\therefore f'(x) = 3x^2 + 8x - 1 \quad \text{M1}$$

$$\therefore f''(x) = 6x + 8 \quad \text{M1}$$

$$6x + 8 = -16 \text{ when } x = -4, \text{ and } -4 \text{ when } x = -2$$

$$\therefore 6x + 8 \leq 0 \text{ for all } -4 \leq x \leq -2 \therefore f(x) \text{ is concave on this interval} \quad \text{A1}$$

$$6x + 8 = 2 \text{ when } x = -1, \text{ and } 14 \text{ when } x = 1$$

$$\therefore 6x + 8 \geq 0 \text{ for all } -1 \leq x \leq 1 \therefore f(x) \text{ is convex on this interval} \quad \text{A1 [4 Marks]}$$

8. $y = 2x^3 - 6x^2 + 6x + 4$

$$\therefore \frac{dy}{dx} = 6x^2 - 12x + 6 \quad \text{M1}$$

$$\therefore \frac{d^2y}{dx^2} = 12x - 12 \quad \text{M1}$$

If $12x - 12 = 0$ then $x = 1$

$$\frac{d^2y}{dx^2} < 0 \text{ if } x < 1 \text{ and } \frac{d^2y}{dx^2} > 0 \text{ if } x > 1 \text{ so second derivative changes sign at } x = 1 \quad \text{A1}$$

At $x = 1$, $y = 2 \times 1^3 - 6 \times 1^2 + 6 \times 1 + 4 \quad \text{M1}$
 $= 2 - 6 + 6 + 4 = 6$

So the coordinates of the point of inflection are $(1, 6) \quad \text{A1 [5 Marks]}$

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9. Prove from first principles that the derivative of $\cos x$ is $-\sin x$.

Let $f(x) = \cos x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \quad \text{M1}$$

$$= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \quad \text{M1}$$

$$= \lim_{h \rightarrow 0} \left(\cos x \left(\frac{\cos h - 1}{h} \right) - \sin x \left(\frac{\sin h}{h} \right) \right) \quad \text{M1}$$

This tends to $\cos x \times 0 - \sin x \times 1$ M1

$$\text{So } \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \rightarrow -\sin x$$

Hence the derivative of $\cos x$ is $-\sin x$ A1

[5 Marks]

10. $x = 4 + \cos 2t$, $y = \sin 2t$

$$\therefore \frac{dx}{dt} = -2 \sin 2t \quad \text{M1}$$

$$\therefore \frac{dy}{dt} = 2 \cos 2t \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 \cos 2t}{-2 \sin 2t} = -\frac{\cos 2t}{\sin 2t} \quad \text{M1}$$

$$\text{So at } t = \frac{\pi}{12}, \text{ gradient of the tangent is } \frac{-\cos 2\pi/12}{\sin 2\pi/12} = -\sqrt{3} \quad \text{M1}$$

$$\text{So at } t = \frac{\pi}{12}, \text{ gradient of the normal is } \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \text{A1}$$

Also at $t = \frac{\pi}{12}$:

$$x = 4 + \cos \frac{2\pi}{12} = 4 + \frac{\sqrt{3}}{2} = \frac{8 + \sqrt{3}}{2} \quad \text{M1}$$

$$y = \sin \frac{2\pi}{12} = \frac{1}{2} \quad \text{M1}$$

So at $t = \frac{\pi}{12}$, equation of the normal to the curve is $y = mx + c$ with $y = \frac{1}{2}$, $m = \frac{\sqrt{3}}{3}$

$$\therefore \frac{1}{2} = \frac{\sqrt{3}}{3} \times \frac{8 + \sqrt{3}}{2} + c$$

$$\therefore c = \frac{1}{2} - \frac{8\sqrt{3} + 3}{6} = -\frac{4\sqrt{3}}{3} \quad \text{A1}$$

So $y = \frac{\sqrt{3}}{3}x - \frac{4\sqrt{3}}{3}$ (or equivalent, e.g. $3y = \sqrt{3}x - 4\sqrt{3}$) A1 [9 Marks]

Technique
cos(A ± B)

Technique
 $\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$
from first principles

Technique
using chain rule

Alternative method
 $y - y_1 = m(x - x_1)$
 $m = \frac{\sqrt{3}}{3}$

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