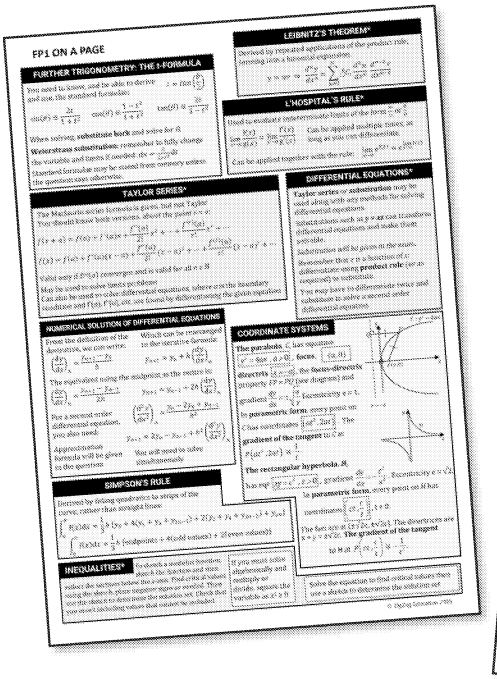
Further Mathematics AS / A Level | Edexcel | 8FM0/9FM0

2017 specification
first exams in 2019 (2018 for AS)



FP1 ON A PAGE

FURTHER TRIGONOMETRY: THE t-FORMULA

LEIBNITZ'S THEOREM*

L'HOSPITAL'S RULE*

TAYLOR SERIES*

DIFFERENTIAL EQUATIONS*

NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

COORDINATE SYSTEMS

SIMPSON'S RULE

INEQUALITIES*

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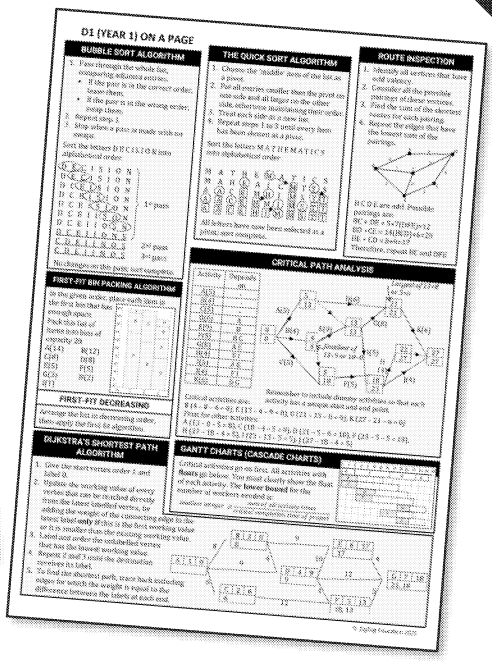

D1 (YEAR 1) ON A PAGE

BUBBLE SORT ALGORITHM

THE QUICK SORT ALGORITHM

ROUTE INSPECTION

FIRST-FIT BIN PACKING ALGORITHM

FIRST-FIT DECREASING

CRITICAL PATH ANALYSIS

DIJKSTRA'S SHORTEST PATH ALGORITHM

GANTT CHARTS (CASCADE CHARTS)

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Module on a Page

A4 and A3 Revision Posters for AS / A Level Edexcel Further Mathematics: FP1, FM1, FS1, D1

zigzageducation.co.uk

POD 12800

Contents

A4 place mats

Single-sided A3 masters

Teacher’s Introduction

These posters have been designed to provide a useful concise summary for the optional papers FP1, FM1, FS1 and D1 of the A Level Edexcel Further Mathematics specification.

The resource contains three double-sided and two single-sided A4 place mats. Our suggestion is that you laminate these and provide one per student.

Remember!

Always check the exam board website for new information, including changes to the specification and sample assessment material.

Each poster is also supplied as a single-sided A3 master. These work as a great classroom wall display and a handy recap.

Formulae given in the standard formula book are included in $\boxed{\text{boxes}}$ for quick reference.

Year 2 (A Level only) content has an asterisk (*) after the title (and is shaded blue in the PDF/Word versions).

December 2025

FP1 ON A PAGE

FURTHER TRIGONOMETRY: THE *t*-FORMULA

You need to know, and be able to derive and use, the standard formulae:

$$t = tan\left(\frac{\theta}{2}\right)$$

$$\sin(\theta) \equiv \frac{2t}{1+t^2} \qquad \cos(\theta) \equiv \frac{1-t^2}{1+t^2} \qquad \tan(\theta) \equiv \frac{2t}{1-t^2}$$

When solving, **substitute back** and solve for θ.

Weierstrass substitution: remember to fully change the variable and limits if needed: $\mathrm{d}x = \frac{2}{1+t^2}\mathrm{d}t$

Standard formulae may be stated from memory unless the question says otherwise.

TAYLOR SERIES

The Maclaurin series formula is given but not Taylor
You should know both versions about the point $x = a$:

$$f(x+a) = f(a) + f'(a)x + \frac{f''(a)}{2!}x^2 + \cdots + \frac{f^{(r)}(a)}{r!}x^r + \cdots$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(r)}(a)}{r!}(x-a)^r + \cdots$$

Valid only if $f^{(n)}(a)$ converges and is valid for all $n \in \mathbb{N}$

May be used to solve limits problems
Can also be used to solve differential equations, where a is the boundary condition and f′(a), f′′(a), etc. are found by differentiating the given equation

NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

From the definition of the derivative, we can write:

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n \approx \frac{y_{n+1} - y_n}{h}$$

Which can be rearranged to the iterative formula:

$$y_{n+1} \approx y_n + h\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n$$

The equivalent using the midpoint as the centre is:

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n \approx \frac{y_{n+1} - y_{n-1}}{2h} \qquad y_{n+1} \approx y_{n-1} + 2h\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n$$

For a second order differential equation, you also need:

$$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_n \approx \frac{y_n - 2y_n + y_{n-1}}{h^2}$$

$$y_{n+1} \approx 2y_n - y_{n-1} + h^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_n$$

Approximation formula will be given in the question

You will need to solve simultaneously

SIMPSON'S RULE

Derived by fitting quadratic strips of the curve, rather than straight lines:

$$\int_a^b \mathrm{f}(x)\mathrm{d}x \approx \frac{1}{3}h\{y_0 + 4(y_1 + y_3 + y_{2n-1}) + 2(y_2 + y_4 + y_{2n-2}) + y_{2n}\}$$

$$\int_a^b \mathrm{f}(x)\mathrm{d}x \approx \frac{1}{3}h\{\text{endpoints} + 4(\text{odd values}) + 2(\text{even values})\}$$

INEQUALITIES*

To sketch a modulus function, sketch the function and then reflect the sections below the *x*-axis. Find critical values using the sketch, place negative signs as needed. Then use the sketch to determine the solution set. Check that you aren't including values that cannot be included.

If you must solve algebraically and multiply or divide, square the variable as $x^2 \geq 0$

LEI

Derived by repeate
forming into a bino

$y = uv \Rightarrow$

L'HO

Used to evaluate indete

$$\lim_{x\to a}\frac{\mathrm{f}(x)}{\mathrm{g}(x)} = \lim_{x\to a}\frac{\mathrm{f}'(x)}{\mathrm{g}'(x)}$$

Can be applied togethe

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COORDINATE SYSTEM

The parabola, *C*, has equat
$\boxed{y^2 = 4ax\ ,\ a>0}$, **focus**,

directrix $\boxed{x=-a}$, the **foc**
property *FP* = *PQ* (see diag

gradient $\frac{dy}{dx} = \pm\sqrt{\frac{a}{x}}$ Eccent

In **parametric form**, every
C has coordinates $\boxed{\left(at^2, 2a\right.}$

gradient of the tangent t
$P\left(at^2, 2at\right)$ is $\frac{1}{t}$.

The rectangular hyperbo
has eq^n^ $\boxed{xy = c^2\ ,\ c>0}$, g

In **param**

coordina

The foci a
x + y = ±√2

to

Solve the
use a ske

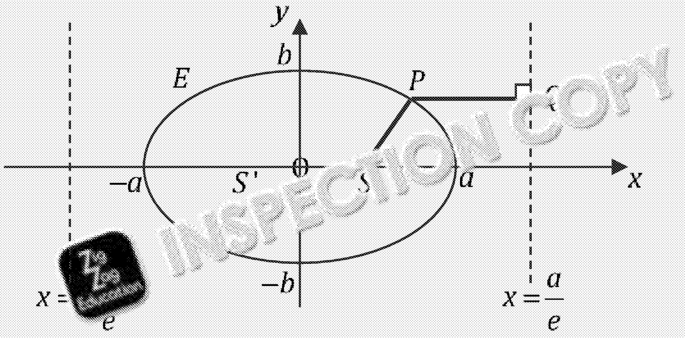
FP1 ON A PAGE

COORDINATE SYSTEMS*

The **ellipse** E has Cartesian equation $\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$.

If $a>b$, E has **eccentricity** e, $0<e<1$, given by $\boxed{b^2=a^2\left(1-e^2\right)}$,

foci $\boxed{(\pm ae,0)}$, **directrices** $\boxed{x=\pm\frac{a}{e}}$ and the **focus-directrix property** $PS=ePQ$ (see diagram)



In parametric form, every point on E has coordinates $\boxed{(a\cos t,b\sin t)}$

The gradient of the tangent to E at $P(a\cos t,b\sin t)$ is $-\frac{b}{a}\cot t$.

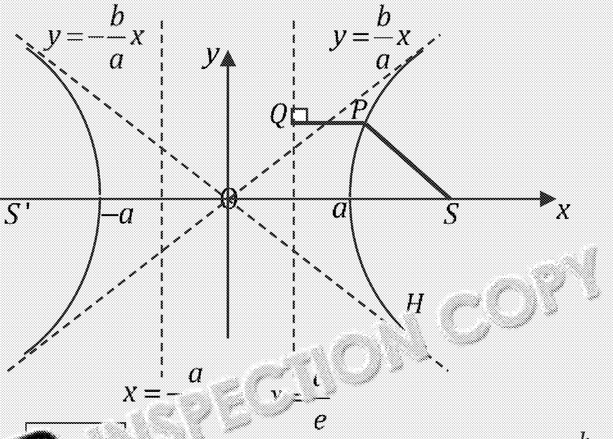
A line with equation $y=mx+c$ is a tangent to E precisely when $m^2a^2+b^2=c^2$.

The **hyperbola** H has Cartesian equation $\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$,

eccentricity e, $e>1$, given by $\boxed{b^2=a^2\left(e^2-1\right)}$, **foci** $\boxed{(\pm ae,0)}$,

directrices $\boxed{x=\pm\frac{a}{e}}$, the **focus-directrix property** $PS=ePQ$

(see diagram)



and asymptotes $\boxed{\frac{x}{a}=\pm\frac{y}{b}}$, which can also be written as $y=\pm\frac{b}{a}x$

In parametric form, every point on H has coordinates $\boxed{(a\sec t,b\tan t)}$ or equivalently $\boxed{(\pm a\cosh t,b\sinh t)}$.

The gradient of the tangent to H at $P(a\sec t,b\tan t)$ is $\frac{b}{a}\operatorname{cosec} t$.

A line with equation $y=mx+c$ is a tangent to H precisely when $m^2a^2-b^2=c^2$.

Locus questions are likely to require solving tangents or normals simultaneously in terms of x, y and t.

VEC

Gives a vector

θ *is the angle*
$\hat{\mathbf{n}}$ ***is a unit ve***

$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$

Vector produ

SC

Scalar triple p

$\mathbf{a}.(\mathbf{b}\times\mathbf{c})=$

$\mathbf{a}.(\mathbf{a}\times\mathbf{p})=\mathbf{a}.$

A

which is the a
area of the tri

Scalar triple p
gives volume
tetrahedron

S

Recall that th

a is a point o
b is the direc
Because $\mathbf{r}-\mathbf{a}$

Because $\mathbf{a}\times\mathbf{b}$
used to find t
shortest dista

DIRECTION

Consider the
$\mathbf{a}=x\mathbf{i}+y\mathbf{j}+z$
are called α, β

Direction ra
$x:y:z$ (This
Direction co
and are given

$l=\cos(\alpha)$

$n=\cos(\alpha)$

l^2+m^2+

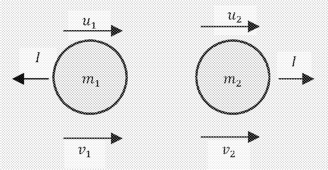
FM1 ON A PAGE

MOMENTUM AND IMPULSE

Momentum = Mass × Velocity, or $p = mv$
In a closed system, total momentum is conserved. Total momentum before collision is equal to total momentum after collision.

Before:

After:



$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Impulse, I (Ns) = Force × Time
Impulse, I (kgm/s) = Final Momentum − Initial Momentum, or change in Momentum

$$I = m(v - u)$$

WORK, ENERGY AN[illegible]OWE[illegible]

Work done = resultant force in [illegible] m[illegible]on × distance moved in direction of force: [illegible]

On a rough horiz[illegible]ce, work done by friction equals distance × force
Friction = μR wher[illegible]s the normal reaction and μ is the coefficient of friction

$W = \mu Rd$ (change in kinetic energy)

Kinetic energy (KE), $E_k = \frac{1}{2}mv^2$

(Gravitational) potential energy (GPE), $E_p = mgh$

WORK-ENERGY PRINCIPLE

Energy change in a system = work done by external force.

Initial energy (J) − Final energy (J) = Work done (Nm)

Without external forces energy is conserved.

Initial energy (J) = Final energy (J)

For gravitational potential energy the lowest height within the system is often stated as the zero level.

$$\left(\frac{1}{2}mu^2 + mgh_i\right) - \left(\frac{1}{2}mv^2 + mgh_f\right) = W$$

POWER

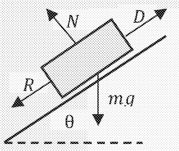
Power (W) = Rate of work (J/s)

Power = force × velocity

So, the driving force of an engine is found by $D = \frac{P}{v}$

The maximum speed of a vehicle is when there is no acceleration, and the forces are balanced.

So D is equal to the resistance plus any components of weight acting on the slope.



At max speed, $\frac{P}{v} = R + mg\sin\theta$

Or if accelerating, $\frac{P}{v} - mg\sin\theta - R = ma$

ELASTIC COLLISIONS IN ON[illegible]

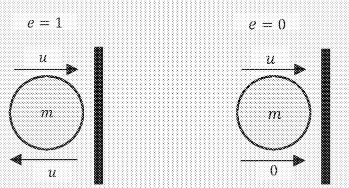
NEWTON'[illegible] RE[illegible]TITUTION

Coefficient of res[illegible] $= \frac{\text{[illegible]eed of separation}}{\text{speed of approach}}, 0 < e < 1$

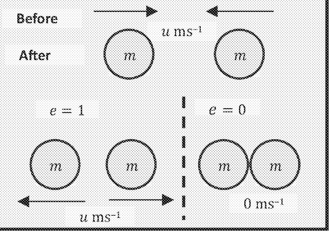
If objects are travelling in **opposite** directions, then speeds are **added**. If they are travelling in the **same** direction, then the speeds are **subtracted**.

If $e = 1$, all energy is conserved in the collision.
If $e = 0$, speed of separation is 0.

Collision with a surface



Collision of two spheres



ELASTIC STRINGS A

Hooke's law states:

Tension = $\frac{\text{modulus of elas}}{\text{natu}}$

Sometimes written as T = kx,

ELAS

Elastic potential energy (EPE)
compressed spring is equal to

Elastic potential

from l

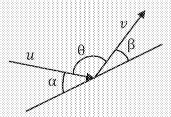
If a spring is suspended verti
weight force.

For no external forces, KE + G

ELASTIC COL

SMOOTH SPHERE

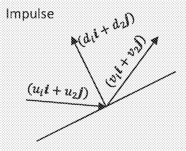
Using angles and magnitudes:



Given
With

With

Using vectors:



The im
perpe

With the law of restitution:

$$-e(u_1\mathbf{i} + u_2\mathbf{j}) \cdot (d_1$$

With the conservation of mo

$$(u_1\mathbf{i} + u_2\mathbf{j}) \cdot (d_2\mathbf{i}$$

i.e. total momentum before im

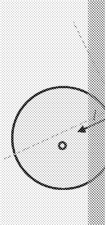
Remember: Angle of deflec

Angle

O

Oblique collisions occur whe
s[illegible]ne straight line.

T[illegible] components of the veloc
centres are unchanged in the



- Newton's law of restitut
of the spheres parallel to
- The principle of conserv

Loss of k
$= \left(\frac{1}{2}mu_1^2 +\right.$

FS1 ON A PAGE

DISCRETE RANDOM VARIABLES

$P(X=x)$ is probability that *X* takes value *x*.
A probability distribution table lists all outcomes and their probabilities.

$$\text{Expectation of } X = E(X) = \mu = \sum x\,P(X=x)$$
$$\text{Variance of } X = \text{Var}(X) = \sigma^2 = E(X^2) - \mu^2$$

$$E(g(X)) = \sum g(x)P(X=x)$$
$$E(X^2) = \sum x^2\,P(X=x)$$

$$E(aX+b) = aE(X)+b$$
$$\text{Var}(aX+b) = a^2\,\text{Var}(X)$$
$$E(X+Y) = E(X)+E(Y)$$

GEOMETRIC DISTRIBUTION

$X \sim G(p)$ where X = number of independent trials until 1st success (special case of NB distribution)

$P(X=x) = p(1-p)^{x-1}$ $\quad x = 1,2,3,\ldots$

$P(X \le x) = 1-(1-p)^x$

$\mu = \dfrac{1}{p}$ $\quad \sigma^2 = \dfrac{1-p}{p^2}$ $\quad$ mode $= 1$

NEGATIVE BINOMIAL DISTRIBUTION*

$X \sim NB(r,p)$ where X = number of independent trials until r^{th} success (general case of geometric distribution)

$$P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r} \quad x = r, r+1, r+2, \ldots$$

$$\mu = \frac{r}{p} \qquad \sigma^2 = \frac{r(1-p)}{p^2}$$

POISSON DISTRIBUTION

If $X \sim Po(\lambda)$ then $\mu = \sigma^2 = \lambda$

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0,1,2,3,\ldots$$

Events that occur randomly, singly, independently and at a constant average rate in space or time can be modelled by a Poisson distribution.

For large *n* and small *p*, binomial distribution can be approximated by Poisson with $\lambda = np$, i.e. $B(n,p) \approx Po(np)$

If *X* and *Y* are independent random variables where $X \sim Po(\lambda)$ and $Y \sim Po(\mu)$ then $X+Y \sim Po(\lambda+\mu)$

CENTRAL LIMIT THEOREM*

If $X_1, X_2, X_3, \ldots, X_n$ is a random sample from a population with mean μ and variance σ^2 then

$\bar{X} \approx N\left(\mu, \dfrac{\sigma^2}{n}\right)$ provided *n* is large.

NOTE: Continuity correction not needed.

CHI-SQUARED TESTS

X^2 = test statistic $\quad v$ = no. of degrees of freedom

$$X^2 = \sum \frac{(O-E)^2}{E} = \sum \frac{O^2}{E} - N$$

$X^2 \approx \chi^2_v$ provided that all $E \ge 5$

Hypothesis test is always a one-tailed upper-tail test. If X^2 exceeds critical value then reject H$_0$ in favour of H$_1$.

TESTING A MODEL

A goodness-of-fit test uses the chi-squared distribution to check whether a particular probability distribution is a suitable model for an observed distribution.

H$_0$: the observed distribution doesn't differ from the theoretical distribution
H$_1$: the observed distribution is different from the theoretical distribution

Combine cells for expected frequencies < 5

v = number of cells − number of restrictions − 1

If the estimate of the parameter is *calculated* using the observed data, it is a restriction.

Discrete uniform
v = number of cells − 1

Binomial, Poisson
v = number of cells − 1 (parameter not calculated)
v = number of cells − 2 (parameter calculated)

GEOMETRIC DISTRIBUTION*

v = number of cells − 1 (parameter not calculated)
v = number of cells − 2 (parameter calculated)

TESTING THE INDEPENDENCE OF 2 VARIABLES

An $r \times c$ contingency table is used to test whether there is an association between two categorical variables.

H$_0$: the two variables are *independent* (or *not associated*)
H$_1$: the two variables are *dependent* (or *associated*)

$$\text{Expected frequency} = \frac{\text{row total} \times \text{column total}}{\text{overall total}}$$

$v = (r-1)(c-1)$ where
r = no. of rows, c = no. of columns

Combine rows/columns for expected frequencies < 5

QUALITY OF TE

ERRORS

Type I – reject H$_0$ when H$_0$ is true
Type II – don't reject H$_0$ when H$_0$ is false

Actual significance level = P(test statistic lies in critical region)
Actual significance level = P(reject H$_0$ when H$_0$ is true)
Actual significance level = P(Type I error)

	H$_0$ true	H$_0$ false
Don't reject H$_0$	✓	Type II error
Reject H$_0$	Type I error	✓

The smaller the rejection region, the less likely it is that a Type I error will occur, but the more likely it is that a Type II error could be made.

A hi

Size
Size

Pow
Pow
Pow

If p
dec
wro

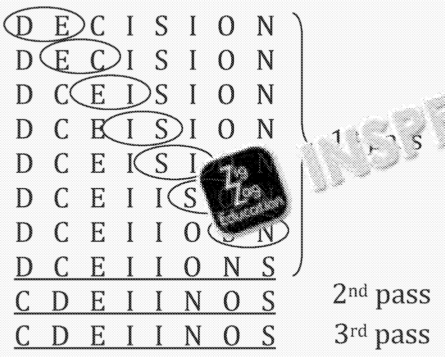
A p
pos
sam

D1 (YEAR 1) ON A PAGE

BUBBLE SORT ALGORITHM

1. Pass through the whole list, comparing adjacent entries.
   - If the pair is in the correct order, leave them.
   - If the pair is in the wrong order, swap them.
2. Repeat step 1.
3. Stop when a pass is made with no swaps.

Sort the letters D E C I S I O N into alphabetical order.



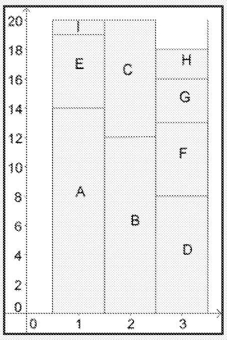
No changes on this pass; sort complete.

FIRST-FIT BIN PACKING ALGORITHM

In the given order, place each item in the first bin that has enough space.

Pack this list of items into bins of capacity 20:

A(14)	B(12)
C(8)	D(8)
E(5)	F(5)
G(3)	H(2)
I(1)	

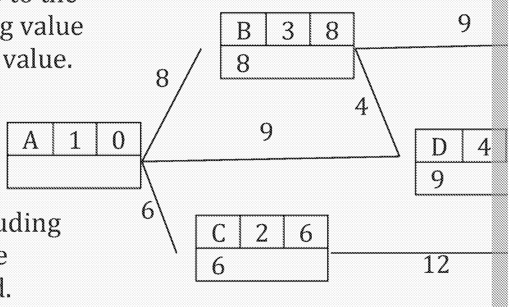


FIRST-FIT DECREASING

Arrange the list in decreasing order, then apply the first-fit algorithm.

DIJKSTRA'S SHORTEST PATH ALGORITHM

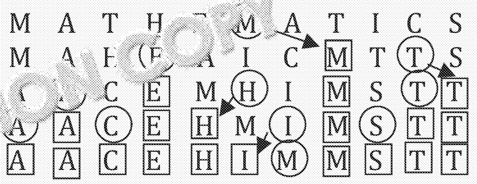
1. Give the start vertex order 1 and label 0.
2. Update the working values of every vertex that can be reached directly from the latest labelled vertex, by adding the weight of the connecting edge to the latest label **only if** this is the first working value or it is smaller than the existing working value.
3. Label and order the unlabelled vertex that has the lowest working value.
4. Repeat 2 and 3 until the destination receives its label.
5. To find the shortest path, trace back including edges for which the weight is equal to the difference between the labels at each end.



THE QUICK SORT ALGORITHM

1. Choose the 'middle' item of the list as a pivot.
2. Put all entries smaller than the pivot on one side and all larger on the other side, otherwise maintaining their order.
3. Treat each side as a new list.
4. Repeat steps 1 to 3 until every item has been chosen as a pivot.

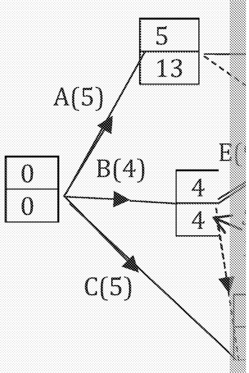
Sort the letters M A T H E M A T I C S into alphabetical order.



All letters have now been selected as a pivot; sort complete.

CRITICAL PATH

Activity	Depends on
A(5)	-
B(4)	-
C(5)	-
D(6)	A
E(9)	B
F(5)	B C
G(8)	A E
H(4)	F I
I(5)	A E
J(4)	F I
K(6)	D G



Remember to incl
activity has a uniq

Critical activities are:
B (4 – 0 – 4 = 0), E (13 – 4 – 9 = 0), G (21 –

Float for other activities:
A (13 – 0 – 5 = 8), C (18 – 4 – 5 = 9), D (21 –
H (27 – 18 – 4 = 5), I (23 – 13 – 5 = 5), J (27 –

GANTT CHARTS (CASCADE CHARTS

Critical activities go on first. All activities w
floats go below. You must clearly show th
of each activity. The **lower bound** for the
number of workers needed is:

$$\text{smallest integer} \geq \frac{\text{sum of all activity tim}}{\text{critical completion time of }}$$

D1 (YEAR 2) ON A PAGE

PLANARITY ALGORITHM*

A **planar graph** is a graph where no edges cross.

The **planarity algorithm** can be used on graphs containing a **Hamiltonian cycle**.

1. Identify the Hamiltonian cycle and draw the nodes as a regular polygon. Draw any remaining edges inside the polygon.
2. List the edges that are drawn inside the polygon (in any order).
3. Choose any unlabelled edge and label it (I). If all edges have been labelled, the graph is now planar.
4. Consider unlabelled edges that cross the edge you have just labelled I.
   - If there are none, return to step 3.
   - If any cross each other the graph is non-planar.
   - If they do not cross each other, label them (O).
5. Return to step 3 until all edges are labelled I or O.

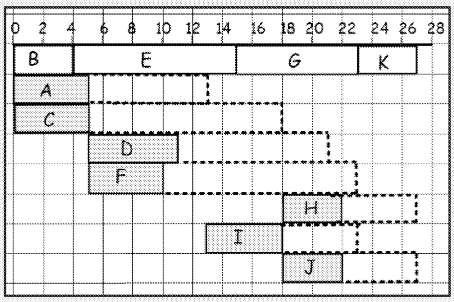
For a planar graph, the algorithm separates the edges into those that can be drawn inside the polygon without crossing (labelled I) and those that can be drawn outside the polygon without crossing (labelled O).

ROUTE INSPECTION*

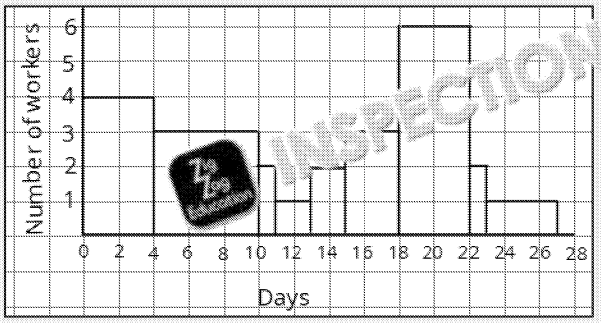
Networks with more than four odd nodes.
You will be given **additional information** such that you end up considering possible pairings of four nodes at most.

RESOURCE HISTOGRAM AND SCHEDULING*

A resource histogram shows the numbers of workers that are operating at any given time. You can use the Gannt chart and any additional information given.



Activity	Number of workers
A	1
B	2
C	1
D	1
E	1
F	1
G	2
H	1
I	1
J	3
k	1



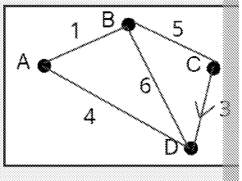
Allocating workers to activities is called **scheduling**.
A scheduling diagram shows which workers do which activities. The **Lower Bound** for the number of workers needed to complete a project within the critical time is

$$\text{smallest integer} \geq \frac{\text{sum of all activity times}}{\text{critical completion time of project}}$$

FLOY

For minimum distance

1. *Complete a distance*
   Where no direct rou
   the table. Complete



2. *First iteration – sha*
   Compare the corres
   he distance table:

Distance table

	A	B	C	D
A	-	1	∞	4
B	1	-	5	6
C	∞	5	-	3
D	4	6	∞	-

3. *Second iteration – d*

Distance table

	A	B	C	D
A	-	1	[6]	4
B	1	-	5	5
C	[6]	5	-	3
D	4	5	[10]	-

4. *Third iteration for t*

Distance table

	A	B	C	D
A	-	1	6	4
B	1	-	5	5
C	6	5	-	3
D	4	5	10	-

5. *Fourth iteration fo*

Distance table

	A	B	C	D
A	-	1	6	4
B	1	-	5	5
C	6	5	-	3
D	4	5	10	-

6. *Algorithm is compl*
   shortest route bet
   route table to find

Distance table

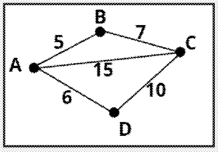
	A	B	C	D
A	-	1	6	4
B	1	-	5	5
C	6	5	-	3
D	4	5	10	-

From the distance table
distance from D to C is
would be DABC

D1 (YEAR 2) ON A PAGE

TRAVELLING SALESMAN PROBLEM*

A **tour** is a walk that starts and finishes at the same vertex. The travelling salesman problem weight. The **classical** travelling salesman problem finds a tour that visits every vertex exact salesman problem finds a tour that visits each vertex at least once. By using a **table of least**



	A	B	C	D
A	-	5	12	6
B	5	-	7	11
C	12	7	-	10
D	6	11	10	-

- The table of least distances shows the shor vertex. It is not always the direct route.
- We only consider undirected graphs, so the
- Fill in the table by inspection – unless Q ask

It is difficult to find the optimal solution to the travelling salesman problem, so we find a lo

The optimal solution lies in the interval: lower bound < optimal solution ≤ upper bound want to find the **lowest upper bound** and the **highest lower bound**.

Finding upper bounds using a minimum spanning tree (MST)

1. Find the MST using Prim or Kruskal.
2. Using the MST and retracing every edge provides an initial upper bound. Upper bound = 2 × weight of MST
3. Use inspection and shortcuts to improve upon the initial upper bound.

Finding lower bounds using a minimum

1. Remove each vertex in turn by deleting from the table of least distances. With t the residual minimum spanning tree (R
2. Find the two shortest distinct edges to c was deleted in step 1.
3. Possible lower bound (LB) = weight of t connecting the vertex deleted in step 1.
4. Return to step 1 for each vertex. Compare

The Nearest Neighbour algorithm for finding upper bounds

1. Choose each vertex in turn as the starting point. Visit the nearest vertex which has no vertices have been visited. Take note of the distance of the tour.
2. Go back to step 1 until all vertices have been selected as a starting point. Select tour w

SIMPLEX METHOD*

Formulate the linear programming problem. Convert the ≤ inequalities into equalities by adding **slack variables**. You can solve problems involving, at most, four decision variables and four constraints using a **simplex tableau**.

Maximise
Subject to
$x + 2y \leq 5$
$2x + y \leq 8$
$x, y \geq 0$

Choose the most negative value in the bottom row and find corresponding θ values. Select the smallest θ value to determine the pivot row.
Perform row operations to change the basic variable

BV	x	y	r	s	value	θ
r	1	2	1	0	5	5÷2 =2.5
s	2	1	0	1	8	8÷1 = 8
P	-2	-3	0	0	0	

BV	x	y	r	s	value	Row ops
y	0.5	1	0.5	0	2.5	R1÷2
s	2	1	0	1	8	
P	-2	-3	0	0	0	

BV	x	y	r	s	value
y	0.5	1	0.5	0	2.5
s	1.5	0	-0.5	1	5.5
P	-0.5	0	1.5	0	7.5

Repeat these steps until there are no negative values left on the bottom row.

BV	x	y	r	s	value	θ
y	0.5	1	0.5	0	2.5	2.5÷0.5=5
s	1.5	0	-0.5	1	5.5	5.5÷ 1.5=11/3
P	-0.5	0	1.5	0	7.5	

BV	x	y	r	s	value	Row ops
y	0.5	1	0.5	0	2.5	
x	1	0	-1/3	2/3	11/3	R2÷1.5
P	-0.5	0	1.5	0	7.5	

BV	x	y	r	s	value
y	0	1	2/3	-1/3	2/3
x	1	0	-1/3	2/3	11/3
P	0	0	4/3	1/3	28/3

Solving problems involving ≥ constraints

For problems which include ≥ constraints, we need to subtract **surplus variables** in order to convert the inequalities into equations. However, for every **surplus variable** used, we also need to introduce an **artificial variable**. This will ensure that the simplex method can still be used to solve the problem.

Maximise P
$x+2y \geq 5$
r is a sur
$2x+y \geq 8$
s is a sur

There are then two methods to solve this kind of problem: the two-stage simplex method

The two-stage simplex method	
1. Write all constraints as equations using slack, surplus and	
2. Define a new object function – to minimise the sum of the artificial variables. Minimise $a_1 + a_2$ which is equivalent to maximising $-(a_1 + a_2)$. 3. Substitute in for the artificial variables and solve using the simplex method. This is the **first stage**. 4. If a solution exists to the first stage, then this becomes the starting point to the **second stage**. Use the simplex method to solve the second stage. 5. If no solution exists to the first stage, then the original problem does not have a solution.	2. Let M r original $M(\sum_{i=1}^{n}$ includi can ma artifici 3. Substitu solve th method.

D1 (YEAR 1) ON A PAGE

ORDER OF AN ALGORITHM

The *size* of a problem indicates its complexity.

The *efficiency* of an algorithm is a measure of the 'run time' (often proportional to the number of operations required).

The *order* of an algorithm is a measure of its efficiency expressed as a function of the problem's size.

For example, to carry out the bubble sort on n items the number of operations required:

$$(n-1)+(n-2)+(n-3)\ldots\ldots+2+1=\frac{n}{2}(n-1)$$

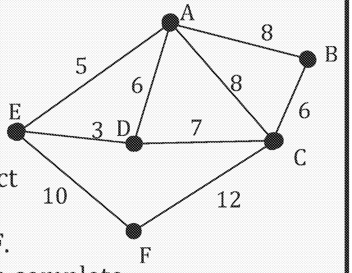
Therefore, the bubble sort has quadratic order.

MINIMUM SPANNING TREE ALGORITHMS

KRUSKAL

1. Sort the edges into ascending order.
2. Start the tree with the smallest edge.
3. Add the next edge if it does not form a cycle; reject it if it does.
4. Repeat 3 until all vertices are connected.

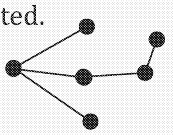
List the edges in order: DE(3), AE(5), AD(6), BC(6), CD(7), AB(8), AC(8), EF(10), CF(12). Start with DE, add AE, reject AD, add BC, add CD, reject AB, reject AC, add EF. All vertices connected; tree complete.



PRIM

1. Choose any vertex to start the tree.
2. Select the smallest edge that will join any vertex in the tree to any vertex that is not in the tree, and add it to the tree.
3. Repeat 2 until all vertices are connected.

(starting at E) ED(3), EA(5), DC(7), CB(6), EF(10)



Prim's tree grows in a connected way, but Kruskal's does not.

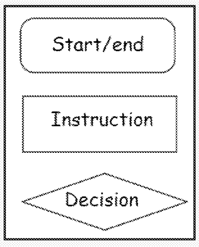
PRIM ON A MATRIX

1. Choose a vertex to start the tree.
2. Delete the row of the chosen vertex.
3. Number the column of the chosen vertex.
4. Put a ring around the lowest undeleted entry of all the numbered columns. This entry becomes the next edge added to the tree.
5. Repeat 2, 3 and 4 until all columns are numbered.

EULERIAN GRAPH – a connected graph where all vertices are even.

SEMI EULERIAN – a connected graph with two odd nodes

FLOW CHARTS



EULER'S HA

In any undirected graph: vertices of a graph = 2 × t

It follows that the numbe

GRAPH TH

- A *graph* G is a collectio are connected by lines
- A *subgraph* of G is a gra edges which belong to
- A graph has a numbe *weighted graph* or *netw*
- The *degree* or *valency* o connected to it. The de depending on the numb
- A *walk* is a sequence of edge being the beginnin
- A *path* is a walk in whic
- A *trail* is a walk where
- A *cycle* is a closed *path* the same as the start ve vertex appears more th
- A *Hamiltonian cycle* is a the graph.
- A *loop* is an arc that sta
- A *simple graph* has no l any two vertices.
- Two vertices are *conne* A graph is connected if
- A *digraph* is a graph in have a direction associ
- A *tree* is a connected gr
- A *spanning tree* of a gra a tree, and includes all
- A *minimum spanning tr* lowest possible total w
- A *complete graph* is a gr connected by one edge by K_n, where *n* is the nu
- *Isomorphic graphs* show drawn differently.

LINE

Maximise P: $3x+2y$ moving line right
Vertex method – find the coordinates of the vertices and check the value of the objective function at each point.