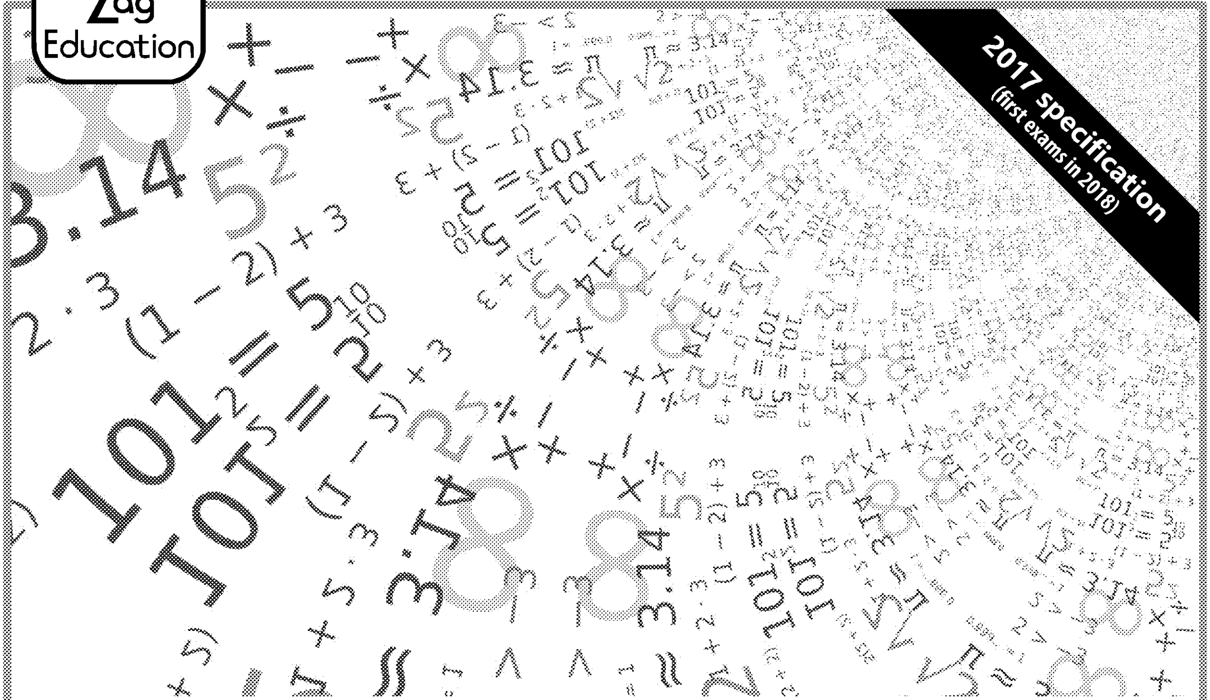
Maths

AS / A Level | AQA | 7356



Topic Tests: Fundamentals Tests – Set A

For AS / A Level Year 1 AQA Pure Mathematics

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POD 8665

Contents

Teacher's Introduction

Content

This pack contains 14 fundamentals level topic tests for the AQA Pure Mathematics AS / Year 1 A Level content.

The tests come with fully worked solutions, containing helpful tips, hints and technique boxes for students struggling on a particular question. Answers should be given to three significant figures unless specified in the question.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the fundamentals tests

These **fundamentals** tests focus on isolating and testing the core skills of each topic. The questions are designed to use simple numbers and contexts **so that students can show what they can do,** and to allow you to easily identify any weaknesses.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

Although students are allowed to use a calculator in their examinations, the first topic (Algebraic Expressions) should be done without a calculator, as indicated by the non-calculator symbol (🚫) at the top of the test. This encourages students to develop their non-calculator skills, saving time in their examinations on basic algebra and arithmetic.

Also available from ZigZag Education

For students who are ready to go beyond the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and progresses the concepts covered.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Cross-referencing Grid

Topic	AQA spec. points	Subtopics	Chapter Reference				
			Edexcel Pearson textbook [ISBN: 9781292183398]	**Edexcel Hodder textbook** [ISBN: 9781471853043]	**Edexcel Collins textbook** [ISBN: 9780008204952]	**AQA Hodder textbook** [ISBN: 9781471852862]	**OCR A Hodder textbook** [ISBN: 9781471853067]
Algebraic Expressions	B1, B2, B6	Index laws, expanding brackets, factorising, negative and fractional indices, surds, rationalising denominators	1	2, 3.1, 7.1	1.1, 1.4, 1.6–1.8	2, 3.1, 7.1	2, 3.1, 7.1
Quadratics	B3	Solving quadratic equations, completing the square, functions, quadratic graphs, discriminants, modelling	2	3, 8.1	2.1–2.4, 3.1, 3.7–3.8, 8.2	3, 8.1	3, 8.1
Simultaneous Equations and Inequalities	B3, B5	Linear simultaneous equations, quadratic simultaneous equations, simultaneous equations on graphs, linear inequalities, quadratic inequalities, inequalities on graphs, regions	3	4	2.5–2.8	4	4
Graphs and Transformations	B7, B9	Cubic graphs, quartic graphs, reciprocal graphs, points of intersection, translations, stretching, transformations	4	8	3	8	8
Straight Line Graphs	C1	Equations of straight lines, parallel and perpendicular lines, length and area, modelling	5	5.1–5.3	3.6, 4	5.1–5.3	5.1–5.3
Circles	C2	Midpoints and perpendicular bisectors, equation of a circle, intersections of straight lines and circles, use tangent and chord properties, circles and triangles	6	5.4–5.5	5	5.4–5.5	5.4–5.5
Algebraic Methods	A1, B6	Algebraic fractions, dividing polynomials, the factor theorem, mathematical proof, methods of proof	7	1, 7	1.5, 11	1, 7	1, 7
Binomial Expansion	D1	Pascal's triangle, factorial notation, binomial expansion, binomial problems, binomial estimation	8	9	1.2–1.3	9	9
Trigonometric Ratios	E1	The cosine rule, the sine rule, areas of triangles, solving triangle problems, graphs of sine, cosine and tangent, transforming trigonometric graphs	9	6.2–6.5, 8.4	6.1–6.5, 3.7–3.8	6.2–6.5, 8.4	6.2–6.5, 8.4
Trigonometric Identities and Equations	E3	Angles in all four quadrants, exact values of trigonometric ratios, trigonometric identities, simple trigonometric equations, harder trigonometric equations, equations and identities	10	6.1–6.2	6.1, 6.4–6.6	6.1–6.2	6.1–6.2
Vectors	J1, J2, J3, J4, J5	Vectors, representing vectors, magnitude and direction, position vectors, solving geometric problems, modelling	11	12	10	12	12
Differentiation	G1, G2, G3	Gradients of curves, finding derivatives, differentiating x^n, differentiating quadratics, gradients, tangents and normals, increasing and decreasing functions, second order derivatives, stationary points, sketching, modelling	12	10	8	10	10
Integration	H1, H2, H3	Integrating x^n, indefinite integrals, finding functions, definite integrals, areas under curves, areas under the x-axis, areas between curves and lines	13	11	9	11	11
Exponentials and Logarithms	F1, F2, F3, F4, F5, F6, F7	Exponential functions, $y = e^x$, exponential modelling, logarithms, laws of logarithms, solving equations using logarithms, working with natural logarithms, logarithms and non-linear data	14	13	7	13	13

Graphs and Transformations – Test A (17 m

Subtopics: Cubic graphs, quartic graphs, reciprocal graphs, points of intersection, trans

1. **Sketch** the following cubics, indicating **all** points where the curves cro
   a) $y = x^3$ b) $y = -x^3$ c)
   d) $y = (x-1)^2(x+3)$ e) $y = (x+4)^3$ f)

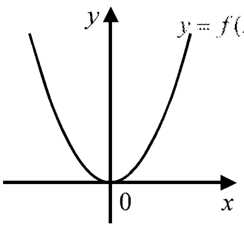
2. **Sketch** the following quartics, indicating **all** points where the curves c
   a) $y = x^4$ b) $y = (x-1)(x-2)(x+2)(x+3)$
   c) $y = -x^4$ d) $y = (x-1)^2(x+3)(x+5)$
   e) $y = (x-2)^2(x+2)^2$ f) $y = -(x-3)(x+1)(x+2)^2$

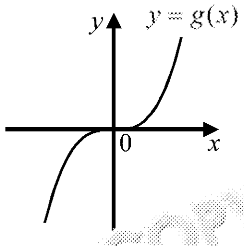
3. **Sketch** each of the following pairs of curves on the same diagram:
   a) $y = \frac{1}{x}$ and $y = \frac{4}{x}$ b) $y = -\frac{1}{x}$ and $y = -\frac{3}{x}$
   c) $y = \frac{2}{x^2}$ and $y = \frac{5}{x^2}$ d) $y = -\frac{8}{x^2}$ and $y = -\frac{1}{x^2}$

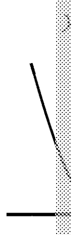
4. a) On the same diagram, **sketch** the curves $y = \frac{2}{x^2}$ and $y = x^2(x-5)$ the curves cross the **axes**.
   b) Using your sketch, state the number of **real** solutions to the equati reason for your answer.

5. a) Give the **vector** that corresponds to the **translation** that takes $y =$
   b) Give the **vector** that corresponds to the **translation** that takes $y =$

6. $f(x) = x^2$ $g(x) = x^3$ $h(x$







Sketch the following graphs, indicating any points where the curves cr
   a) $y = f(x+2)$ b) $y = g(x) - 2$ c)
   d) $y = f(2x)$ e) $y = -h(x)$ f)

Solutions to Differentiation – Test A

1. a) The gradient can be calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

i) $\dfrac{36-25}{6-5} = \dfrac{11}{1} = 11$ **A1**

ii) $\dfrac{30.25-25}{5.5-5} = \dfrac{5.25}{0.5} = 10.5$ **A1**

iii) $\dfrac{26.01-25}{5.1-5} = \dfrac{1.01}{0.1} = 10.1$ **A1**

iv) $\dfrac{25.1001-25}{5.01-5} = \dfrac{0.1001}{0.01} = 10.01$ **A1**

v) $\dfrac{(5+h)^2-25}{5+h-5} = \dfrac{10h+h^2}{h} = 10+h$ **A1**

b) As the points get closer to (5, 25) the gradient tends to 10 **A1** **[6 Marks]**

2. a) $f'(3) = \lim\limits_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$

$= \lim\limits_{h\to 0} \dfrac{(3+h)^2-3^2}{h} = \lim\limits_{h\to 0} \dfrac{9+6h+h^2-9}{h}$ **M1**

$= \lim\limits_{h\to 0}(6+h) = 6$ **A1**

b) $f'(-2) = \lim\limits_{h\to 0} \dfrac{f(-2+h)-f(-2)}{h}$

$= \lim\limits_{h\to 0} \dfrac{(-2+h)^2-(-2)^2}{h} = \lim\limits_{h\to 0} \dfrac{4-4h+h^2-4}{h}$ **M1**

$= \lim\limits_{h\to 0}(-4+h) = -4$ **A1**

c) $f'(0) = \lim\limits_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{(0+h)^2-0^2}{h} = \lim\limits_{h\to 0} \dfrac{h^2}{h}$ **M1**

$= \lim\limits_{h\to 0} h = 0$ **A1** **[6 Marks]**

3. a) $y = 2x^3$

$\dfrac{dy}{dx} = 6x^2$ **A1**

b) $y = \dfrac{1}{3}x^9$

$\dfrac{dy}{dx} = 3x^8$ **A1**

c) $y = 4x^{-1}$

$\dfrac{dy}{dx} = -4x^{-2}$ or $-\dfrac{4}{x^2}$ **A1** **[3 Marks]**

4. a) $y = 4x^2 \therefore \dfrac{dy}{dx} = 8x$ **A1**

When $x = 1$, the gradient is 8(1) = 8 **A1**

b) $y = 2x^2 - 3x + 1 \therefore \dfrac{dy}{dx} = 4x - 3$ **A1**

When $x = 2$, the gradient is 4(2) – 3 = 5 **A1** **[4 Marks]**

5. $y = x^2 - 5x + 3 \;\therefore\; \dfrac{dy}{dx} = 2x - 5$ **M1**

When $x = 3$, the gradient is 2(3) – 5 = 1 **A1**

So the equation of the tangent at (3, –3) is $y - (-3) = 1(x - 3)$ **M1**

$y + 3 = x - 3 \;\therefore\; y = x - 6$ **A1**

When $x = 2$, the gradient is 2(2) – 5 = –1 **A1**

So the gradient of the normal at $x = 2$ is $\dfrac{-1}{-1} = 1$ **M1**

So the equation of the normal at (2, –3) is $y - (-3) = 1(x - 2)$ **M1**

$y + 3 = x - 2 \;\therefore\; y = x - 5$ **A1** **[8 Marks]**

Tech… norm… tang…

6. Increasing **A1** **[1 Mark]**

7. $f(x) = 2x^3 + 4x + 1 \;\therefore\; f'(x)$ [illegible] **A1**

$x^2 \geq 0$ for all real [illegible] **M1**

So 6[illegible] 0 [illegible] real values of x **A1**

So f [illegible] increasing for all real values of x **[3 Marks]**

8. a) $y = 4x^2 + 7x + 3$

$\dfrac{dy}{dx} = 8x + 7$ **A1**

$\dfrac{d^2y}{dx^2} = 8$ **A1**

Hint: $\dfrac{d^2y}{dx^2}$ … y and is eq… respect to …

b) $y = 12x + 4 + \dfrac{1}{x} = 12x + 4 + x^{-1}$

$\dfrac{dy}{dx} = 12 - x^{-2} = 12 - \dfrac{1}{x^2}$ **M1A1**

$\dfrac{d^2y}{dx^2} = 2x^{-3} = \dfrac{2}{x^3}$ **A1**

c) $y = (2x + 1)(x + 4) = 2x^2 + 8x + x + 4 = 2x^2 + 9x + 4$ **M1**

$\dfrac{dy}{dx} = 4x + 9$ **A1**

$\dfrac{d^2y}{dx^2} = 4$ **A1** **[8 Marks]**

Alter… quad… is zer… comp… coord…

9. To find the gradient function we differentiate $y = x^2 - 12x + 1$

$\dfrac{dy}{dx} = 2x - 12$ **A1**

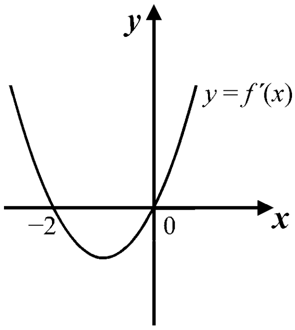
So the gradient is zero when $2x - 12 = 0$ [illegible] he [illegible] $= 6$ **A1**

At $x = 6$, $y = 6^2 - 12(6) + 1 =$ [illegible]

So the gradient is [illegible] point (6, –35) **A1** **[4 Marks]**

10. $f(x) = x^2(x+3) = x^3 + 3x^2$ **A1**

$f'(x) = 3x^2 + 6x = 3x(x+2)$ **A1**



A2 **[… arks]**

11. Rate of change of displacement: $\frac{dr}{d…}$ … $2t$ … **A1**

The rate of change … 3 can be found by substituting $t = 3$ into the equati…
displ…
2(3) + … (metres per second) **M1A1** **[3 Marks]**

Techn… touche… does … repe… at x …

Techn… functio… x = −…

Techn… vario… m s⁻¹ …

Preview of Answers Ends Here