Maths **AS / A Level | Edexcel | 8MA0/9MA0**

2017 specification
(first exams in 2018)

Topic Tests: Expert Tests – Set A

For AS / A Level Year 1 Edexcel Pure Mathematics

zigzageducation.co.uk

POD 8657

Publish your own work... Write to a brief...
Register at **publishmenow.co.uk**

Photocopiable/digital resources may only be copied by the purchasing institution on a single site and for their own use

Contents

Thank You for Choosing ZigZag Education ii

Teacher Feedback Opportunity iii

Terms and Conditions of Use iv

Teacher's Introduction 1

Cross-referencing Grid 2

Tests

- Test 1.3a – Algebraic Expressions
- Test 2.3a – Quadratics
- Test 3.3a – Simultaneous Equations and Inequalities
- Test 4.3a – Graphs and Transformations
- Test 5.3a – Straight Line Graphs
- Test 6.3a – Circles
- Test 7.3a – Algebraic Methods
- Test 8.3a – Binomial Expansion
- Test 9.3a – Trigonometric Ratios
- Test 10.3a – Trigonometric Identities and Equations
- Test 11.3a – Vectors
- Test 12.3a – Differentiation
- Test 13.3a – Integration
- Test 14.3a – Exponentials and Logarithms

Solutions

Teacher's Introduction

Content

This pack contains 14 expert level topic tests for the Edexcel Pure Mathematics AS / Year 1 A Level content.

The tests come with fully worked solutions, containing helpful tips, hints and technique boxes for students struggling on a particular question. Answers should be given to three significant figures unless specified in the question.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Timings

The recommended times for students to complete each test are given at the top of individual tests.

Calculator use

Although students are allowed to use a calculator in their examinations, the first topic (Algebraic Expressions) should be done without a calculator, as indicated by the non-calculator symbol (🚫) at the top of the test. This encourages students to develop their non-calculator skills, saving time in their examinations on basic algebra and arithmetic.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who are ready to go beyond the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and progresses the concepts covered.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Free Updates!

Register your email address to receive any future free updates* made to this resource or other Maths resources your school has purchased, and details of any promotions for your subject.

** resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers*

Go to **zzed.uk/freeupdates**

Cross-referencing Grid

Topic	Edexcel spec. points	Subtopics	Chapter Reference				
			Edexcel Pearson textbook [ISBN: 9781292183398]	**Edexcel Hodder textbook** [ISBN: 9781471853043]	**Edexcel Collins textbook** [ISBN: 9780008204952]	**AQA Hodder textbook** [ISBN: 9781471852862]	**OCR A Hodder textbook** [ISBN: 9781471853067]
Algebraic Expressions	2.1–2.2, 2.6	Index laws, expanding brackets, factorising, negative and fractional indices, surds, rationalising denominators	1	2, 3.1, 7.1	1.1, 1.4, 1.6–1.8	2, 3.1, 7.1	2, 3.1, 7.1
Quadratics	2.3	Solving quadratic equations, completing the square, functions, quadratic graphs, discriminants, modelling	2	3, 8.1	2.1–2.4, 3.1, 3.7–3.8, 8.2	3, 8.1	3, 8.1
Simultaneous Equations and Inequalities	2.4–2.5	Linear simultaneous equations, quadratic simultaneous equations, simultaneous equations on graphs, linear inequalities, quadratic inequalities, inequalities on graphs, regions	3	4	2.5–2.8	4	4
Graphs and Transformations	2.7–2.8	Cubic graphs, quartic graphs, reciprocal graphs, points of intersection, translations, stretching, transformations	4	8	3	8	8
Straight Line Graphs	3.1	Equations of straight lines, parallel and perpendicular lines, length and area, modelling	5	5.1–5.3	3.6, 4	5.1–5.3	5.1–5.3
Circles	3.2	Midpoints and perpendicular bisectors, equation of a circle, intersections of straight lines and circles, use tangent and chord properties, circles and triangles	6	5.4–5.5	5	5.4–5.5	5.4–5.5
Algebraic Methods	1.1, 2.6	Algebraic fractions, dividing polynomials, the factor theorem, mathematical proof, methods of proof	7	1, 7	1.5, 11	1, 7	1, 7
Binomial Expansion	4.1	Pascal's triangle, factorial notation, binomial expansion, binomial problems, binomial estimation	8	9	1.2–1.3	9	9
Trigonometric Ratios	5.1–5.2	The cosine rule, the sine rule, areas of triangles, solving triangle problems, graphs of sine, cosine and tangent, transforming trigonometric graphs	9	6.2–6.5, 8.4	6.1–6.5, 3.7–3.8	6.2–6.5, 8.4	6.2–6.5, 8.4
Trigonometric Identities and Equations	5.3–5.4	Angles in all four quadrants, exact values of trigonometric ratios, trigonometric identities, simple trigonometric equations, harder trigonometric equations, equations and identities	10	6.1–6.2	6.1, 6.4–6.6	6.1–6.2	6.1–6.2
Vectors	9.1–9.5	Vectors, representing vectors, magnitude and direction, position vectors, solving geometric problems, modelling	11	12	10	12	12
Differentiation	7.1–7.3	Gradients of curves, finding derivatives, differentiating x^n, differentiating quadratics, gradients, tangents and normals, increasing and decreasing functions, second order derivatives, stationary points, sketching, modelling	12	10	8	10	10
Integration	8.1–8.3	Integrating x^n, indefinite integrals, finding functions, definite integrals, areas under curves, areas under the x-axis, areas between curves and lines	13	11	9	11	11
Exponentials and Logarithms	6.1– 6.7	Exponential functions, $y = e^x$, exponential modelling, logarithms, laws of logarithms, solving equations using logarithms, working with natural logarithms, logarithms and non-linear data	14	13	7	13	13

 © ZigZag Education, 2018

Circles – Test A (11 mins)

Subtopics: Midpoints and perpendicular bisectors, equation of a circle, intersections of s
and chord properties, circles and triangles
For this test you should leave your answers in surd form where a

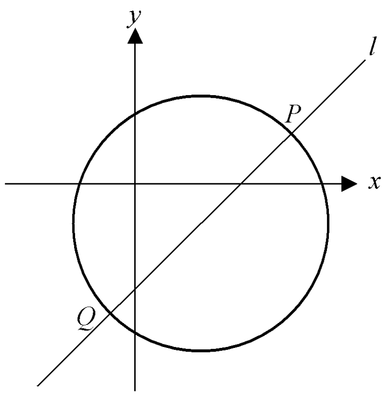
1. Find the **centre** and **radius** of each of the following circles:

   a) $x^2 - 8x + y^2 + 4y = 0$ b) $x^2 + y^2 + 7x - 3y + \frac{5}{2} = 0$

2. The points $P = (3, 8)$ and $Q = (-5, 11)$ are two points on the circumfere **diameter** of C.
   a) Find the **centre** and **radius** of C.
   b) Hence write down an equation that describes the circle C.

3. The circle C has equation $x^2 + y^2 - 5x + 2y = k$. Find the **range** of pos

4. The line with equation $y = x - 4$ intersects the circle C with equation points P and Q as shown in the diagram below. The line PQ is a **diam**



   a) Find the **coordinates** of P and Q.
   b) Find the length of PQ.
   c) Find the equation of the line that is a **perpendicular bisector** of P

5. The circle C has equation $x^2 + y^2 = 4$ and the point $P = \left(-\sqrt{3}, 1\right)$ lies o to C at P, and meets the x-axis at the point A and the y-axis at the poin

INSPECTION COPY
INSPECTION COPY
INSPECTION COPY
COPYRIGHT PROTECTED
Zig Zag Education

Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Preview of Answers Ends Here

This is a limited inspection copy. Sample of answers ends here to stop students looking up answers to their assessments. See contents page for details of the rest of the resource.

Solutions to Trigonometric Identities and Equations – Test A

1. $\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta\tan\theta} = \dfrac{\sqrt{\sin^2\theta}}{\cos\theta\tan\theta}$ **M1**

$= \dfrac{\sin\theta}{\cos\theta\tan\theta} = \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\tan\theta}$ **M1**

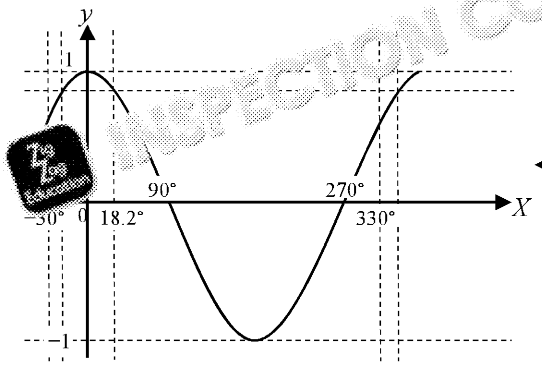
$= \tan\theta \times \dfrac{1}{\tan\theta} = 1$ **A1** **[3 Marks]**

Techni
$\sin^2\theta +$
$\tan\theta =$

2. a) $\cos(\theta - 30°) = 0.95$

$\therefore \theta - 30° = \arccos(0.95) = 18.1948... = 18.2°$ (1 d.p.) **M1**

Let $X = \theta - 30°$, then $-30° \le X \le 330°$



Techni
$y = $ co
find th
given.
used fo
questio
questio

So $X = 18.2°$ is a solution in the interval $-30° \le X \le 330°$ **A1**

There is a second solution x in the interval $-30° \le X \le 330°$

From the graph can see that $x = 0 - 18.1948... = -18.1948...° = -18.2°$ (1 d.p

So $X = 18.2°$, or $X = -18.2°$, therefore $\theta = 48.2°$, or $\theta = 11.8°$ **A1**

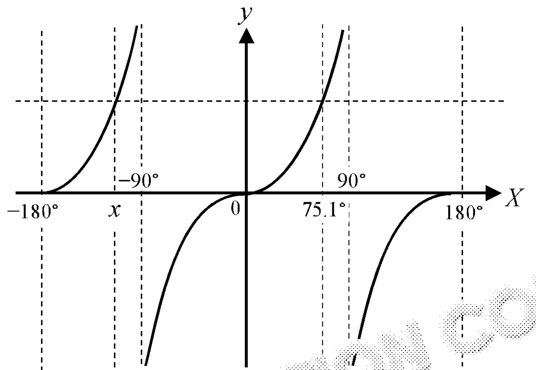
b) $4\sin 3\theta - 15\cos 3\theta = 0 \therefore 4\sin 3\theta = 15\cos 3\theta$

$\therefore \dfrac{4\sin 3\theta}{\cos 3\theta} = 15 \therefore 4\tan 3\theta = 15 \therefore \tan 3\theta = \dfrac{15}{4}$ **M1**

Techni
$\tan 3\theta$

$\therefore 3\theta = \arctan\left(\dfrac{15}{4}\right) = 75.0685... = 75.1°$ (1 d.p.) **M1**

Let $X = 3\theta$, then $-180° \le X \le 180°$



So $X = 75.1°$ is a solution in the interval $-180° \le X \le 180°$

There is a second solution x in the interval $-180° \le X \le 180°$

As the graph has a period of 180° we can see that $x = 75.0685... - 180 = -1$

So $X = 75.1°$, or $X = -104.9°$; therefore, $\theta = 25.0°$, or $\theta = -35.0°$ **A1**

INSPECTION COPY
COPYRIGHT PROTECTED
Zig Zag Education

3. $7\sin^2\theta + 3\cos\theta - 5 = 0 \therefore 7\left(1-\cos^2\theta\right) + 3\cos\theta - 5 = 0$ **M1**

$7 - 7\cos^2\theta + 3\cos\theta - 5 = 0 \therefore 7\cos^2\theta - 3\cos\theta - 2 = 0$ **A1** ←

Techni... $\sin^2\theta$... the qu... quadr...

Using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

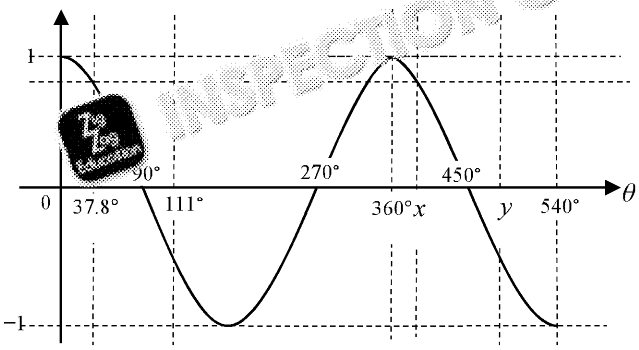
with $a = 7$, $b = -3$, $c = -2$, $x = \cos\theta$

$\cos\theta = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4\times 7\times(-2)}}{2\times 7} = \dfrac{3\pm\sqrt{65}}{14}$ **M1**

$\therefore \theta = \arccos\left(\dfrac{3+\sqrt{65}}{14}\right) = 37.7994... = 37.8°$ (3 s.f.) **A1**

or $\theta = \arccos\left(\dfrac{3-\sqrt{65}}{14}\right) = 111.197... = 111°$ (3 s.f.) **A1**

Neither of these values are in the interval given.



There are two solutions x and y in the interval $0° \le \theta \le 360°$

From the symmetry of the graph, you can see that $x = 360 + 37.7994... = 397.799...$

and $y = 360 + 111.197... = 471.197...° = 471°$ (3 s.f.) **A1** **[7 Mark**

4. a) $\dfrac{1}{2}\sin^2\theta + 3 - \dfrac{7}{2}\cos^2\theta = \dfrac{1}{2}\sin^2\theta + 3 - \dfrac{7}{2}\left(1-\sin^2\theta\right)$ **M1** ←

Techni... $\sin^2\theta +$...

$= \dfrac{1}{2}\sin^2\theta + 3 - \dfrac{7}{2} + \dfrac{7}{2}\sin^2\theta$ **M1**

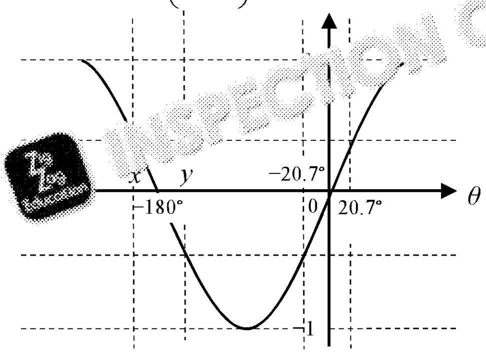
$= 4\sin^2\theta - \dfrac{1}{2}$ **A1**

b) $\dfrac{1}{2}\sin^2\theta + 3 - \dfrac{7}{2}\cos^2\theta = 0 \therefore 4\sin^2\theta - \dfrac{1}{2} = 0$ by part a) **M1**

$\therefore \sin^2\theta = \dfrac{1}{8} \therefore \sin\theta = \pm\sqrt{\dfrac{1}{8}} = \pm\dfrac{\sqrt{2}}{4}$ **M1**

$\therefore \theta = \arcsin\left(\dfrac{\sqrt{2}}{4}\right) = 20.7048... = 20.7°$ (3 s.f.) **A1**

or $\theta = \arcsin\left(-\dfrac{\sqrt{2}}{4}\right) = -20.7048... = -20.7°$ (3 s.f.) **A1**



So $\theta = 20.7°$ and $\theta = -20.7°$ are two solutions in the interval $-270° \le \theta \le 90°$

There are two additional solutions x and y in the interval $-270° \le \theta \le 90°$

From the symmetry of the graph can see that $x = -180 - 20.7048... = -200.7$

and $y = -180 + 20.7048... = -159.295... = -159°$ (3 s.f.)

So solutions are $\theta = 20.7°$, $\theta = -20.7°$, $\theta = -201°$, $\theta = -159°$ **A1** **[9 Mark**

INSPECTION COPY

INSPECTION COPY

INSPECTION COPY

COPYRIGHT PROTECTED

5. a) Cosine rule for angles: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ ← Techni…

$a = 9, b = 4, c = 6, A = Q$

$\therefore \cos Q = \dfrac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6}$ **M1**

$= \dfrac{16 + 36 - 81}{48} = -\dfrac{29}{48}$ **A1**

b) Using the identity $\sin^2\theta + \cos^2\theta = 1$

$\sin^2 Q + \cos^2 Q = 1 \therefore \sin^2 Q + \left(-\dfrac{29}{48}\right)^2 = 1$ **M1**

$\therefore \sin^2 Q = 1 - \dfrac{841}{2304} = \dfrac{1463}{2304}$ **M1**

$\therefore \sin Q = \sqrt{\dfrac{1463}{2304}} = \dfrac{\sqrt{1463}}{48}$ **A1** **[5 Marks]**

Techni… positiv… an ang… be bet… Theref…

6. a) The graph of $y = \sin(x - 30°)$ has been drawn incorrectly. **M1**
[illegible] should have been translated 30° to the right, not 30° to the left. **A1**
The second solution is, therefore, wrong.

b) The correct proof is shown below:

$3\sin(x - 30°) = 1$

$\sin(x - 30°) = \dfrac{1}{3}$

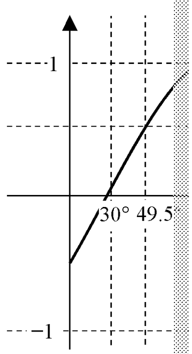
$x - 30° = \arcsin\left(\dfrac{1}{3}\right) = 19.4712...°$

$\therefore x = 19.4712... + 30 = 49.4712...° = 49.5°$ (3 s.f.)

There is another solution to the equation at y.

From the graph $y = 210 - (49.4712... - 30)$ **M1**

$= 190.528...° = 191°$ (3 s.f.) **A1**



[5 Marks]

7. $\sin^3\theta - 2\sin\theta + 1 = \cos^2\theta \therefore \sin^3\theta - 2\sin\theta + 1 = 1 - \sin^2\theta$ **M1**

$\therefore \sin^3\theta + \sin^2\theta - 2\sin\theta = 0$ **M1**

$\therefore \sin\theta(\sin^2\theta + \sin\theta - 2) = 0$

$\therefore \sin\theta(\sin\theta + 2)(\sin\theta - 1) = 0$ **M1**

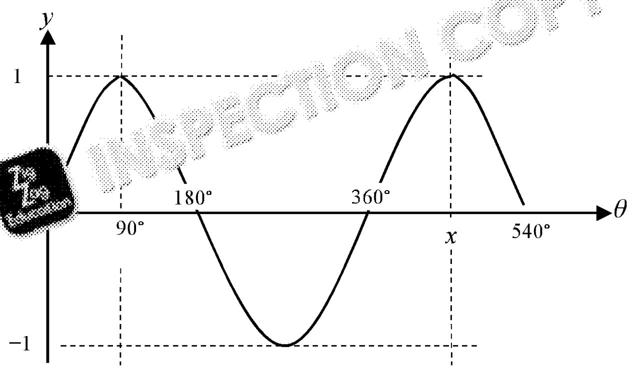
$\therefore \sin\theta = 0$ or $\sin\theta = -2$ or $\sin\theta = 1$

$\therefore \theta = \arcsin(0) = 0°$ **A1**

or $\theta = \arcsin(1) = 90°$ **A1**

$\sin\theta = -2$ has no solutions as $-1 \le \sin\theta \le 1$ for all values of θ **A1**

Techni… $\sin^2\theta +$…



From the graph can see that other solutions to $\sin\theta = 0$ are $\theta = 180°$ and θ …
(not 540° as strict inequality)
There is a second solution to $\sin\theta = 1$ at x, where $x = 360 + 90 = 450°$
So the solutions are $\theta = 0, 180°, 360°, 90°, 450°$ **A1** **[8 Marks]**

INSPECTION COPY
COPYRIGHT PROTECTED
Zig Zag Education