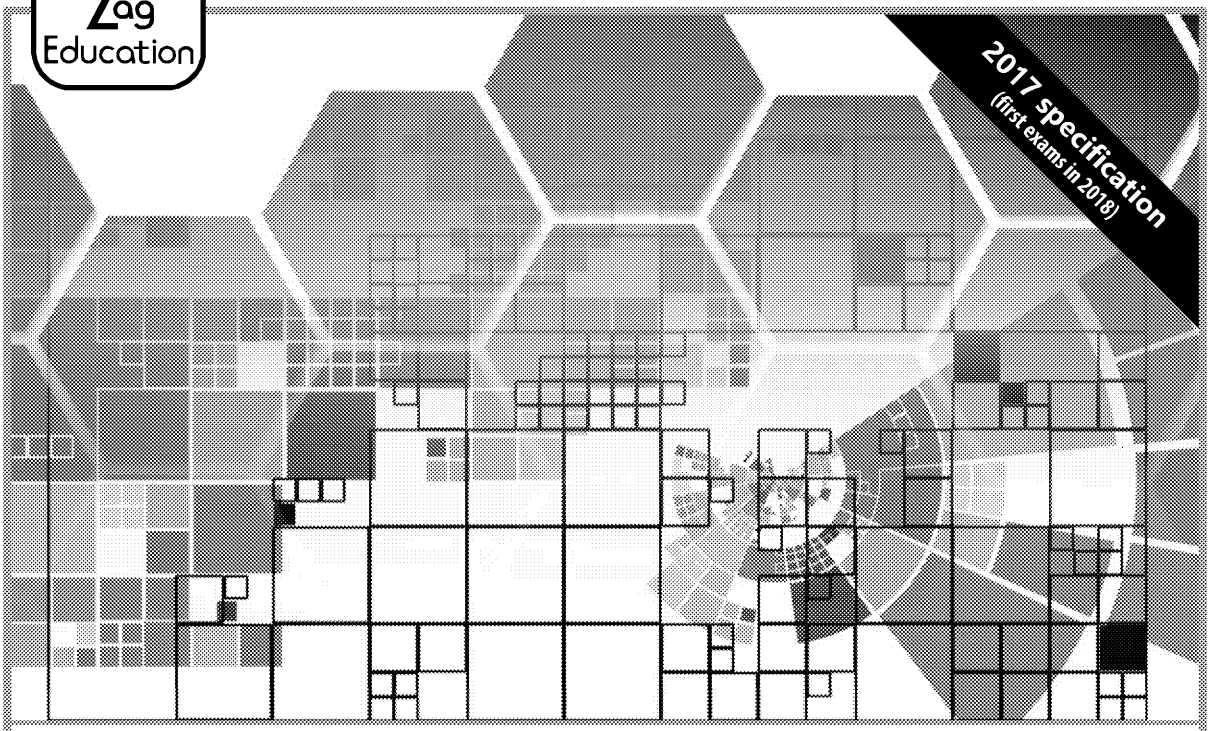
Zig Zag Education

Maths **AS / A Level | AQA | 7356/7357**



Topic Tests: Challenge Tests – Set A

For AS / A Level Year 1 AQA Statistics and Mechanics

Update v1.1 April 2018

zigzageducation.co.uk

POD 8673

Contents

Teacher's Introduction

Content

This pack contains 9 challenge level topic tests and solutions for the AQA Applied Mathematics AS / Year 1 A Level content.

Each test comes with fully worked solutions, containing helpful tips, hints, and technique boxes for students who are struggling on a particular question.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the challenge tests

These **challenge** tests have been designed to **stretch and challenge** your students. 50% of the marks come from questions similar in style to our fundamentals tests. These questions isolate and test the core skills in each topic. The other 50% of the marks come from questions of increased difficulty that progress and start to combine the concepts in the topic.

Timings

The recommended times for students to complete each test are given at the top of individual tests. This pack is an updated version of the Applied AS / Year 1 topic tests – the suggested times have been increased to provide students with a more reasonable amount of time to complete each test.

Calculator use

The effective use of a calculator is one of the objectives of the new specification and is encouraged for all the enclosed tests. In particular, students should be comfortable using the statistical functions on their calculator.

Large data set questions

As part of their assessment, students will be tested on data from a **large data set** provided by AQA. This data set contains data on vehicles registered in various locations in England between 2002 and 2016. These topic tests make use of the original large data set provided by AQA, which contains data on household food and drink purchases; familiarity with these is not assumed and is not needed to take these tests.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

For each collection of Set A tests we also offer a corresponding collection of Set B duplicated tests with the same styles of questions but different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework**: Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test**: Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork**: Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test**: Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

For total flexibility, the Set A and Set B tests of all three levels can be run on a rolling basis, using the fundamentals tests as starters, with a time interval between them, leaving one expert level test to use at the end of the course for topic revision.

Update v1.1, April 2018
Error corrected in Test 7.2a, question 6a. 20 seconds corrected to 2 seconds

Update v1.2, September 2018
Improved and increased suggested times to complete each test. Additionally reduced number of points 3.2a Q3, added 'Graph Paper Needed' flag where needed.

Cross-referencing Grid

Topic	AQA spec. points	Sub-topics	Edexcel Pearson textbook [ISBN: 9781292232539]
Data Collection	K1	Populations, samples, sampling, non-random sampling, types of data, the large data set	1
Measures of Location and Spread	L3	Measures of central tendency, other measures of location, measures of spread, variance and standard deviation, *coding (Edexcel only)*	2
Representations of Data & Correlation	L1 – L2, L4	Outliers, box plots, cumulative frequency, histograms, comparing data, correlation, linear regression	3 - 4
Probability	M1	Calculations, mutually exclusive and independent events, Venn diagrams, tree diagrams	5
Statistical distributions	N1	Probability distributions, binomial distribution, cumulative probabilities	6
Hypothesis testing	O1 – O2	Hypothesis testing, finding critical values, one-tailed tests, two-tailed tests	7
Constant Acceleration	Q1 – Q3, R3	Displacement-time graphs, velocity-time graphs, constant acceleration formulae, vertical motion under gravity	9
Modelling in Mechanics & Forces and Motion	P1, R1 – R4	Force diagrams, forces as vectors, forces and acceleration, motion in 2 dimensions, connected particles, pulleys	8, 10
Variable Acceleration	Q4	Functions of time, using differentiation, maxima and minima problems, using integration, constant acceleration formulae	11

Statistical Distributions – Test A (24 mins)

Subtopics: Probability distributions, binomial distribution, cumulati
Rounded answers should be given to three significant figures unless

1. The discrete random variable X has the probability distribution given i value of y.

x	1	2	3	4	5
$P(X = x)$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	y	y

2. Celeste rolls an eight-sided dice. The result of the dice throw is repres X. Write down one condition that the dice must satisfy if X is to be mod **distribution**.

3. A fair eight-sided dice is thrown. The result of the dice throw is repres X. W w **probability distribution** of X as:
   a) A bility function
   b) A table

4. The random variable X has a **discrete uniform distribution**. X can ta 10. Find:
   a) $P(X \geq 6)$ b) $P(3 \leq X \leq 6)$

5. The random variable $X \sim B(25, 0.17)$. Find:
   a) $P(X = 7)$ b) $P(X \leq 6)$ c)
   d) The largest **integer** value of k such that $P(X \leq k) < 0.3$

6. Rosie has five red sweets and three blue sweets in her pocket. She ran pocket, notes its colour, and replaces it. She does this a total of four ti the number of times that she takes a red sweet from her pocket.
   a) Write down the possible values of R.
   b) By modelling R as a binomial distribution, write down the probab table.

7. The random variable X has a probability function
$$P(X = x) = ax \qquad x = 1, 2, 3, 4, 5$$
where a is a constant.
   a) Find the exact value of a.
   b) Find $P(X > 3)$.

8. Jeffrey s a e factory. One in every 100 bicycles produced in decide t a sample of 50 bicycles.
   a) Suggest a suitable distribution to model the number of faulty bicy reasons for your answer.
   b) Find the probability that at least three bicycles in the sample are fa
   c) Find the probability that the number of faulty bicycles in the samp than 5.

Preview of Questions Ends Here

Solutions to Variable Acceleration – Test A

1. a) When $t = 0$, $v = 0^2 - 6\times 0 + 8 = 8 \text{ m s}^{-1}$ **A1**

   b) Need to solve $t^2 - 6t + 8 = 0$ in the range $0 \le t \le 10$ **M1**

   $t^2 - 6t + 8 = 0$

   $\therefore (t-2)(t-4) = 0$

   So $t = 2$ or $t = 4$ **M1**

   Both of these satisfy $0 \le t \le 10$ so solutions are $t = 2$ seconds and $t = 4$ seconds

   c) Need to solve $t^2 - 6t + 8 = 15$ in the range $0 \le t \le 10$ **M1**

   $t^2 - 6t + 8 = 15$

   $\therefore t^2 - 6t - 7 = 0$

   $\therefore (t-7)(t+1) = 0$

   So $t = 7$ or $t = -1$ **M1**

   Since $0 \le t \le 10$ the solution is $t = 7$ seconds **A1**

2. Total distance travelled is the area under the velocity–time graph **M1**

   The velocity is positive between $t = 0$ and $t = 1$

   So the area under the graph between $t = 0$ and $t = 1$ is $\int_0^1 \left(4t^3 - 24t^2 + 20t\right)\mathrm{d}t$

   $$\int_0^1 \left(4t^3 - 24t^2 + 20t\right)\mathrm{d}t = \left[t^4 - 8t^3 + 10t^2\right]_0^1 \quad \textbf{M1}$$

   $$= (1 - 8 + 10) - (0 - 0 + 0) \quad \textbf{M1}$$

   $$= 3 \text{ m}$$

   So the total distance travelled is 3 m **A1**

3. Need to solve $\dfrac{\mathrm{d}s}{\mathrm{d}t} = 0$ to find stationary points **M1**

   $\dfrac{\mathrm{d}s}{\mathrm{d}t} = 20 - 10t$ **M1**

   This is zero when $20 - 10t = 0$, so $t = 2$ **M1**

   To check this is a maximum, confirm $\dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2} < 0$

   $\dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2} = -10 < 0$ so this is a maximum **M1**

   When $t = 2$, $s = 20\times 2 - 5\times 2^2 = 20$ m **A1**

4. a) $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$ **M1**

   $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 6t - 3$ **A1**

   b) $s = \int v\,\mathrm{d}t = \int (3t^2 - 3t + 36)\,\mathrm{d}t = t^3 - \dfrac{3t^2}{2} + 36t + c$ **M1**

   When $t = 2$, $s = 84$, so $84 = 2^3 - \dfrac{3\times 2^2}{2} + 36\times 2 + c = 74 + c$

   So $c = 84 - 74 = 10$ **M1**

   So $s = t^3 - \dfrac{3t^2}{2} + 36t + 10$ **A1**

5. a) $v = \int a\,dt = \int (2t-7)\,dt = t^2 - 7t + c$ **M1**

When $t = 0$, $v = 10$, so $10 = 0^2 - 7\times 0 + c = c$

So $c = 10$ **M1**

So at time t, $v = t^2 - 7t + 10$ **A1**

b) Need to solve $t^2 - 7t + 10 = 0$ **M1**

$t^2 - 7t + 10 = 0$

$\therefore (t-2)(t-5) = 0$

So $t = 2$ or $t = 5$ **M1**

Both of these satisfy $t \geq 0$, so the solutions are $t = 2$ and $t = 5$ seconds **A1**

Tech
integ
shou
well.

6. $v = \int a\,dt = at + c$ **M1**

When $t = 0$, $v = u$, so $u = a\times 0 + c = c$

So $c = u$ **M1**

So at time t, $v = at + u$ **A1**

Tip:
integ

7. a) $v = \int a\,dt = \int 3.75t^{-1/2}\,dt$

$= \frac{3.75t^{1/2}}{1/2} + c_1 = 7.5t^{1/2} + c_1$ **M1**

When $t = 1$, $v = 7.5$, so $7.5 = 7.5\times 1^{1/2} + c_1 = 7.5 + c_1$

So $c_1 = 0$ **M1**

$v = 7.5t^{1/2}$ **A1**

b) $s = \int v\,dt = \int 7.5t^{1/2}\,dt$

$= \frac{7.5t^{3/2}}{3/2} + c_2 = 5t^{3/2} + c_2$ **M1**

When $t = 1$, $s = 5$, so $s = 5\times 1^{3/2} + c_2 = 5 + c_2$

So $c_2 = 0$ and $s = 5t^{3/2}$ **M1**

Need to solve $s = 4\times 10^{13}$

$5t^{3/2} = 4\times 10^{13}$

$t^{3/2} = 0.8\times 10^{13}$

$t^{3/2} = 8\times 10^{12}$

$t = 8^{2/3}\times\left(10^{12}\right)^{2/3}$

$t = 4\times 10^8$ s **M1**

To convert this to days, divide by $60\times 60\times 24 = 86400$

So probe takes $\frac{4\times 10^8}{86\,400} = 4629.62... = 4630$ days (to nearest day) **A1**

Tec
in c
and
are
seco

8. a) $a = \frac{dv}{dt}$ **M1**

$= 6t - 14$ **A1**

b) $s = \int v\,dt = \int (3t^2 - 14t + 7)\,dt = t^3 - 7t^2 + 7t + c$ **M1**

When $t = 0$, $s = 15$, so $15 = 0^3 - 7\times 0^2 + 7\times 0 + c = c$

So $c = 15$

So $s = t^3 - 7t^2 + 7t + 15$ **M1**

When $t = 3$, $s = 3^3 - 7\times 3^2 + 7\times 3 + 15 = 0$ m

Displacement is measured from hive so bee is at hive when $t = 3$ s **A1**

c) Need to solve $t^3 - 7t^2 + 7t + 15 = 0$

$t^3 - 7t^2 + 7t + 15 = (t-3)(t^2 - 4t - 5)$

$= (t-3)(t-5)(t+1)$ **M1**

So $s = 0$ when $t = -1$, $t = 3$, $t = 5$

Need $0 \leq t \leq 8$ and $t \neq 3$, so solution is $t = 5$ s **A1**

Preview of Answers Ends Here