

Topic Tests: Challenge Tests – Set B

For AS / A Level Year 1 AQA
Pure Mathematics

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Contents

Thank You for Choosing ZigZag Education.....	ii
Teacher Feedback Opportunity	iii
Terms and Conditions of Use	iv
Teacher’s Introduction.....	1
Cross-referencing Grid	2
Timings Sheet.....	3

Tests

- Test 1.2b – Algebraic Expressions
- Test 2.2b – Quadratics
- Test 3.2b – Simultaneous Equations and Inequalities
- Test 4.2b – Graphs and Transformations
- Test 5.2b – Straight Line Graphs
- Test 6.2b – Circles
- Test 7.2b – Algebraic Methods
- Test 8.2b – Binomial Expansion
- Test 9.2b – Trigonometric Ratios
- Test 10.2b – Trigonometric Identities and Equations
- Test 11.2b – Vectors
- Test 12.2b – Differentiation
- Test 13.2b – Integration
- Test 14.2b – Exponentials and Logarithms

Solutions

Teacher's Introduction

Content

This pack contains 14 challenge level topic tests, which together form 'Set B' in a paired range of tests for the AQA Pure Mathematics AS / Year 1 A Level content.

Each test comes with fully worked solutions, containing helpful tips, hints and technique boxes for students struggling on a particular question.

These topic tests have been **fully cross-referenced** to the Pearson, Hodder and Collins textbooks for your convenience (see reference sheet on page 2). Each test has been designed to reflect the specification fully.

About the challenge tests

These **challenge** tests have been designed to **stretch and challenge** your students. 50% of the marks come from questions similar in style to our fundamentals tests. These questions isolate and test the core skills in each topic. The other 50% of the marks come from questions of increased difficulty that progress and start to combine the concepts in the topic.

Suggested use of the A and B tests


Each test in Set A has a corresponding test in Set B that features the same styles of questions but with different numbers. This allows for a variety of **flexible** uses including:

- **Test → Homework:** Students use test B as a homework to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Homework → Test:** Students revise as homework using test A before doing test B in class under test conditions.
- **Test → Classwork:** Students work through test B with teacher input to consolidate on areas of weakness identified from completing test A under test conditions in class.
- **Classwork → Test:** Students work through test A with teacher input, before checking their learning by completing test B under test conditions.

Timings

The recommended times for students to complete each test are given at the top of individual tests. Suggested times for our entire range of topic tests are also compiled in a table on the timings sheet for convenience (see page 3). For these challenge tests, the relevant times are the third and fourth times listed under each topic.

Calculator use

Although students are allowed to use a calculator in their examinations, the first topic (Algebraic Expressions) should be done without a calculator, as indicated by the non-calculator symbol () at the top of the tests. This encourages students to develop their non-calculator skills, saving time in their examinations on basic algebra and arithmetic.

Also available from ZigZag Education

The perfect starting point for students of all abilities are our **fundamentals** tests. These isolate and test the core skills in each topic so that your students can show what they can do. They get a confidence boost and you can see at a glance where each student's weaknesses lie.

To prepare students for the exam itself, our **expert** tests contain 25% repeated marks from the fundamentals and challenge tests, and 75% exam-style material with compound/multistep questions.

Free Updates!

Register your email address to receive any future free updates* made to this resource or other Maths resources your school has purchased, and details of any promotions for your subject.

* resulting from minor specification changes, suggestions from teachers and peer reviews, or occasional errors reported by customers

Go to zzed.uk/freeupdates

Cross-referencing Grid

Topic	AQA spec. points	Subtopics	Chapter Reference				
			Edexcel Pearson textbook [ISBN: 9781292183398]	Edexcel Hodder textbook [ISBN: 9781471853043]	Edexcel Collins textbook [ISBN: 9780008204952]	AQA Hodder textbook [ISBN: 9781471852862]	OCR A Hodder textbook [ISBN: 9781471853067]
Algebraic Expressions	B1, B2, B6	Index laws, expanding brackets, factorising, negative and fractional indices, surds, rationalising denominators	1	2, 3.1, 7.1	1.1, 1.4, 1.6–1.8	2, 3.1, 7.1	2, 3.1, 7.1
Quadratics	B3	Solving quadratic equations, completing the square, functions, quadratic graphs, discriminants, modelling	2	3, 8.1	2.1–2.4, 3.1, 3.7–3.8, 8.2	3, 8.1	3, 8.1
Simultaneous Equations and Inequalities	B3, B5	Linear simultaneous equations, quadratic simultaneous equations, simultaneous equations on graphs, linear inequalities, quadratic inequalities, inequalities on graphs, regions	3	4	2.5–2.8	4	4
Graphs and Transformations	B7, B9	Cubic graphs, quartic graphs, reciprocal graphs, points of intersection, translations, stretching, transformations	4	8	3	8	8
Straight Line Graphs	C1	Equations of straight lines, parallel and perpendicular lines, length and area, modelling	5	5.1–5.3	3.6, 4	5.1–5.3	5.1–5.3
Circles	C2	Midpoints and perpendicular bisectors, equation of a circle, intersections of straight lines and circles, use tangent and chord properties, circles and triangles	6	5.4–5.5	5	5.4–5.5	5.4–5.5
Algebraic Methods	A1, B6	Algebraic fractions, dividing polynomials, the factor theorem, mathematical proof, methods of proof	7	1, 7	1.5, 11	1, 7	1, 7
Binomial Expansion	D1	Pascal's triangle, factorial notation, binomial expansion, binomial problems, binomial estimation	8	9	1.2–1.3	9	9
Trigonometric Ratios	E1	The cosine rule, the sine rule, areas of triangles, solving triangle problems, graphs of sine, cosine and tangent, transforming trigonometric graphs	9	6.2–6.5, 8.4	6.1–6.5, 3.7–3.8	6.2–6.5, 8.4	6.2–6.5, 8.4
Trigonometric Identities and Equations	E3	Angles in all four quadrants, exact values of trigonometric ratios, trigonometric identities, simple trigonometric equations, harder trigonometric equations, equations and identities	10	6.1–6.2	6.1, 6.4–6.6	6.1–6.2	6.1–6.2
Vectors	J1, J2, J3, J4, J5	Vectors, representing vectors, magnitude and direction, position vectors, solving geometric problems, modelling	11	12	10	12	12
Differentiation	G1, G2, G3	Gradients of curves, finding derivatives, differentiating x^n , differentiating quadratics, gradients, tangents and normals, increasing and decreasing functions, second order derivatives, stationary points, sketching, modelling	12	10	8	10	10
Integration	H1, H2, H3	Integrating x^n , indefinite integrals, finding functions, definite integrals, areas under curves, areas under the x-axis, areas between curves and lines	13	11	9	11	11
Exponentials and Logarithms	F1, F2, F3, F4, F5, F6, F7	Exponential functions, $y = e^x$, exponential modelling, logarithms, laws of logarithms, solving equations using logarithms, working with natural logarithms, logarithms and non-linear data	14	13	7	13	13

Timings Sheet

For the **fundamentals** tests, refer to the tests marked X.1a and X.1b.

For the **challenge** tests, refer to the tests marked X.2a and X.2b.

For the **expert** tests, refer to the tests marked X.3a and X.3b.

Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)	Topic test reference	Recommended time (minutes)
Algebraic Expressions		Circles		Vectors	
1.1.a	14	6.1a	15	11.1a	9
1.1b	15	6.1b	11	11.1b	9
1.2a	21	6.2a	11	11.2a	17
1.2b	17	6.2b	11	11.2b	15
1.3a	14	6.3a	11	11.3a	11
1.3b	12	6.3b	13	11.3b	11
Quadratics		Algebraic Methods		Differentiation	
2.1a	19	7.1a	5	12.1a	13
2.1b	16	7.1b	5	12.1b	9
2.2a	23	7.2a	7	12.2a	17
2.2b	19	7.2b	5	12.2b	13
2.3a	26	7.3a	7	12.3a	17
2.3b	25	7.3b	7	12.3b	17
Simultaneous Equations and Inequalities		Binomial Expansion		Integration	
3.1a	10	8.1a	11	13.1a	15
3.1b	8	8.1b	9	13.1b	13
3.2a	13	8.2a	21	13.2a	15
3.2b	11	8.2b	17	13.2b	13
3.3a	23	8.3a	13	13.3a	13
3.3b	17	8.3b	13	13.3b	11
Graphs and Transformations		Trigonometric Ratios		Exponentials and Logarithms	
4.1a	17	9.1a	11	14.1a	19
4.1b	9	9.1b	11	14.1b	11
4.2a	17	9.2a	19	14.2a	15
4.2b	15	9.2b	13	14.2b	11
4.3a	19	9.3a	23	14.3a	15
4.3b	13	9.3b	21	14.3b	11
Straight Line Graphs		Trigonometric Identities and Equations			
5.1a	9	10.1a	17		
5.1b	7	10.1b	17		
5.2a	7	10.2a	15		
5.2b	7	10.2b	13		
5.3a	13	10.3a	19		
5.3b	11	10.3b	15		

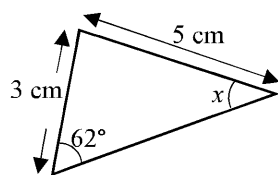
Trigonometric Ratios – Test B (13 mins)

Subtopics: The cosine rule, the sine rule, areas of triangles, solving triangle problems, transforming trigonometric graphs

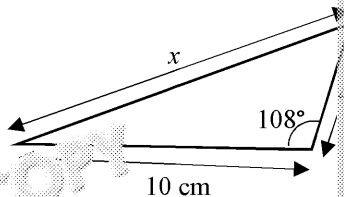
For this test you should give your answers to three significant figures

1. Find the value of x in the following triangles:

a)



b)



2. Given that the **area** of the triangle on the right is 6.4 cm^2 and the angle x is acute, find the value of x .



3. **Sketch** the graphs of the following equations, indicating **all** points of **maximum** and **minimum** points where appropriate:

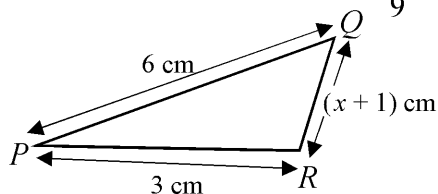
a) $y = \tan \theta$, $-90^\circ \leq \theta \leq 90^\circ$

b) $y = \cos\left(\frac{1}{2}\theta\right)$, $0^\circ \leq \theta \leq 360^\circ$

4. The triangle PQR has side lengths $PQ = 11 \text{ cm}$, $PR = 6 \text{ cm}$, and angle Q . Find the possible values of the angle PRQ .

5. The triangle ABC has side lengths $AB = 4 \text{ cm}$, $AC = 9 \text{ cm}$, and $BC = 10 \text{ cm}$. Find the **largest** angle in the triangle ABC .

6. In the triangle below, $\cos P = \frac{5}{9}$



a) Calculate the value of x .

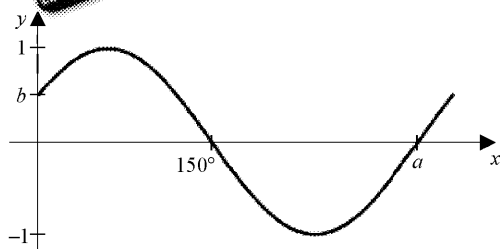
b) Find the **area** of the triangle.

7. **Sketch** the graphs of the following equations, indicating **all** points of **maximum** and **minimum** points where appropriate:

a) $y = -\tan \theta$, $0^\circ \leq \theta \leq 180^\circ$

b) $y = \sin(\theta + 180^\circ)$, $0^\circ \leq \theta \leq 360^\circ$

8. The diagram below shows part of the graph of $y = f(x)$. It crosses the x -axis at $(a, 0)$ and the y -axis at the point $(0, b)$. Given that $f(x) = \cos(x - \pi)$



- a) Find the value of a .
b) Find the minimum value of $f(x)$.
c) Find the value of b .

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Preview of Questions Ends Here

This is a limited inspection copy. Sample of questions ends here to avoid students previewing questions before they are set. See contents page for details of the rest of the resource.

Solutions to Quadratics – Test B

1. a) $2x^2 - 6x = 20$
 $\therefore 2x^2 - 6x - 20 = 0$ **M1**
 $\therefore (2x+4)(x-5) = 2(x+2)(x-5) = 0$ **M1**
 $\therefore x+2 = 0$ or $x-5 = 0$
 $\therefore x = -2$ or $x = 5$ **A1**

b) $(x-3)^2 = -16x$
 $\therefore x^2 - 6x + 9 = -16x$
 $\therefore x^2 + 10x + 9 = 0$ **M1**
 $\therefore (x+9)(x+1) = 0$ **M1**
 $\therefore x = -9$ or $x = -1$ **A1**

c) $x + 4 = \frac{5}{x}$
 $\therefore x(x+4) = 5$
 $\therefore x^2 + 4x - 5 = 0$ **M1**
 $\therefore (x+5)(x-1) = 0$ **M1**
 $\therefore x = -5$ or $x = 1$ **A1**

d) $x^3 - 12x^2 + 20x = 0$
 $\therefore x(x^2 - 12x + 20) = 0$ **M1**
 $\therefore x(x-10)(x-2) = 0$ **M1**
 $\therefore x = 0$ or $x = 10$ or $x = 2$ **A1**

[12 Marks]

2. a) $a = 3, b = -4, c = 1$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(1)}}{2(3)} = \frac{4 \pm \sqrt{16-12}}{6} = \frac{4 \pm \sqrt{4}}{6} = \frac{4 \pm 2}{6}$$
 M1
 $\therefore x = 1$ or $x = \frac{1}{3}$ **A1**

b) $7x^2 = 2x + 4 \therefore 7x^2 - 2x - 4 = 0$
 $a = 7, b = -2, c = -4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(-4)}}{2(7)} = \frac{2 \pm \sqrt{4+112}}{14} = \frac{2 \pm \sqrt{116}}{14} = \frac{2 \pm 2\sqrt{29}}{14} = \frac{1 \pm \sqrt{29}}{7}$$

 $\therefore x = \frac{1+\sqrt{29}}{7}$ or $x = \frac{1-\sqrt{29}}{7}$ **A1**

c) $a = 3, b = -2, c = -4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(3)} = \frac{2 \pm \sqrt{4+48}}{6} = \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm 2\sqrt{13}}{6} = \frac{1 \pm \sqrt{13}}{3}$$

 $\therefore x = \frac{1+\sqrt{13}}{3}$ or $x = \frac{1-\sqrt{13}}{3}$ **A1**

d) $7 - 4x = \frac{1}{4x^2}$
 $4x^2 - 7x + 1 = 0$ **M1**
 $a = 4, b = -7, c = 1$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(1)}}{2(4)} = \frac{7 \pm \sqrt{33}}{8}$$
 M1
 $\therefore x = \frac{7+\sqrt{33}}{8}$ or $x = \frac{7-\sqrt{33}}{8}$ **A1**

[9 Marks]

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3. a) $x^2 - 4x - 5 = \left(x - \frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 5 = 0$

$$\therefore (x-2)^2 - 4 - 5 = 0$$

$$\therefore (x-2)^2 - 9 = 0$$

$$\therefore (x-2)^2 = 9 \quad \text{M1}$$

$$\therefore x-2 = \pm\sqrt{9} = \pm 3$$

$$\therefore x = 2 \pm 3$$

$$\therefore x = 2+3 = 5 \text{ or } x = 2-3 = -1 \quad \text{A1}$$

b) $2x^2 - 5x + 1 = 0$

$$\therefore 2\left(x^2 - \frac{5}{2}x\right) = -1$$

$$\therefore x^2 - \frac{5}{2}x = -\frac{1}{2}$$

$$\therefore \left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 = -\frac{1}{2}$$

$$\therefore \left(x - \frac{5}{4}\right)^2 = \frac{25}{16} - \frac{1}{2} = \frac{17}{16} \quad \text{M1}$$

$$\therefore x - \frac{5}{4} = \pm\sqrt{\frac{17}{16}} = \pm\frac{\sqrt{17}}{4}$$

$$\therefore x = \frac{5}{4} + \frac{\sqrt{17}}{4} \text{ or } x = \frac{5}{4} - \frac{\sqrt{17}}{4} \quad \text{A1}$$

c) $(2x-4)^2 = 8x$

$$\therefore (2x-4)(2x-4) = 8x$$

$$\therefore 4x^2 - 8x - 8x + 16 = 8x$$

$$\therefore 4x^2 - 24x + 16 = 0 \quad \text{M1}$$

$$\therefore x^2 - 6x + 4 = 0$$

$$\therefore x^2 - 6x = -4$$

$$\therefore (x-3)^2 - 3^2 = -4$$

$$\therefore (x-3)^2 = 9-4 = 5 \quad \text{M1}$$

$$\therefore x-3 = \pm\sqrt{5}$$

$$\therefore x = 3 \pm \sqrt{5}$$

$$\therefore x = 3 + \sqrt{5} \text{ or } x = 3 - \sqrt{5} \quad \text{A1}$$

[7 Marks]

4. $x^2 - 6x + 9 = (x-3)^2 - 3^2 + 9$

$$= (x-3)^2 \text{ (where } p=1, q=-3 \text{ and } r=0) \quad \text{A1} \quad [1 \text{ Mark}]$$

5. $3x^2 - 2x + 7 = 3\left(x^2 - \frac{2}{3}x\right) + 7$

$$= 3\left(\left(x - \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right) + 7$$

$$= 3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3} + 7 \quad \text{M1}$$

$$= 3\left(x - \frac{1}{3}\right)^2 + \frac{20}{3} \text{ (where } p=3, q=-\frac{1}{3} \text{ and } r=\frac{20}{3}) \quad \text{A1} \quad [3 \text{ Marks}]$$

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6. a) $f(x) = -x - 2 \therefore f(11) = -11 - 2 = -13$ **A1**
 $g(x) = (x+7)(x+2) \therefore g(-1) = (-1+7)(-1+2) = (6)(1) = 6$ **A1**
- b) $3f(x) = g(x)$
 $3(-x-2) = (x+7)(x+2)$
 $\therefore -3x-6 = x^2+9x+14$
 $\therefore x^2+12x+20 = 0$ **M1**
 $\therefore (x+10)(x+2) = 0$
 $\therefore x = -10$ or $x = -2$ **A1**

[4 Marks]

7. a) $a = 1, b = -8, c = 21$
 $b^2 - 4ac = (-8)^2 - 4(1)(21) = 64 - 84 = -20$ **A1**
 $-20 < 0 \therefore$ no real roots **A1**
- b) $g(x) = (-2x+1)(x-5)$
 $= -2x^2 + 10x - x + 5 = -2x^2 + 9x + 5$ **M1**
 $b^2 - 4ac = 81 - 4(-2)(-5) = 81 - 40 = 41$ **A1**
 $41 > 0 \therefore$ two real roots **A1**
- c) $a = 5, b = -1, c = 7$
 $b^2 - 4ac = (-1)^2 - 4(5)(7) = 1 - 140 = -139$ **A1**
 $-139 < 0 \therefore$ no real roots **A1**
- d) $j(x) = (4x-1)(3x+2) = 12x^2 + 8x - 3x - 2$
 $= 12x^2 + 5x - 2$ **M1**
 $a = 12, b = 5, c = -2$
 $b^2 - 4ac = 25 - 4(12)(-2) = 25 + 96 = 121$ **A1**
 $121 > 0 \therefore$ two real roots **A1**

[10 Marks]

8. $\sqrt{4x+5} = x$
 $\therefore 4x+5 = x^2$
 $\therefore x^2 - 4x - 5 = 0$ **M1**
- Quadratic formula says $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Here, $a = 1, b = -4, c = -5$
- So $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-5)}}{2 \times 1} = \frac{4 \pm \sqrt{36}}{2} = \frac{4 \pm 6}{2}$ **M1**
- $\therefore x = 5$ or $x = -1$ (both solutions allowed as $x > -\frac{5}{4}$) **A1** [3 Marks]

9. $a = 1, b = 5, c = k$
 $b^2 - 4ac > 0$ for two distinct real roots **M1**
 So there are two distinct real roots if $(k) = 25 - 4k > 0$ **M1**
 $\therefore 25 > 4k$
 $\therefore \frac{25}{4} > k$ for k so accept $k \in \left(-\infty, \frac{25}{4}\right)$ **A1** [3 Marks]

10. $a = 3, b = -4, c = k$
 $b^2 - 4ac < 0$ for no real solutions **M1**
 So there are no real solutions if $(-4)^2 - 4(3)k = 16 - 12k < 0$ **M1**
 $\therefore 16 < 12k$
 $\therefore k > \frac{4}{3}$ (for OCR also accept $k \in \left(\frac{4}{3}, \infty\right)$) **A1** [3 Marks]

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