2017 specification
(first exams in 2018)

Topic Tests:
Expert Tests – Set B

For AS / A Level Year 1 OCR A Pure Mathematics

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Contents

Teacher's Introduction

Content

This pack contains 14 expert level topic tests, which together form 'Set B' in a paired range of tests for the OCR A Pure Mathematics AS / Year 1 A Level content.

Each test comes with fully worked solutions, containing helpful tips, hints and technique boxes for students struggling on a particular question.

> These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the expert tests

These **expert** tests have been designed to **prepare your students** for success in their exam. 25% of the marks come from questions similar in style to our fundamentals and challenge tests, giving all of your students a chance to show what they can do. The other 75% of the marks come from examination-style material, including compound and multistep questions that bring all parts of the topic together.

Suggested use of the A and B tests

Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these expert tests, the relevant times are the fifth and sixth times listed under each topic.

Calculator use

Although students are allowed to use a calculator in their examinations, the first topic (Algebraic Expressions) should be done without a calculator, as indicated by the non-calculator symbol (🚫) at the top of the tests. This encourages students to develop their non-calculator skills, saving time in their examinations on basic algebra and arithmetic.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

For students who are ready to go beyond the fundamentals, a complete set of **challenge** tests are available. 50% of the marks in these tests come from concepts covered in the fundamentals tests in order to reinforce learning and boost students' confidence, while the other 50% increases in difficulty and progresses the concepts covered.

Free Updates!

Register your email address to receive any future free updates* made to this resource or other Maths resources your school has purchased, and details of any promotions for your subject.

resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers

Go to **zzed.uk/freeupdates**

Cross-referencing Grid

Topic	OCR A spec. points	Subtopics	Edexcel Pearson textbook [ISBN: 9781292183398]	Edexcel Hodder textbook [ISBN: 9781471853043]	Edexcel Collins textbook [ISBN: 9780008204952]	AQA Hodder textbook [ISBN: 9781471852862]	OCR A Hodder textbook [ISBN: 9781471853067]
					Chapter Reference		
Algebraic Expressions	1.02a–b, 1.02j	Index laws, expanding brackets, factorising, negative and fractional indices, surds, rationalising denominators	1	2, 3.1, 7.1	1.1, 1.4, 1.6–1.8	2, 3.1, 7.1	2, 3.1, 7.1
Quadratics	1.02f	Solving quadratic equations, completing the square, functions, quadratic graphs, discriminants, modelling	2	3, 8.1	2.1–2.4, 3.1, 3.7–3.8, 8.2	3, 8.1	3, 8.1
Simultaneous Equations and Inequalities	1.02c, 1.02g	Linear simultaneous equations, quadratic simultaneous equations, simultaneous equations on graphs, linear inequalities, quadratic inequalities, inequalities on graphs, regions	3	4	2.5–2.8	4	4
Graphs and Transformations	1.02m–r, 1.02w	Cubic graphs, quartic graphs, reciprocal graphs, points of intersection, translations, stretching, transformations	4	8	3	8	8
Straight Line Graphs	1.03a–c	Equations of straight lines, parallel and perpendicular lines, length and area, modelling	5	5.1–5.3	3.6, 4	5.1–5.3	5.1–5.3
Circles	1.03d–f	Midpoints and perpendicular bisectors, equation of a circle, intersections of straight lines and circles, use tangent and chord properties, circles and triangles	6	5.4–5.5	5	5.4–5.5	5.4–5.5
Algebraic Methods	1.01a–c, 1.02j	Algebraic fractions, dividing polynomials, the factor theorem, mathematical proof, methods of proof	7	1, 7	1.5, 11	1, 7	1, 7
Binomial Expansion	1.04a–b	Pascal's triangle, factorial notation, binomial expansion, binomial problems, binomial estimation	8	9	1.2–1.3	9	9
Trigonometric Ratios	1.05a–c	The cosine rule, the sine rule, areas of triangles, solving triangle problems, graphs of sine, cosine and tangent, transforming trigonometric graphs	9	6.2–6.5, 8.4	6.1–6.5, 3.7–3.8	6.2–6.5, 8.4	6.2–6.5, 8.4
Trigonometric Identities and Equations	1.05f	Angles in all four quadrants, exact values of trigonometric ratios, trigonometric identities, simple trigonometric equations, harder trigonometric equations, equations and identities	10	6.1–6.2	6.1, 6.4–6.6	6.1–6.2	6.1–6.2
Vectors	1.10a, 1.10c–g	Vectors, representing vectors, magnitude and direction, position vectors, solving geometric problems, modelling	11	12	10	12	12
Differentiation	1.07a–d, 1.07g, 1.07i, 1.07m–n, 1.07o	Gradients of curves, finding derivatives, differentiating x^n, differentiating quadratics, gradients, tangents and normals, increasing and decreasing functions, second order derivatives, stationary points, sketching, modelling	12	10	8	10	10
Integration	1.08a–b, 1.08d–e	Integrating x^n, indefinite integrals, finding functions, definite integrals, areas under curves, areas under the x-axis, areas between curves and lines	13	11	9	11	11
Exponentials and Logarithms	1.06a–i	Exponential functions, $y = e^x$, exponential modelling, logarithms, laws of logarithms, solving equations using logarithms, working with natural logarithms, logarithms and non-linear data	14	13	7	13	13

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.
For the **challenge** tests, refer to the tests marked X.2a and X.2b.
For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Algebraic Expressions		**Circles**		**Vectors**	
1.1.a	14	6.1a	15	11.1a	9
1.1b	15	6.1b	11	11.1b	9
1.2a	21	6.2a	11	11.2a	17
1.2b	17	6.2b	11	11.2b	15
1.3a	14	6.3a	11	11.3a	11
1.3b	12	6.3b	13	11.3b	11
Quadratics		**Algebraic Methods**		**Differentiation**	
2.1a	19	7.1a	5	12.1a	13
2.1b	16	7.1b	5	12.1b	9
2.2a	23	7.2a	7	12.2a	17
2.2b	19	7.2b	5	12.2b	13
2.3a	26	7.3a	7	12.3a	17
2.3b	25	7.3b	7	12.3b	17
Simultaneous Equations and Inequalities		**Binomial Expansion**		**Integration**	
3.1a	10	8.1a	11	13.1a	15
3.1b	8	8.1b	9	13.1b	13
3.2a	13	8.2a	21	13.2a	15
3.2b	11	8.2b	17	13.2b	13
3.3a	23	8.3a	13	13.3a	13
3.3b	17	8.3b	13	13.3b	11
Graphs and Transformations		**Trigonometric Ratios**		**Exponentials and Logarithms**	
4.1a	17	9.1a	11	14.1a	19
4.1b	9	9.1b	11	14.1b	11
4.2a	17	9.2a	19	14.2a	15
4.2b	15	9.2b	13	14.2b	11
4.3a	19	9.3a	23	14.3a	15
4.3b	13	9.3b	21	14.3b	11
Straight Line Graphs		**Trigonometric Identities and Equations**			
5.1a	9	10.1a	17		
5.1b	7	10.1b	17		
5.2a	7	10.2a	15		
5.2b	7	10.2b	13		
5.3a	13	10.3a	19		
5.3b	11	10.3b	15		

Subtopics: Solving quadratic equations, completing the square, functions, quadratic

1. Solve $2x^2 - 3x - 1 = 0$, leaving your answer in simplified surd form.

2. Given that x is **positive**, solve $\dfrac{1}{x} - \dfrac{2x}{1-x} = 1$

3. This trapezium has an area of 100 m². Show that the **height** of the tra
   $\left(-10 + 10\sqrt{3}\right)$ m. Explain why this is the only solution.
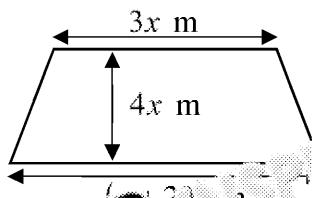
4. By **completing the square**, show that the solutions to the equation x^2
   formula $x = -2b \pm \sqrt{(2b)^2 - c}$

5. The function f is defined as $f(x) = 4x - 2x^2 - 3$
   a) Write $f(x)$ in the form $A + B(x + C)^2$, where A, B and C are real
   b) Explain why $f(x) < 0$ for all values of x and find the maximum v

6. Sketch the graphs of the following equations **on separate diagrams**.
   curve crosses the axes, label the coordinates of the turning point, and
   line of symmetry:
   a) $y = 25 - x^2$ 	b) $y = x^2 + 4x + 4$ 	c)

7. The graph of $y = ax^2 + bx + c$ has a **minimum** at $\left(-\dfrac{7}{4}, -\dfrac{169}{8}\right)$ and pas
   Find the values of a, b and c.

8. Find all roots of the following functions, leaving your answers in surd
   a) $g(x) = 2x^3 + 8x^2 + 6x$ 	b) $j(x) = x - 5\sqrt{x} + 6$ 	c)

9. $f(x) = 3x^2 + (k + 6)x + k$ where k is a real constant
   a) Find the **discriminant** of $f(x)$ in te ms of
   b) Using part a), or otherwi , ro that $f(x)$ has two **distinct** real

10. Jack is at the t ill. At time $t = 0$ he starts to cycle down to the
    the bc h four minutes. He then stops cycling. Jack's height, in m
    hill afte minutes is modelled by the function $g(t) = t^2 - 8t + c$, for so
    (*In this question we are only considering vertical displacement; horizontal d*
    a) Find the value of c.
    b) At what time was Jack 9 m above the bottom of the hill?
    c) Rearrange $g(t)$ into the form $q + (t + p)^2$ and state the values of

Preview of Questions Ends Here

Solutions to Straight Line Graphs – Test B

1. The distance between (x_1, y_1) and (x_2, y_2) can be found using the formula $d = \sqrt{(}$

$$d = \sqrt{(4-(-1))^2 + (7-3)^2} \quad \textbf{M1}$$

$$= \sqrt{5^2 + 4^2} = \sqrt{25+16} = \sqrt{41} \quad \textbf{A1} \qquad \textbf{[2 Marks]}$$

2. The gradient of the line that joins the points $(-2,-1)$ and $(2,7)$ is

$$m = \frac{7-(-1)}{2-(-2)} = \frac{8}{4} = 2 \quad \textbf{M1}$$

The line passing through the point $(-2,-1)$ with gradient 2 has the equation $y-($

$$\therefore 2x - y + 3 = 0 \quad \textbf{A1} \qquad \qquad \text{arks]}$$

3. $12x - 4y - 16 = 0$ can be rearranged t $2x - 16$ and simplified to $y = 3x - 4$
    The gradient is 3 **M1**
    Parallel lines have   a   ents.
    The li   sin   the origin with gradient 3 has the equation $y = 3x$ **A1** [

4. Let A , 3), B be (2, 6) and C be (4, 8).
    Points are collinear if gradients of lines AB and BC are equal.

    The line joining (x_1, y_1) and (x_2, y_2) has gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

    For line AB: $m = \dfrac{6-3}{2-(-1)} = \dfrac{3}{3} = 1$ **M1**

    For line BC: $m = \dfrac{8-6}{4-2} = \dfrac{2}{2} = 1$ **M1**

    Same gradient and common point so the three points lie on the same straight
    line and are, therefore, collinear. **A1**   **[3 Marks]**

5. Find equation of line that passes through point $(0,3)$ and has gradient -3
    Can see $m = -3$, $c = 3$
    Substitute into $y = mx + c$
    $y = -3x + 3$ **M1**
    Substitute $y = -3x + 3$ into $3x - 3y + 6 = 0$
    $3x - 3(-3x+3) + 6 = 3x + 9x - 9 + 6 = 12x - 3 = 0$ **M1**

    $$\therefore x = \frac{3}{12} = \frac{1}{4} \quad \textbf{M1}$$

    Substitute $x = \dfrac{1}{4}$ into $y = -3x + 3$

    $$y = -3\left(\frac{1}{4}\right) + 3 = -\frac{3}{4} + 3 = \frac{9}{4} \quad \textbf{M1}$$

    Intersects at point $\left(\dfrac{1}{4}, \dfrac{9}{4}\right)$ **A1**   **[5 Marks]**

6. Substitute $y = 2x + 4$ i -3 $= 0$
    $3x - 3(2   4)$ $5x - 12 - 9 = -3x - 21 = 0$ **M1**
    $\therefore x =$
    Substit $= -7$ into $y = 2x + 4$
    $y = 2(-7) + 4 = -10$ **M1**
    $\therefore X$ is the point $(-7, -10)$

    Line that passes through X and Y has gradient $m = \dfrac{-2-(-10)}{1-(-7)} = \dfrac{8}{8} = 1$ **M1**

    The equation of the line with gradient 1, passing through point X (and Y) is
    $y - (-10) = 1(x - (-7))$
    $\therefore x - y - 3 = 0$ **A1**   **[5 Marks]**

7. The gradient of the line $y = 2x - 3$ is 2 **M1**

The gradient of the line perpendicular to it is $-\dfrac{1}{2}$ **M1**

The line that passes through the point $(2, -3)$ with gradient $-\dfrac{1}{2}$ can be found usin̶

$y - y_1 = m(x - x_1)$

$y - (-3) = -\dfrac{1}{2}(x - 2)$ **M1**

$\therefore 2y + 6 = -x + 2$

$\therefore x + 2y + 4 = 0$ **A1**　　　　　　　　**[4 Marks]**

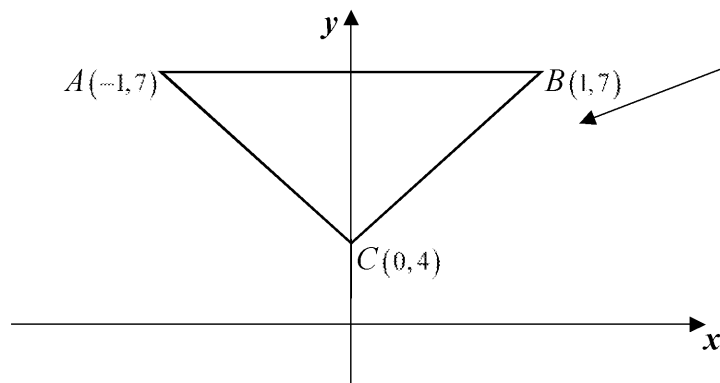8. The distance between (x_1, y_1) and (x_2, y_2) can be f̶ ̶ ̶si̶ ̶ the formula $d = \sqrt{}$

a) The distance between B and C: $\sqrt{(0-1)^2 + (4-7)^2} = \sqrt{(-1)^2 + (-3)^2}$

The distance ̶ ̶ ̶d̶ C: $d_{AC} = \sqrt{(0-(-1))^2 + (4-7)^2} = \sqrt{1^2 + (-3)^2}$

st̶a̶ce between A and B: $d_{AB} = \sqrt{(1-(-1))^2 + (7-7)^2} = \sqrt{2^2 + 0} = 2$

$d_{BC} = d_{AC} \neq d_{AB}$ so triangle is isosceles **A1**

b)

The base of the triangle has length 2
The height of the triangle is 3 **M1**

Area of triangle $= \dfrac{1}{2} \times \text{base} \times \text{height}$

Area of triangle $= \dfrac{1}{2} \times 2 \times 3 = 3 \text{ units}^2$ **A1**　　　　**[6 Marks]**

9. a) $a = \dfrac{2100 - 1200}{14 - 4} = 90$ **M1**

Substitute $(4, 1200)$ into $C = 90D + b$

$1200 = 90(4) + b$

$\therefore b = 840$ **M1**

$C = 90D + 840$ **A1**

b) a is the dail̶y̶ ̶ ̶ ̶ ̶ ̶ by the designer. **A1**
̶ ̶ ̶ ̶he ̶ ̶ ̶ ̶arged by the designer. **A1**

c) $= 90D + 840$

$= 17$ **A1**　　　　　　　　**[6 Marks]**

Preview of Answers Ends Here