

Topic Tests:

Fundamentals Tests – Set A

For AS / A Level Year 1 OCR A
Pure Mathematics

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Solutions

Teacher's Introduction

Content

This pack contains 14 fundamentals level topic tests for the OCR A Pure Mathematics AS / Year 1 A Level content.

The tests come with fully worked solutions, containing helpful tips, hints and technique boxes for students struggling on a particular question. Answers should be given to three significant figures unless specified in the question.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the fundamentals tests

These **fundamentals** tests focus on isolating and testing the core skills of each topic. The questions are designed to use simple numbers and contexts **so that students can show what they can do**, and to allow you to easily identify any weaknesses.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

Although students are allowed to use a calculator in their examinations, the first topic (Algebraic Expressions) should be done without a calculator, as indicated by the non-calculator symbol () at the top of the test. This encourages students to develop their non-calculator skills, saving time in their examinations on basic algebra and arithmetic.

Also available from ZigZag Education

For students who are ready to go beyond the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and progresses the concepts covered.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework:** Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test:** Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork:** Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test:** Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Free Updates!

Register your email address to receive any future free updates* made to this resource or other Maths resources your school has purchased, and details of any promotions for your subject.

* resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers

Go to zzed.uk/freeupdates

Cross-referencing Grid

Topic	OCR A spec. points	Subtopics	Chapter Reference				
			Edexcel Pearson textbook [ISBN: 9781292183398]	Edexcel Hodder textbook [ISBN: 9781471853043]	Edexcel Collins textbook [ISBN: 9780008204952]	AQA Hodder textbook [ISBN: 9781471852862]	OCR A Hodder textbook [ISBN: 9781471853067]
Algebraic Expressions	1.02a–b, 1.02j	Index laws, expanding brackets, factorising, negative and fractional indices, surds, rationalising denominators	1	2, 3.1, 7.1	1.1, 1.4, 1.6–1.8	2, 3.1, 7.1	2, 3.1, 7.1
Quadratics	1.02f	Solving quadratic equations, completing the square, functions, quadratic graphs, discriminants, modelling	2	3, 8.1	2.1–2.4, 3.1, 3.7–3.8, 8.2	3, 8.1	3, 8.1
Simultaneous Equations and Inequalities	1.02c, 1.02g	Linear simultaneous equations, quadratic simultaneous equations, simultaneous equations on graphs, linear inequalities, quadratic inequalities, inequalities on graphs, regions	3	4	2.5–2.8	4	4
Graphs and Transformations	1.02m–r, 1.02w	Cubic graphs, quartic graphs, reciprocal graphs, points of intersection, translations, stretching, transformations	4	8	3	8	8
Straight Line Graphs	1.03a–c	Equations of straight lines, parallel and perpendicular lines, length and area, modelling	5	5.1–5.3	3.6, 4	5.1–5.3	5.1–5.3
Circles	1.03d–f	Midpoints and perpendicular bisectors, equation of a circle, intersections of straight lines and circles, use tangent and chord properties, circles and triangles	6	5.4–5.5	5	5.4–5.5	5.4–5.5
Algebraic Methods	1.01a–c, 1.02j	Algebraic fractions, dividing polynomials, the factor theorem, mathematical proof, methods of proof	7	1, 7	1.5, 11	1, 7	1, 7
Binomial Expansion	1.04a–b	Pascal's triangle, factorial notation, binomial expansion, binomial problems, binomial estimation	8	9	1.2–1.3	9	9
Trigonometric Ratios	1.05a–c	The cosine rule, the sine rule, areas of triangles, solving triangle problems, graphs of sine, cosine and tangent, transforming trigonometric graphs	9	6.2–6.5, 8.4	6.1–6.5, 3.7–3.8	6.2–6.5, 8.4	6.2–6.5, 8.4
Trigonometric Identities and Equations	1.05f	Angles in all four quadrants, exact values of trigonometric ratios, trigonometric identities, simple trigonometric equations, harder trigonometric equations, equations and identities	10	6.1–6.2	6.1, 6.4–6.6	6.1–6.2	6.1–6.2
Vectors	1.10a, 1.10c–g	Vectors, representing vectors, magnitude and direction, position vectors, solving geometric problems, modelling	11	12	10	12	12
Differentiation	1.07a–d, 1.07g, 1.07i, 1.07m–n, 1.07o	Gradients of curves, finding derivatives, differentiating x^n , differentiating quadratics, gradients, tangents and normals, increasing and decreasing functions, second order derivatives, stationary points, sketching, modelling	12	10	8	10	10
Integration	1.08a–b, 1.08d–e	Integrating x^n , indefinite integrals, finding functions, definite integrals, areas under curves, areas under the x-axis, areas between curves and lines	13	11	9	11	11
Exponentials and Logarithms	1.06a–i	Exponential functions, $y = e^x$, exponential modelling, logarithms, laws of logarithms, solving equations using logarithms, working with natural logarithms, logarithms and non-linear data	14	13	7	13	13

Graphs and Transformations – Test A (17 marks)

Subtopics: Cubic graphs, quartic graphs, reciprocal graphs, points of intersection, transformations

1. **Sketch** the following cubics, indicating **all** points where the curves cross the axes.

a) $y = x^3$ b) $y = -x^3$ c) $y = (x-1)^3$
 d) $y = (x-1)^2(x+3)$ e) $y = (x+4)^3$ f) $y = (x-2)^3$

2. **Sketch** the following quartics, indicating **all** points where the curves cross the axes.

a) $y = x^4$ b) $y = (x-1)(x-2)(x+2)(x+3)$
 c) $y = -x^4$ d) $y = (x-1)^2(x+3)(x+5)$
 e) $y = (x-2)^2(x+2)^2$ f) $y = -(x-3)(x+1)(x+2)^2$

3. **Sketch** each of the following pairs of curves on the same diagram:

a) $y = \frac{4}{x}$ and $y = \frac{4}{x}$ b) $y = -\frac{1}{x}$ and $y = -\frac{3}{x}$
 c) $y = \frac{2}{x^2}$ and $y = \frac{5}{x^2}$ d) $y = -\frac{8}{x^2}$ and $y = -\frac{1}{x^2}$

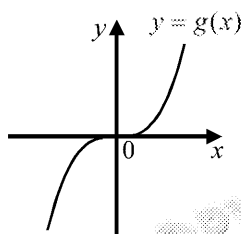
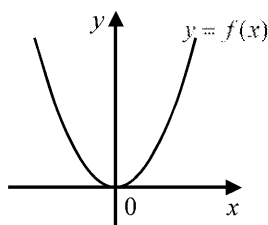
4. a) On the same diagram, **sketch** the curves $y = \frac{2}{x^2}$ and $y = x^2(x-5)$. Indicate **all** points where the curves cross the axes.

- b) Using your sketch, state the number of **real** solutions to the equation $x^2(x-5) = \frac{2}{x^2}$. Give a reason for your answer.

5. a) Give the **vector** that corresponds to the **translation** that takes $y = x^2$ to $y = (x-2)^2$.

- b) Give the **vector** that corresponds to the **translation** that takes $y = x^2$ to $y = (x+3)^2$.

6. $f(x) = x^2$ $g(x) = x^3$ $h(x) = x^4$



- Sketch** the following graphs, indicating **all** points where the curves cross the axes.

a) $y = f(x+2)$ b) $y = g(x)-2$ c) $y = h(x-3)$
 d) $y = f(2x)$ e) $y = -h(x)$ f) $y = g(x+1)$

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Solutions to Differentiation – Test A

1. a) The gradient can be calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ ← **Technique**
Line

i) $\frac{36 - 25}{6 - 5} = \frac{11}{1} = 11$ **A1**

ii) $\frac{30.25 - 25}{5.5 - 5} = \frac{5.25}{0.5} = 10.5$ **A1**

iii) $\frac{26.01 - 25}{5.1 - 5} = \frac{1.01}{0.1} = 10.1$ **A1**

iv) $\frac{25.1001 - 25}{5.01 - 5} = \frac{0.1001}{0.01} = 10.01$ **A1**

v) $\frac{(5+h)^2 - 25}{5+h-5} = \frac{10h+h^2}{h} = 10+h$ **A1**

- b) As the point (x, y) approaches $(5, 25)$ the gradient tends to 10 **A1 [6 Marks]**

2. a) $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ ← **Technique**
definition of derivative at point $f'(x)$
$$= \lim_{h \rightarrow 0} \frac{(3+h)^2 - 3^2}{h} = \lim_{h \rightarrow 0} \frac{9+6h+h^2-9}{h}$$
 M1
$$= \lim_{h \rightarrow 0} (6+h) = 6$$
 A1

b) $f'(-2) = \lim_{h \rightarrow 0} \frac{f(-2+h) - f(-2)}{h}$
$$= \lim_{h \rightarrow 0} \frac{(-2+h)^2 - (-2)^2}{h} = \lim_{h \rightarrow 0} \frac{4-4h+h^2-4}{h}$$
 M1
$$= \lim_{h \rightarrow 0} (-4+h) = -4$$
 A1

c) $f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$
$$= \lim_{h \rightarrow 0} \frac{(0+h)^2 - 0^2}{h} = \lim_{h \rightarrow 0} \frac{h^2}{h}$$
 M1
$$= \lim_{h \rightarrow 0} h = 0$$
 A1

[6 Marks]

3. a) $y = 2x^3$
 $\frac{dy}{dx} = 6x^2$ **A1**

b) $y = \frac{1}{3}x^9$
 $\frac{dy}{dx} = 3x^8$ **A1**

c) $y = 4x^{-1}$
 $\frac{dy}{dx} = -4x^{-2}$ or $-\frac{4}{x^2}$ **A1**

[3 Marks]

4. a) $y = 4x^2 \therefore \frac{dy}{dx} = 8x$ **A1**

When $x = 1$, the gradient is $8(1) = 8$ **A1**

b) $y = 2x^2 - 3x + 1 \therefore \frac{dy}{dx} = 4x - 3$ **A1**

When $x = 2$, the gradient is $4(2) - 3 = 5$ **A1 [4 Marks]**

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5. $y = x^2 - 5x + 3 \therefore \frac{dy}{dx} = 2x - 5$ M1

When $x = 3$, the gradient is $2(3) - 5 = 1$ A1

So the equation of the tangent at $(3, -3)$ is $y - (-3) = 1(x - 3)$ M1

$y + 3 = x - 3 \therefore y = x - 6$ A1

When $x = 2$, the gradient is $2(2) - 5 = -1$ A1

So the gradient of the normal at $x = 2$ is $\frac{-1}{-1} = 1$ M1

So the equation of the normal at $(2, -3)$ is $y - (-3) = 1(x - 2)$ M1

$y + 3 = x - 2 \therefore y = x - 5$ A1

[8 Marks]

Technical
normal
tangent

6. Increasing A1

[1 Mark]

7. $f(x) = 2x^3 + 4x + 1 \therefore f'(x) = 6x^2 + 4$ A1

$x^2 \geq 0$ for all real values of x M1

So $6x^2 + 4 \geq 0$ for all real values of x A1

So $f(x)$ is increasing for all real values of x

[3 Marks]

8. a) $y = 4x^2 + 7x + 3$

$\frac{dy}{dx} = 8x + 7$ A1

$\frac{d^2y}{dx^2} = 8$ A1

Hint: $\frac{d^2y}{dx^2}$
y and is equal to
respect to x

b) $y = 12x + 4 + \frac{1}{x} = 12x + 4 + x^{-1}$

$\frac{dy}{dx} = 12 - x^{-2} = 12 - \frac{1}{x^2}$ M1A1

$\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$ A1

c) $y = (2x + 1)(x + 4) = 2x^2 + 8x + x + 4 = 2x^2 + 9x + 4$ M1

$\frac{dy}{dx} = 4x + 9$ A1

$\frac{d^2y}{dx^2} = 4$ A1

[8 Marks]

Alternative
quadratic
is zero
complete
coordinate

9. To find the gradient function we differentiate $y = x^2 - 12x + 1$

$\frac{dy}{dx} = 2x - 12$ A1

So the gradient is zero when $2x - 12 = 0$, hence $x = 6$ A1

At $x = 6$, $y = 6^2 - 12(6) + 1 = -35$ A1

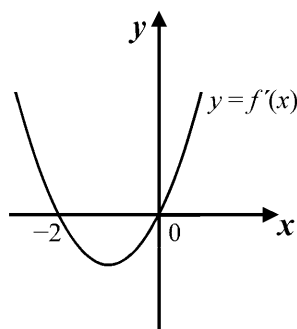
So the gradient is zero at point $(6, -35)$ A1

[4 Marks]

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10. $f(x) = x^2(x+3) = x^3 + 3x^2$ A1
 $f'(x) = 3x^2 + 6x = 3x(x+2)$ A1



A2 [3 Marks]

Technique touches does not repeat at x

Technique function x = -2

11. Rate of change of displacement $\frac{dr}{dt}$ A1

The rate of change of displacement can be found by substituting $t = 3$ into the equation displacement

$2(3) + \dots$ (metres per second) M1A1 [3 Marks]

Technique various m s⁻¹

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